# STAT 224 F-Distributions and F-tests

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# F-Test for Comparing Reduced & Full Model

For testing

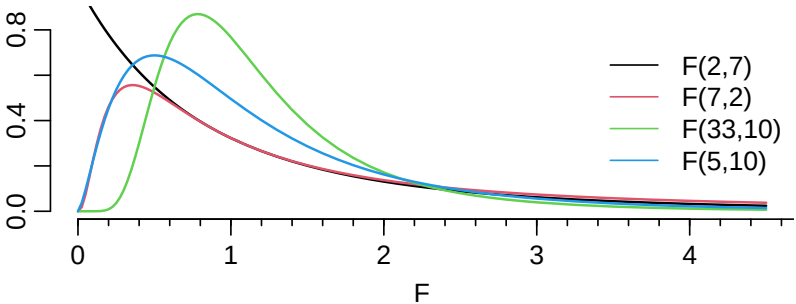
$$\boxed{\mathrm{H}_0\text{: reduced model is true}} \text{ v.s. } \boxed{\mathrm{H}_1 \text{ : full model is true}}$$

The test statistic is

$$\begin{aligned} F &= \frac{(\mathrm{SSE}_{\text{reduced}} - \mathrm{SSE}_{\text{full}})/(\mathrm{dfE}_{\text{reduced}} - \mathrm{dfE}_{\text{full}})}{\mathrm{MSE}_{\text{full}}} \\ &= \frac{(\mathrm{SSE}_{\text{reduced}} - \mathrm{SSE}_{\text{full}})/(\mathrm{dfE}_{\text{reduced}} - \mathrm{dfE}_{\text{full}})}{\mathrm{SSE}_{\text{full}}/\mathrm{dfE}_{\text{full}}} \end{aligned}$$

- Under $\mathrm{H}_0$, $\mathrm{SSE}_{\text{reduced}} \approx \mathrm{SSE}_{\text{full}}$, and thus $F$-stat would be small.
- A large $F$-statistic is evidence against $\mathrm{H}_0$ and hence should reject $\mathrm{H}_0$.
- $F \geq 0$ since $\mathrm{SSE}_{\text{reduced}} \geq \mathrm{SSE}_{\text{full}}$
- Under $\mathrm{H}_0$, the $F$-statistic has an *$F$-distribution* with $\mathrm{dfE}_{\text{reduced}}-\mathrm{dfE}_{\text{full}}$ and $\mathrm{dfE}_{\text{full}}$ degrees of freedom.

# The $F$ Distributions




- An $F$-distribution has two parameters df1 and df2.
- There is one $F$-density for each pair of df1 and df2.
- The order of df1 and df2 matters.
  e.g., $F(2,7)$ and $F(7,2)$ are different $F$-distributions.

# Example (Salary Survey Data)

To test whether the salary growth rate per year of experience $(X)$ depends on education $(E)$, we should test `E*X` interactions.

```
p130 = read.table("P130.txt", header=TRUE)
p130$E = as.factor(p130$E)
lm1 = lm(S ~ E + X + E*X, data=p130)
lm2 = lm(S ~ E + X, data=p130)
anova(lm2, lm1)
Analysis of Variance Table

Model 1: S ~ E + X
Model 2: S ~ E + X + E * X
  Res.Df       RSS Df Sum of Sq    F Pr(>F)
1     42 550853135
2     40 497897342  2  52955792 2.13   0.13
```

```
anova(lm2, lm1)
Analysis of Variance Table

Model 1: S ~ E + X
Model 2: S ~ E + X + E * X
  Res.Df       RSS Df Sum of Sq    F Pr(>F)
1     42 550853135
2     40 497897342  2  52955792 2.13   0.13
```

The $F$-statistic 2.13 is computed from the SSE's (RSS) as follows

$$F = \frac{(\text{SSE}_{\text{reduced}} - \text{SSE}_{\text{full}})/(\text{dfE}_{\text{reduced}} - \text{dfE}_{\text{full}})}{\text{SSE}_{\text{full}}/\text{dfE}_{\text{full}}}$$
$$= \frac{(550853134.6991 - 497897342.452)/(42 - 40)}{497897342.452/40} = 2.1272$$

What are the degrees of freedom of the F statistic?

a. 42 and 40
b. 40 and 42
c. 2 and 40
d. 2 and 42

```
anova(lm2, lm1)
Analysis of Variance Table

Model 1: S ~ E + X
Model 2: S ~ E + X + E * X
  Res.Df       RSS Df Sum of Sq    F Pr(>F)
1     42 550853135
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```

The $F$-statistic 2.13 is computed from the SSE's (RSS) as follows

$$F = \frac{(\text{SSE}_{\text{reduced}} - \text{SSE}_{\text{full}})/(\text{dfE}_{\text{reduced}} - \text{dfE}_{\text{full}})}{\text{SSE}_{\text{full}}/\text{dfE}_{\text{full}}}$$
$$= \frac{(550853134.6991 - 497897342.452)/(42 - 40)}{497897342.452/40} = 2.1272$$

What are the degrees of freedom of the F statistic?

a. 42 and 40
b. 40 and 42
c. 2 and 40 ⟵ Ans
d. 2 and 42

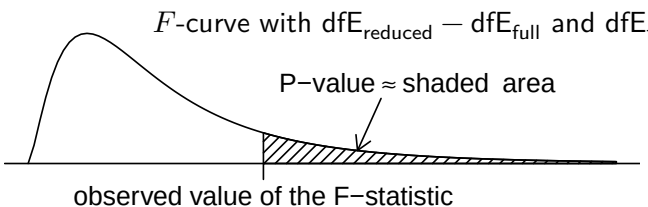
# $P$-value of an $F$-Test

The $F$-statistic

$$F = \frac{(\mathsf{SSE}_{\mathsf{reduced}} - \mathsf{SSE}_{\mathsf{full}})/(\mathsf{dfE}_{\mathsf{reduced}} - \mathsf{dfE}_{\mathsf{full}})}{\mathsf{SSE}_{\mathsf{full}}/\mathsf{dfE}_{\mathsf{full}}}$$

which has an $F$ distribution with $\mathsf{dfE}_{\mathsf{reduced}} - \mathsf{dfE}_{\mathsf{full}}$ and $\mathsf{dfE}_{\mathsf{full}}$ degrees of freedom.

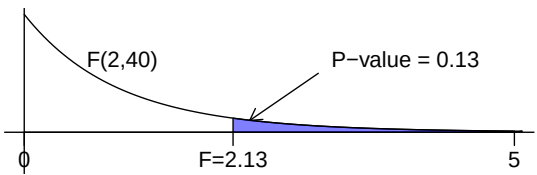
Under $\mathsf{H}_0$: Reduced Model is true, the $P$-value is the area of the upper-tail under the $F$-curve with $\mathsf{dfE}_{\mathsf{reduced}} - \mathsf{dfE}_{\mathsf{full}}$ and $\mathsf{dfE}_{\mathsf{full}}$ degrees of freedom beyond the $F$ statistic.



# Finding the $P$-value in R

To test `E*X` interaction, the $P$-value for the $F$-statistic 2.13 is

$$P\text{-value} = P(F_{2,40} \geq 2.13) \approx 0.13.$$




```
pf(2.13, df1=2, df2=40, lower.tail = FALSE)
[1] 0.1321
```

Conclusion: Little evidence of `E*X` interaction.

# Finding $P$-values using the $F$-table

Table entries are $F_{.05,\nu_1,\nu_2}$ where $P_{\nu_1,\nu_2}(F > F_{.05,\nu_1,\nu_2}) = .05$ .

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 25 | 30 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 | 244 | 246 | 248 | 249 | 250 | 251 |
| 2 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 |
| 3 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.63 | 8.62 | 8.59 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.52 | 4.50 | 4.46 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.83 | 3.81 | 3.77 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.40 | 3.38 | 3.34 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.11 | 3.08 | 3.04 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.89 | 2.86 | 2.83 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.73 | 2.70 | 2.66 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.60 | 2.57 | 2.53 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.50 | 2.47 | 2.43 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.28 | 2.25 | 2.20 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 |

- The $F$-table below gives the **critical value** at 0.05 significance level.
- For df1 $= 5$, df2 $= 18$, if $F$-stat $> F_{0.05,df1=5,df2=18} = 2.77$, then $P$-value $< 0.05$ and $H_0$ is rejected at 0.05 level

# Finding $P$-values using the $F$-table

Table entries are $F_{.05,\nu_1,\nu_2}$ where $P_{\nu_1,\nu_2}(F > F_{.05,\nu_1,\nu_2}) = .05$ significance level

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 25 | 30 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 | 244 | 246 | 248 | 249 | 250 | 251 |
| 2 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 |
| 3 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.63 | 8.62 | 8.59 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.52 | 4.50 | 4.46 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.83 | 3.81 | 3.77 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.40 | 3.38 | 3.34 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.11 | 3.08 | 3.04 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.89 | 2.86 | 2.83 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.73 | 2.70 | 2.66 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.60 | 2.57 | 2.53 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.50 | 2.47 | 2.43 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.28 | 2.25 | 2.20 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 |

- The $F$-table below gives the **critical value** at 0.05 significance level.
- For df1 $= 5$, df2 $= 18$, if $F$-stat $> F_{0.05,df1=5,df2=18} = 2.77$, then $P$-value $< 0.05$ and $H_0$ is rejected at 0.05 level

# Finding $P$-values using the $F$-table

Table entries are $F_{.01,\nu_1,\nu_2}$ where $P_{\nu_1,\nu_2}(F > F_{.01,\nu_1,\nu_2}) = .01$ .

| | $\nu_1$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 25 | 30 | 40 |
| 2 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 |
| 3 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.2 | 27.1 | 26.9 | 26.7 | 26.6 | 26.5 | 26.4 |
| 4 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.5 | 14.4 | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 |
| 5 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 | 9.89 | 9.72 | 9.55 | 9.45 | 9.38 | 9.29 |
| 6 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.56 | 7.40 | 7.30 | 7.23 | 7.14 |
| 7 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.31 | 6.16 | 6.06 | 5.99 | 5.91 |
| 8 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.52 | 5.36 | 5.26 | 5.20 | 5.12 |
| 9 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 4.96 | 4.81 | 4.71 | 4.65 | 4.57 |
| 10 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.56 | 4.41 | 4.31 | 4.25 | 4.17 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.25 | 4.10 | 4.01 | 3.94 | 3.86 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.01 | 3.86 | 3.76 | 3.70 | 3.62 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 |

- The $F$-table above gives the **critical value** at 0.01 significance level

# Finding $P$-values using the $F$-table

Table entries are $F_{.01,\nu_1,\nu_2}$ where $P_{\nu_1,\nu_2}(F > F_{.01,\nu_1,\nu_2}) = .01$ significance level

| | $\nu_1$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 25 | 30 | 40 |
| 2 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 |
| 3 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.2 | 27.1 | 26.9 | 26.7 | 26.6 | 26.5 | 26.4 |
| 4 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.5 | 14.4 | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 |
| 5 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 | 9.89 | 9.72 | 9.55 | 9.45 | 9.38 | 9.29 |
| 6 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.56 | 7.40 | 7.30 | 7.23 | 7.14 |
| 7 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.31 | 6.16 | 6.06 | 5.99 | 5.91 |
| 8 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.52 | 5.36 | 5.26 | 5.20 | 5.12 |
| 9 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 4.96 | 4.81 | 4.71 | 4.65 | 4.57 |
| 10 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.56 | 4.41 | 4.31 | 4.25 | 4.17 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.25 | 4.10 | 4.01 | 3.94 | 3.86 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.01 | 3.86 | 3.76 | 3.70 | 3.62 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 |

- The $F$-table above gives the **critical value** at 0.01 significance level
- For df1 $= 5$, df2 $= 18$, if the $F$-statistic exceeds $F_{0.01,df1=5,df2=18} = 4.25$, $P$-value $< 0.01$ and $H_0$ is rejected at 0.01 level