STAT 224 Lecture 7 Interactions of Three or More Predictors

Yibi Huang Department of Statistics University of Chicago

Various Interactions of X, E, and M

With 3 or more predictors, there are various interaction terms we can include in the model.

- \bullet X + E + M
- \bullet X + E + M + E:X
- \bullet X + E + M + M:X
- \bullet X + E + M + E:M
- \bullet X + E + M + E:X + M:X
- \bullet X + E + M + E:X + M:X + E:M
- \bullet X + E + M + E:X + M:X + E:M + E:M:X

Model X + E + M + E:X

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \gamma_2 (E_2 \cdot X) + \gamma_3 (E_3 \cdot X) + \varepsilon$$

	E(S)					
Education (E)	Other $(M=0)$			Manager $(M = 1)$		
1 (HS, $E_2 = E_3 = 0$)	β_0	+ (β)X	$\beta_0 + \alpha$	+ (β) <i>X</i>
2 (BA/BS, $E_2 = 1, E_3 = 0$)	$\beta_0 + \delta$	$\delta_2 + (\beta + 1)$	$\gamma_2)X$	$\beta_0 + \alpha +$	$\delta_2 + (\beta +$	$\gamma_2)X$
3 (Adv, $E_2 = 0, E_3 = 1$)	$\beta_0 + \delta$	$\hat{b}_3 + (\beta +$	$\gamma_3)X$	$\beta_0 + \alpha +$	$\delta_3 + (\beta +$	$\gamma_3)X$

- Effect (Slope) of X depends on E only but not M
- No E:M or M:X interaction: Effect of M doesn't depend on E or X
 - If equally experienced and educated, managers are paid α more than others on average, and the gap α doesn't change with X or E.
- Effect of E depends on X but not on M
 - College graduates are paid $\delta_2 + \gamma_2 X$ more than HS graduates w/ the same experience (X years) and management status

Model X + E + M + E:M

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \theta_2 (E_2 \cdot M) + \theta_3 (E_3 \cdot M) + \varepsilon$$

$$E(S)$$
Education (E) Other (M = 0) Manager (M = 1)
$$1 \text{ (HS, } E_2 = E_3 = 0) \qquad \beta_0 + \beta X \quad \beta_0 + \alpha \qquad + \beta X$$

$$2 \text{ (BA/BS, } E_2 = 1, E_3 = 0) \qquad \beta_0 + \delta_2 + \beta X \qquad \beta_0 + \alpha + \delta_2 + \theta_2 + \beta X$$

$$3 \text{ (Adv, } E_2 = 0, E_3 = 1) \qquad \beta_0 + \delta_3 + \beta X \qquad \beta_0 + \alpha + \delta_3 + \theta_3 + \beta X$$

- No E:X or M:X interactions: same slope of X for all levels of E & M
- Effect of M depends on E but not on X
 - If equally experienced and educated, how much are managers paid more than others?

- Effect of E depends on M but not on X
 - How much do college graduates earn more than HS graduates
 w/ the same experience and management status?

Model X + E + M + E:M

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \theta_2 (E_2 \cdot M) + \theta_3 (E_3 \cdot M) + \varepsilon$$

$$E(S)$$
Education (E) Other (M = 0) Manager (M = 1)
$$1 \text{ (HS, } E_2 = E_3 = 0) \qquad \beta_0 + \beta X \quad \beta_0 + \alpha \qquad + \beta X$$

$$2 \text{ (BA/BS, } E_2 = 1, E_3 = 0) \qquad \beta_0 + \delta_2 + \beta X \qquad \beta_0 + \alpha + \delta_2 + \theta_2 + \beta X$$

$$3 \text{ (Adv, } E_2 = 0, E_3 = 1) \qquad \beta_0 + \delta_3 + \beta X \qquad \beta_0 + \alpha + \delta_3 + \theta_3 + \beta X$$

- No E:X or M:X interactions: same slope of X for all levels of E & M
- Effect of M depends on E but not on X
 - If equally experienced and educated, how much are managers paid more than others?

It depends on education level but not experience, α if HS, α + θ_2 if BA/BS, α + θ_3 if adv.

- Effect of E depends on M but not on X
 - How much do college graduates earn more than HS graduates
 w/ the same experience and management status?

Model X + E + M + E:M

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \theta_2 (E_2 \cdot M) + \theta_3 (E_3 \cdot M) + \varepsilon$$

$$E(S)$$
Education (E) Other (M = 0) Manager (M = 1)
$$1 \text{ (HS, } E_2 = E_3 = 0) \qquad \beta_0 + \beta X \quad \beta_0 + \alpha \qquad + \beta X$$

$$2 \text{ (BA/BS, } E_2 = 1, E_3 = 0) \qquad \beta_0 + \delta_2 + \beta X \qquad \beta_0 + \alpha + \delta_2 + \theta_2 + \beta X$$

$$3 \text{ (Adv, } E_2 = 0, E_3 = 1) \qquad \beta_0 + \delta_3 + \beta X \qquad \beta_0 + \alpha + \delta_3 + \theta_3 + \beta X$$

- No E:X or M:X interactions: same slope of X for all levels of E & M
- Effect of M depends on E but not on X
 - If equally experienced and educated, how much are managers paid more than others?

It depends on education level but not experience, α if HS, α + θ_2 if BA/BS, α + θ_3 if adv.

- Effect of E depends on M but not on X
 - How much do college graduates earn more than HS graduates w/ the same experience and management status?

 $\delta_2 + \theta_2$ for managers and δ_2 for others

Model X + E + M + E:M + E:X

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \theta_2 (E_2 \cdot M) + \theta_3 (E_3 \cdot M) + \gamma_2 (E_2 \cdot X) + \gamma_3 (E_3 \cdot X) + \varepsilon$$

	E(S)					
Education (E)	Other $(M=0)$		Mana	Manager $(M = 1)$		
1 (HS, $E_2 = E_3 = 0$)	β_0 + (3)X	$\beta_0 + \alpha$	+ (β)X	
2 (BA/BS, $E_2 = 1, E_3 = 0$)	$\beta_0 + \delta_2 + \zeta$	$(3+\gamma_2)X$	$\beta_0 + \alpha + \delta_2$	$+\theta_2+(\beta+\gamma)$	$_2)X$	
3 (Adv, $E_2 = 0, E_3 = 1$)	$\beta_0 + \delta_3 + \zeta$	$(3+\gamma_3)X$	$\beta_0 + \alpha + \delta_3$	$+\theta_3+(\beta+\gamma)$	3)X	

- includes E:X but not M:X: slope of X changes w/ E but not w/ M
- Effect of M depends on E but not on X
 - If equally experienced and educated, how much are managers paid more than others?

- Effect of E depends on both M and X
 - How much are college graduates paid more than HS graduates w/ the same experience and management status?

Model X + E + M + E:M + E:X

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \theta_2 (E_2 \cdot M) + \theta_3 (E_3 \cdot M) + \gamma_2 (E_2 \cdot X) + \gamma_3 (E_3 \cdot X) + \varepsilon$$

	E(S)					
Education (E)	Other $(M=0)$		Mana	Manager $(M = 1)$		
1 (HS, $E_2 = E_3 = 0$)	$\beta_0 + (\beta$) <i>X</i>	$\beta_0 + \alpha$	+ (β) <i>X</i>	
2 (BA/BS, $E_2 = 1, E_3 = 0$)	$\beta_0 + \delta_2 + (\beta$	$+ \gamma_2)X$	$\beta_0 + \alpha + \delta_2$	$+\theta_2+(\beta+\gamma)$	₂)X	
3 (Adv, $E_2 = 0, E_3 = 1$)	$\beta_0 + \delta_3 + (\beta$	$+ \gamma_3)X$	$\beta_0 + \alpha + \delta_3$	$+\theta_3+(\beta+\gamma)$	_{'3})X	

- includes E:X but not M:X: slope of X changes w/ E but not w/ M
- Effect of M depends on E but not on X
 - If equally experienced and educated, how much are managers paid more than others?

It depends on education level but not experience, α if HS, $\alpha + \theta_2$ if BA/BS, $\alpha + \theta_3$ if adv.

- Effect of E depends on both M and X
 - How much are college graduates paid more than HS graduates w/ the same experience and management status?

Model X + E + M + E:M + E:X

$$S = \beta_0 + \beta X + \delta_2 E_2 + \delta_3 E_3 + \alpha M + \theta_2 (E_2 \cdot M) + \theta_3 (E_3 \cdot M) + \gamma_2 (E_2 \cdot X) + \gamma_3 (E_3 \cdot X) + \varepsilon$$

	E(S)					
Education (E)	Other $(M=0)$		Mana	Manager $(M = 1)$		
1 (HS, $E_2 = E_3 = 0$)	β_0 + (3)X	$\beta_0 + \alpha$	+ (β)X	
2 (BA/BS, $E_2 = 1, E_3 = 0$)	$\beta_0 + \delta_2 + \zeta$	$(3+\gamma_2)X$	$\beta_0 + \alpha + \delta_2$	$+\theta_2+(\beta+\gamma)$	$_2)X$	
3 (Adv, $E_2 = 0, E_3 = 1$)	$\beta_0 + \delta_3 + \zeta$	$(3+\gamma_3)X$	$\beta_0 + \alpha + \delta_3$	$+\theta_3+(\beta+\gamma)$	3)X	

- includes E:X but not M:X: slope of X changes w/ E but not w/ M
- Effect of M depends on E but not on X
 - If equally experienced and educated, how much are managers paid more than others?

It depends on education level but not experience, α if HS, $\alpha + \theta_2$ if BA/BS, $\alpha + \theta_3$ if adv.

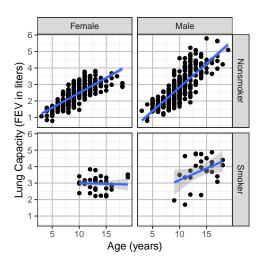
- Effect of E depends on both M and X
 - How much are college graduates paid more than HS graduates w/ the same experience and management status?
 δ₂ + θ₂ + γ₂X for managers and δ₂ + γ₂X for others

Lung Capacity Data Revisit

Sample of 654 youths, aged 3 to 19, in the area of East Boston during middle to late 1970's. The variables are

- age: Subject's age in years
- fev: Lung capacity of subject, measured by forced expiratory volume (abbreviated as FEV), the amount of air an individual can exhale in the first second of forceful breath in liters
- ht: Subject's height in inches
- **sex**: Gender of the subject coded as: 0 = Female, 1 = Male
- **smoke**:Smoking status coded as: 0 = Nonsmoker, 1 = Smoker

```
fevdata = read.table("fevdata.txt", header = TRUE)
fevdata$sex = factor(fevdata$sex, labels=c("Female","Male"))
fevdata$smoke = factor(fevdata$smoke, labels=c("Nonsmoker","Smoker"))
```



Does the slope change with smoking status? Does the slope change with gender?

```
lmfull2 = lm(fev \sim age + sex + smoke + age*sex +
              age*smoke + sex*smoke, data=fevdata)
# or simply
lmfull2 = lm(fev \sim age*sex + age*smoke + sex*smoke, data=fevdata)
summary(lmfull2)$coef
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)
                   0.69028
                             0.10775 6.40614 2.873e-10
                   0.18033 0.01104 16.33231 1.832e-50
age
                -0.76220 0.14633 -5.20868 2.559e-07
sexMale
smokeSmoker 2.14912
                             0.37905 5.66977 2.156e-08
age:sexMale 0.10936
                             0.01474 7.41873 3.729e-13
age:smokeSmoker -0.17079
                             0.02838 -6.01744 2.967e-09
sexMale:smokeSmoker 0.01048
                             0.14886 0.07037 9.439e-01
```

```
summary(lmfull2)$coef
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                     0.69028
                                0.10775 6.40614 2.873e-10
                     0.18033
                                0.01104 16.33231 1.832e-50
age
sexMale
                   -0.76220
                                0.14633 -5.20868 2.559e-07
smokeSmoker
                     2.14912
                                0.37905 5.66977 2.156e-08
age:sexMale
                    0.10936
                                0.01474 7.41873 3.729e-13
age:smokeSmoker -0.17079
                                0.02838 - 6.01744 2.967e - 09
sexMale:smokeSmoker 0.01048
                                0.14886 0.07037 9.439e-01
```

- for female nonsmokers?
- for female smokers?
- for male nonsmokers?
- for male smokers?

```
summary(lmfull2)$coef
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                     0.69028
                                0.10775 6.40614 2.873e-10
                     0.18033
                                0.01104 16.33231 1.832e-50
age
sexMale
                   -0.76220
                                0.14633 -5.20868 2.559e-07
smokeSmoker
                     2.14912
                                0.37905 5.66977 2.156e-08
age:sexMale
                    0.10936
                                0.01474 7.41873 3.729e-13
age:smokeSmoker -0.17079
                                0.02838 - 6.01744 2.967e - 09
sexMale:smokeSmoker 0.01048
                                0.14886 0.07037 9.439e-01
```

- for female nonsmokers? 0.18
- for female smokers?
- for male nonsmokers?
- for male smokers?

```
summary(lmfull2)$coef
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    0.69028
                               0.10775 6.40614 2.873e-10
                    0.18033
                               0.01104 16.33231 1.832e-50
age
sexMale
                   -0.76220
                               0.14633 -5.20868 2.559e-07
smokeSmoker
                    2.14912
                               0.37905 5.66977 2.156e-08
age:sexMale
                    0.10936
                               0.01474 7.41873 3.729e-13
age:smokeSmoker -0.17079
                               0.02838 - 6.01744 2.967e - 09
sexMale:smokeSmoker 0.01048
                               0.14886 0.07037 9.439e-01
```

- for female nonsmokers? 0.18
- for female smokers? 0.18 + (-0.171)
- for male nonsmokers?
- for male smokers?

```
summary(lmfull2)$coef
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    0.69028
                               0.10775 6.40614 2.873e-10
                    0.18033
                               0.01104 16.33231 1.832e-50
age
sexMale
                   -0.76220
                               0.14633 -5.20868 2.559e-07
smokeSmoker
                    2.14912
                               0.37905 5.66977 2.156e-08
                    0.10936
age:sexMale
                               0.01474 7.41873 3.729e-13
age:smokeSmoker -0.17079
                               0.02838 - 6.01744 2.967e - 09
sexMale:smokeSmoker 0.01048
                               0.14886 0.07037 9.439e-01
```

- for female nonsmokers? 0.18
- for female smokers? 0.18 + (-0.171)
- for male nonsmokers? 0.18 + 0.109
- for male smokers?

```
summary(lmfull2)$coef
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    0.69028
                               0.10775 6.40614 2.873e-10
                    0.18033
                               0.01104 16.33231 1.832e-50
age
sexMale
                   -0.76220
                               0.14633 -5.20868 2.559e-07
smokeSmoker
                    2.14912
                               0.37905 5.66977 2.156e-08
                    0.10936
age:sexMale
                               0.01474 7.41873 3.729e-13
age:smokeSmoker -0.17079
                               0.02838 - 6.01744 2.967e - 09
sexMale:smokeSmoker 0.01048
                               0.14886 0.07037 9.439e-01
```

- for female nonsmokers? 0.18
- for female smokers? 0.18 + (-0.171)
- for male nonsmokers? 0.18 + 0.109
- for male smokers? 0.18 + 0.109 + (-0.171)

Model w/ 3-Way Interactions

```
lmfull = lm(fev \sim age + sex + smoke + age*sex + age*smoke +
             sex*smoke + sex*smoke*age, data=fevdata)
# or simply
lmfull = lm(fev ~ age*sex*smoke, data=fevdata)
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
                                  0.10978 6.1384 1.455e-09
(Intercept)
                        0.67387
                        0.18209 0.01127 16.1618 1.355e-49
age
sexMale
                      -0.73143 0.15149 -4.8281 1.722e-06
smokeSmoker
                       2.41025 0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                  0.01531 6.9336 9.973e-12
age:smokeSmoker -0.19100
                                  0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker -0.58904
                                  0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker 0.04497
                                  0.05707 0.7879 4.310e-01
```

```
summary(lmfull)$coef
                        Estimate Std. Error t value Pr(>|t|)
                         0.67387
                                              6.1384 1.455e-09
(Intercept)
                                     0.10978
                         0.18209
                                     0.01127 16.1618 1.355e-49
age
sexMale
                        -0.73143
                                     0.15149 -4.8281 1.722e-06
smokeSmoker
                         2.41025
                                     0.50359
                                              4.7861 2.109e-06
age:sexMale
                         0.10613
                                     0.01531
                                              6.9336 9.973e-12
age:smokeSmoker
                        -0.19100
                                     0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker
                        -0.58904
                                     0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                         0.04497
                                     0.05707
                                              0.7879 4.310e-01
```

- for female nonsmokers?
- for female smokers?
- for male nonsmokers?
- for male smokers?

```
summary(lmfull)$coef
                        Estimate Std. Error t value Pr(>|t|)
                         0.67387
                                              6.1384 1.455e-09
(Intercept)
                                     0.10978
                         0.18209
                                     0.01127 16.1618 1.355e-49
age
sexMale
                        -0.73143
                                     0.15149 -4.8281 1.722e-06
smokeSmoker
                         2.41025
                                     0.50359 4.7861 2.109e-06
age:sexMale
                         0.10613
                                     0.01531 6.9336 9.973e-12
age:smokeSmoker
                       -0.19100
                                     0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker
                        -0.58904
                                     0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                         0.04497
                                     0.05707
                                              0.7879 4.310e-01
```

- for female nonsmokers? 0.182
- for female smokers?
- for male nonsmokers?
- for male smokers?

```
summary(lmfull)$coef
                        Estimate Std. Error t value Pr(>|t|)
                                             6.1384 1.455e-09
(Intercept)
                         0.67387
                                    0.10978
                         0.18209
                                    0.01127 16.1618 1.355e-49
age
sexMale
                        -0.73143
                                    0.15149 -4.8281 1.722e-06
smokeSmoker
                         2.41025
                                    0.50359 4.7861 2.109e-06
age:sexMale
                         0.10613
                                    0.01531 6.9336 9.973e-12
age:smokeSmoker
                     -0.19100
                                    0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker
                   -0.58904
                                    0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                        0.04497
                                    0.05707
                                             0.7879 4.310e-01
```

- for female nonsmokers? 0.182
- for female smokers? 0.182 + (-0.191)
- for male nonsmokers?
- for male smokers?

```
summary(lmfull)$coef
                        Estimate Std. Error t value Pr(>|t|)
                         0.67387
                                             6.1384 1.455e-09
(Intercept)
                                    0.10978
                         0.18209
                                    0.01127 16.1618 1.355e-49
age
sexMale
                        -0.73143
                                    0.15149 -4.8281 1.722e-06
smokeSmoker
                         2.41025
                                    0.50359 4.7861 2.109e-06
age:sexMale
                         0.10613
                                    0.01531 6.9336 9.973e-12
age:smokeSmoker
                      -0.19100
                                    0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker
                    -0.58904
                                    0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                         0.04497
                                    0.05707
                                             0.7879 4.310e-01
```

- for female nonsmokers? 0.182
- for female smokers? 0.182 + (-0.191)
- for male nonsmokers? 0.182 + 0.106
- for male smokers?

```
summary(lmfull)$coef
                        Estimate Std. Error t value Pr(>|t|)
                                             6.1384 1.455e-09
(Intercept)
                         0.67387
                                    0.10978
                         0.18209
                                    0.01127 16.1618 1.355e-49
age
sexMale
                        -0.73143
                                    0.15149 -4.8281 1.722e-06
smokeSmoker
                         2.41025
                                    0.50359 4.7861 2.109e-06
age:sexMale
                         0.10613
                                    0.01531 6.9336 9.973e-12
age:smokeSmoker
                     -0.19100
                                    0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker
                    -0.58904
                                    0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                         0.04497
                                    0.05707
                                             0.7879 4.310e-01
```

- for female nonsmokers? 0.182
- for female smokers? 0.182 + (-0.191)
- for male nonsmokers? 0.182 + 0.106
- for male smokers? 0.182 + 0.106 + (-0.191) + 0.045

```
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
                                           6.1384 1.455e-09
(Intercept)
                        0.67387
                                  0.10978
                        0.18209
                                  0.01127 16.1618 1.355e-49
age
sexMale
                       -0.73143
                                  0.15149 -4.8281 1.722e-06
smokeSmoker
                        2.41025
                                  0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                  0.01531 6.9336 9.973e-12
age:smokeSmoker
               -0.19100
                                  0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker -0.58904
                                  0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                        0.04497
                                  0.05707
                                           0.7879 4.310e-01
```

- female nonsmokers v.s. female smokers
- male nonsmokers v.s. female nonsmokers

```
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
                                           6.1384 1.455e-09
(Intercept)
                        0.67387
                                   0.10978
                        0.18209
                                   0.01127 16.1618 1.355e-49
age
sexMale
                       -0.73143
                                  0.15149 -4.8281 1.722e-06
smokeSmoker
                        2.41025
                                  0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                   0.01531 6.9336 9.973e-12
age:smokeSmoker
               -0.19100
                                   0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker -0.58904
                                   0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                        0.04497
                                   0.05707
                                           0.7879 4.310e-01
```

- female nonsmokers v.s. female smokers
 Look at age:smokeSmoker. Yes, P-value = 7.71 × 10⁻⁷
- male nonsmokers v.s. female nonsmokers

```
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
                                           6.1384 1.455e-09
(Intercept)
                        0.67387
                                   0.10978
                        0.18209
                                   0.01127 16.1618 1.355e-49
age
sexMale
                       -0.73143
                                   0.15149 -4.8281 1.722e-06
smokeSmoker
                        2.41025
                                   0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                   0.01531 6.9336 9.973e-12
age:smokeSmoker
                -0.19100
                                   0.03826 - 4.9916 7.709e - 07
sexMale:smokeSmoker
                   -0.58904
                                   0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                        0.04497
                                   0.05707
                                           0.7879 4.310e-01
```

- female nonsmokers v.s. female smokers
 Look at age:smokeSmoker. Yes, P-value = 7.71 × 10⁻⁷
- male nonsmokers v.s. female nonsmokers
 Look at age:sexMale. Yes, P-value = 9.97 × 10⁻¹²

```
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)
                        0.67387
                                   0.10978
                                           6.1384 1.455e-09
                                   0.01127 16.1618 1.355e-49
                        0.18209
age
sexMale
                       -0.73143
                                  0.15149 -4.8281 1.722e-06
smokeSmoker
                        2.41025
                                   0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                   0.01531 6.9336 9.973e-12
age:smokeSmoker
                -0.19100
                                   0.03826 -4.9916 7.709e-07
sexMale:smokeSmoker -0.58904
                                   0.77534 - 0.7597 4.477e - 01
age:sexMale:smokeSmoker
                                           0.7879 4.310e-01
                        0.04497
                                   0.05707
```

male nonsmokers v.s. male smokers

male smokers v.s. female smokers?

```
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)
                        0.67387
                                   0.10978
                                            6.1384 1.455e-09
                                   0.01127 16.1618 1.355e-49
                        0.18209
age
sexMale
                       -0.73143
                                   0.15149 -4.8281 1.722e-06
smokeSmoker
                        2.41025
                                   0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                   0.01531 6.9336 9.973e-12
age:smokeSmoker
                       -0.19100
                                   0.03826 -4.9916 7.709e-07
sexMale:smokeSmoker
                                   0.77534 - 0.7597 4.477e - 01
                   -0.58904
age:sexMale:smokeSmoker
                                            0.7879 4.310e-01
                        0.04497
                                   0.05707
```

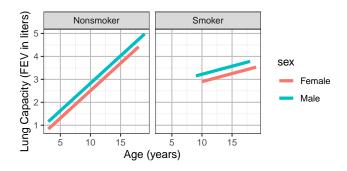
male nonsmokers v.s. male smokers
 The sum of coefficients of age:smokeSmoker and age:sexMale:smokeSmoker

male smokers v.s. female smokers?

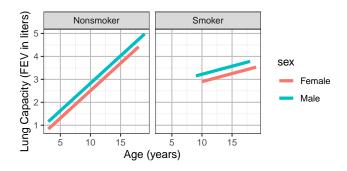
```
summary(lmfull)$coef
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)
                        0.67387
                                   0.10978
                                            6.1384 1.455e-09
                                   0.01127 16.1618 1.355e-49
                        0.18209
age
sexMale
                       -0.73143
                                   0.15149 -4.8281 1.722e-06
smokeSmoker
                        2.41025
                                   0.50359 4.7861 2.109e-06
age:sexMale
                        0.10613
                                   0.01531 6.9336 9.973e-12
age:smokeSmoker
                       -0.19100
                                   0.03826 -4.9916 7.709e-07
sexMale:smokeSmoker
                                   0.77534 - 0.7597 4.477e - 01
                   -0.58904
                                            0.7879 4.310e-01
age:sexMale:smokeSmoker
                        0.04497
                                   0.05707
```

male nonsmokers v.s. male smokers
 The sum of coefficients of age:smokeSmoker and age:sexMale:smokeSmoker

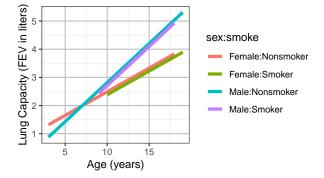
male smokers v.s. female smokers?
 The sum of coefficients of age:sexMale and age:sexMale:smokeSmoker



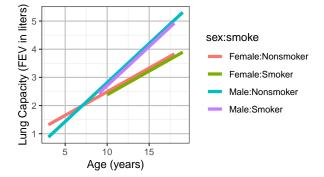
- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke



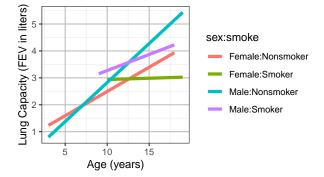
- fev ~ age + smoke + sex + age*smoke ← Correct Ans
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke



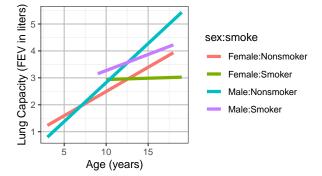
- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke



- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex ← Correct Ans
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke

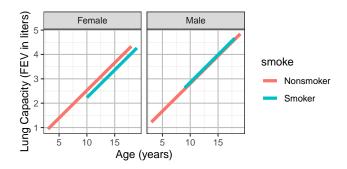


- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke

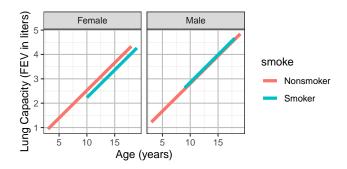


- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke ←

Correct Ans



- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex
- fev ~ age + smoke + sex + age*sex + age*smoke



- fev ~ age + smoke + sex + age*smoke
- fev ~ age + smoke + sex + age*sex
- fev ~ age + smoke + sex + smoke*sex ← Correct Ans
- fev ~ age + smoke + sex + age*smoke