

STAT 222 Lecture 17

Analysis of Single Replicate Data

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Coverage

- ▶ Section 7.5 One Observation Per Cell

Single Replicate = One Observation Per Cell

Some factorial experiments have only ONE replicate per treatment.

- ▶ # of observations = # of treatments, $N = g$
- ▶ No degree of freedom for error ($dfE = N - g = 0$), cannot estimate σ^2
- ▶ fitted value = observed value for all cells/treatments
 - ▶ all the residuals are 0 \Rightarrow $SSE = 0$
- ▶ All parameters can be estimated as usual
- ▶ All sum of squares (SS) can be computed as usual except that $SSE = 0$.
- ▶ ANOVA F -tests for main effects and interactions of all orders cannot be done!

Remedies

1. Pooling higher order interactions into error
2. Half-normal probability plot (Skip)
 - ▶ We will get back to Half-normal probability plot if we have time at the end of the quarter

Problem 8.6 (p. 201, Oehlert's Textbook)

Response: dry matter yield in hundreds of pounds per acre over a 54-week study period

Factors:

- ▶ height of cut (1, 3, or 6 inches)
- ▶ cutting interval (1, 3, 6, or 9 weeks)
- ▶ amount of nitrogen fertilizer (0, 8, 16, or 32 hundred pounds of ammonium sulfate per acre per year).

		Cutting Interval			
		1 wks.	3 wks.	6 wks.	9 wks.
Ht 1	F 0	74.1	65.4	96.7	147.1
	F 8	87.4	117.7	190.2	188.6
	F 16	96.5	122.2	197.9	232.0
	F 32	107.6	140.5	241.3	192.0
Ht 3	F 0	61.7	83.7	88.8	155.6
	F 8	112.5	129.4	145.0	208.1
	F 16	102.3	137.8	173.6	203.2
	F 32	115.3	154.3	211.2	245.2
Ht 6	F 0	49.9	72.7	113.9	143.4
	F 8	92.9	126.4	175.5	207.5
	F 16	100.8	153.5	184.5	194.2
	F 32	115.8	160.0	224.8	197.5

Let's first load the data and fit a full 3-way model.

```
pr8.6 = read.table(  
  "http://www.stat.uchicago.edu/~yibi/s222/pr8_6.txt", h=T)  
pr8.6$HT = as.factor(pr8.6$ht)  
pr8.6$FERT = as.factor(pr8.6$fert)  
pr8.6$INT = as.factor(pr8.6$int)  
lm1 = lm(y ~ HT*FERT*INT, data=pr8.6); anova(lm1)  
Warning in anova.lm(lm1): ANOVA F-tests on an essentially perfect  
fit are unreliable  
Analysis of Variance Table
```

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
HT	2	29	15	NaN	NaN
FERT	3	42072	14024	NaN	NaN
INT	3	73887	24629	NaN	NaN
HT:FERT	6	406	68	NaN	NaN
HT:INT	6	3005	501	NaN	NaN
FERT:INT	9	5352	595	NaN	NaN
HT:FERT:INT	18	3155	175	NaN	NaN
Residuals	0	0	NaN		

We can pool the 3-way interaction terms as errors.

```
lm2 = lm(y ~ (HT+FERT+INT)^2, data=pr8.6)
```

```
anova(lm2)
```

Analysis of Variance Table

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
HT	2	29	15	0.08	0.921
FERT	3	42072	14024	80.02	1.3e-10
INT	3	73887	24629	140.52	1.1e-12
HT:FERT	6	406	68	0.39	0.878
HT:INT	6	3005	501	2.86	0.039
FERT:INT	9	5352	595	3.39	0.013
Residuals	18	3155	175		

When pooling *ABC* interactions into error, the F-stat for testing A main effects is

$$F_A = \frac{MS_A}{MS_{ABC}} \quad \text{rather than} \quad \frac{MS_A}{MSE}.$$

Justification of Pooling Higher Order Terms into Error

While the expected value for MSE is always σ^2 , the expected value for MS_{ABC} is

$$E[MS_{ABC}] = \sigma^2 + \frac{n \sum_{ijk} (\alpha\beta\gamma_{ijk})^2}{(a-1)(b-1)(c-1)} \geq \sigma^2.$$

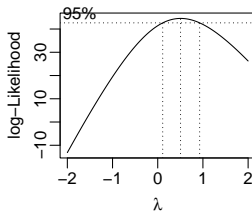
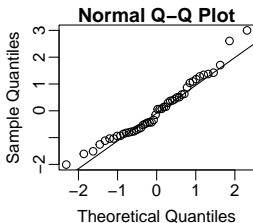
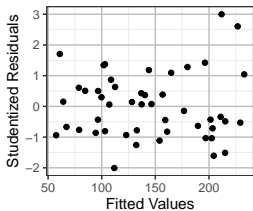
- ▶ If ABC interactions are indeed zero,
 - ▶ $F_A = \frac{MS_A}{MS_{ABC}} \sim F_{a-1, (a-1)(b-1)(c-1)}$ is valid
 - ▶ the P-value for A main effects obtained would be accurate
- ▶ If ABC interactions are nonzero,
 - ▶ MS_{ABC} would overestimate σ^2
 - ▶ $F_A = \frac{MS_A}{MS_{ABC}}$ would tend to be **smaller** than a random variable w/ an $F_{a-1, (a-1)(b-1)(c-1)}$ distribution
 - ▶ the P-value obtained would be larger than the actual one
 - ▶ A main effect would appear less significant than it should be

Justification of Pooling Higher Order Terms into Error (2)

- ▶ The same argument applies to the F-statistics for other effects (B main effects, AB interactions, AC interactions) as well
- ▶ If the df for highest order interactions is not big enough (better to be ≥ 10), one might need to pool more interaction terms (like all 2-way and 3-way interactions) into error.
- ▶ Should pool higher order terms first to maintain *hierarchy*
- ▶ Drawback of pooling terms into error: not able to test the significance of terms that are pooled into error

Always check model assumptions before interpreting the ANOVA table (and further analysis).

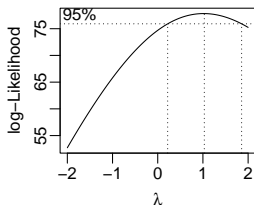
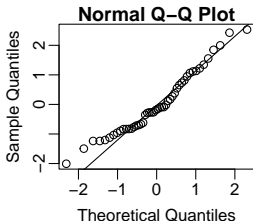
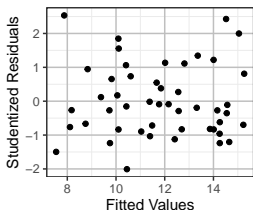
```
library(ggplot2)
ggplot(pr8.6, aes(x=fitted(lm2), y=rstudent(lm2)))+geom_point()+
  labs(x="Fitted Values", y="Studentized Residuals")
par(mai=c(.56,.56,.2,.1),mgp=c(1.8,.5,0))
qqnorm(rstudent(lm2))
qqline(rstudent(lm2))
library(MASS)
boxcox(lm2)
```



Box-Cox suggest a square-root transformation of the response since 0.5 is in the 95% confidence interval for λ .

Should check model assumption for the new model.

```
lm3= lm(sqrt(y) ~ (HT+FERT+INT)^2, data=pr8.6)
ggplot(pr8.6, aes(x=fitted(lm3), y=rstudent(lm3)))+geom_point()+
  labs(x="Fitted Values", y="Studentized Residuals")
par(mai=c(.56,.56,.2,.1),mgp=c(1.8,.5,0))
qqnorm(rstudent(lm3))
qqline(rstudent(lm3))
boxcox(lm3)
```



Though Box-Cox suggest square-root transformation, the original model shows no severe violation of the constant variance and normality assumption. One might adopt the original model for its simplicity in interpretation.

```
anova(lm(y ~ (HT+FERT+INT)^2, data=pr8.6))
```

Analysis of Variance Table

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
HT	2	29	15	0.08	0.921
FERT	3	42072	14024	80.02	1.3e-10
INT	3	73887	24629	140.52	1.1e-12
HT:FERT	6	406	68	0.39	0.878
HT:INT	6	3005	501	2.86	0.039
FERT:INT	9	5352	595	3.39	0.013
Residuals	18	3155	175		

- ▶ Strong **FERT** and **INT** effects

We can be confident about their significance since their actual P-value could be even smaller

- ▶ Borderline evidence of **HT:INT** and **FERT:INT** interactions

Their actual P-values could be even smaller, not sure.

```
anova(lm(sqrt(y) ~ (HT+FERT+INT)^2, data=pr8.6))  
Analysis of Variance Table
```

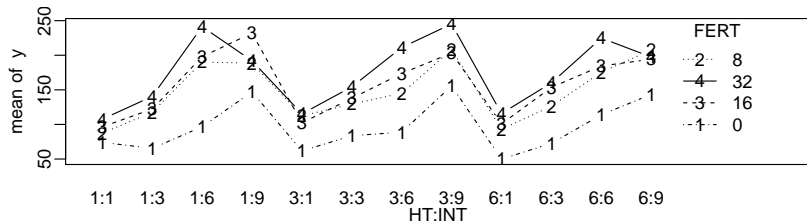
Response: sqrt(y)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
HT	2	0.1	0.1	0.18	0.840
FERT	3	82.2	27.4	93.82	3.5e-11
INT	3	132.7	44.2	151.46	5.9e-13
HT:FERT	6	0.5	0.1	0.31	0.926
HT:INT	6	4.9	0.8	2.78	0.043
FERT:INT	9	6.9	0.8	2.61	0.040
Residuals	18	5.3	0.3		

Similar conclusion if using sqrt(y) as the response.

3-Way Interaction Plot

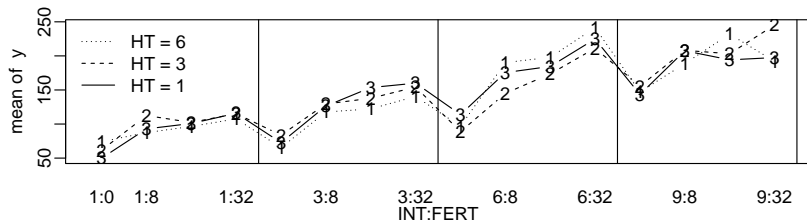
```
par(mai=c(.56,.56,.1,.1),mgp=c(1.8,.5,0))  
with(pr8.6, interaction.plot(HT:INT,FERT,y,type="b"))
```



- ▶ Little **INT:FERT** interactions for the 3 HT levels \Rightarrow little evidence of **INT:FERT:HT** interactions
- ▶ Large gap between lines \Rightarrow significant **FERT** main effect
- ▶ Lines are not horizontal \Rightarrow significant **INT** main effect

3-Way Interaction Plot

```
par(mai=c(.56,.56,.12,.1),mgp=c(1.8,.5,0))  
with(pr8.6, interaction.plot(INT:FERT,HT,y,legend=F, type="b"))  
legend("topleft",lty=c(3,2,1),paste("HT =", c(6,3,1)),bty="n")  
for(i in 1:4){abline(v=4*i+0.5)}
```



- ▶ The 3 lines entangled \Rightarrow little HT main effect
- ▶ Lines are closed to parallel \Rightarrow Little HT:FERT interactions

Warning About Pooling Terms Into Errors

One must decide which terms to pool into error before looking at data.

If one look at the ANOVA table first, and pool terms with small Sum of Squares into error, that would count as *data snooping*.

If one pool terms w/ small SS into error, the SS for “error” would tend to be small and the resulting MS might underestimate σ^2 . We wouldn't know which way the significance is biased.

For example, one cannot further pool [HT:FERT](#) into error. Note MSE is reduced from 175 to 148 and all terms become more significant.

```
anova(lm(y ~ HT*INT+FERT*INT, data=pr8.6))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
HT	2	29	15	0.10	0.9069
INT	3	73887	24629	166.00	3.5e-16
FERT	3	42072	14024	94.52	2.0e-13
HT:INT	6	3005	501	3.38	0.0148
INT:FERT	9	5352	595	4.01	0.0031
Residuals	24	3561	148		

```
anova(lm(y ~ (HT+FERT+INT)^2, data=pr8.6))
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
HT	2	29	15	0.08	0.921
FERT	3	42072	14024	80.02	1.3e-10
INT	3	73887	24629	140.52	1.1e-12
HT:FERT	6	406	68	0.39	0.878
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Residuals	18	3155	175		

Example 7.5.1 Drill Advance Experiment (p.220 Dean & Voss)

A $2 \times 2 \times 2 \times 2$ experiment to study the effects of 4 factors on the rate of advance of a small stone drill.

- ▶ A: load on the drill
- ▶ B: flow rate through the drill
- ▶ C: speed of rotation
- ▶ D: type of mud used in drilling

Each factor was observed at two levels, coded 1 and 2.

Response = $\log_{10}(\text{Advance})$

Data:

```
drill = read.table(  
  "http://www.stat.uchicago.edu/~yibi/s222/drill.txt", h=T)
```

Example: Drill Advance

As the 4 factors all have 2 levels only, all effects have 1 df only.

- ▶ If just pooling ABCD into error, 1 df for errors only
- ▶ If pooling 3-way and 4 way interactions into error, 5 df for errors

ABCD, ABC, ABD, ACD, BCD

- ▶ Need to pool all 2-way, 3-way, and 4-way interactions into error, giving 11 df for error

```
anova(lm(log10(Advance) ~ A+B+C+D, data=drill))
```

Analysis of Variance Table

Response: log10(Advance)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
A	1	0.013	0.013	7.02	0.023
B	1	0.254	0.254	139.74	1.4e-07
C	1	1.005	1.005	553.46	9.3e-11
D	1	0.080	0.080	44.28	3.6e-05
Residuals	11	0.020	0.002		

B, C, D main effects are highly significant.

A is at the borderline of significance

We are not able to test the significance of interactions.

Contrasts and Multiple Comparisons for Single Replicate Data

One can conduct inference for contrasts and adjust for multiple comparisons for Single Replicate Data just like for other factorial data.

The only difference is that the base model is not the full model with all the main effects and interactions of all order, but the model with some terms pooled into error.