# STAT22200 Spring 2014 Chapter 14

Yibi Huang

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# Incomplete Block Designs

## — A Brief Introduction to a Class of Most Useful Designs in Practice

Recall that for a randomized complete block design (RCBD) of $g$ treatments, the size $k$ of each block has to be $g$ (or multiples of $g$). Under complete block design, each treatment occurs once and only once in each block, so that we can randomize and run all treatment combinations in each block.

However, in practice, it often happens that the size of available blocks is smaller than the numbers of treatments ($k < g$). We cannot apply every treatment to every block.

We then have **Incomplete Block Design (IBD)**. IBD makes analysis harder, but sometimes cannot be avoided.

The next best thing to a randomized complete block design (RCBD) is a **Balanced Incomplete Block Design (BIBD)**.

# Balanced Incomplete Block Designs (BIBD)

BIBD not balanced in the general sense that all treatment-block combinations occur equally often. Rather they are balanced in the looser sense by the criteria described below.

A **balanced incomplete block design** with

- $g$ treatments,
- $b$ blocks,
- $k$ as the size of each block,
- $r$ replications of each treatment,

is a design satisfying the following:

Incomplete:
- $k < g$.

Balanced:
- Each treatment appears at most once per block.
- Each pair of treatments appear in a block the same number of times $r$

E.g., if treatments A and C appear in the same block 5 times, then treatments B and C must show up together in exactly 5 blocks.

The following design table is a BIBD.
Block size $k = 3$, there are $g = 5$ treatments and $b = 10$ blocks.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | B | E | A | C | D | B | E | D | C |
| B | D | A | C | A | E | C | B | E | D |
| C | A | B | D | E | A | D | C | B | E |

## First Balancing Condition of BIBD

- If there are $b$ blocks of size $k$ each,
  then the total number of experimental units is $N = bk$.
- If there are $g$ treatments, and each appears $r$ times,
  then the total number of experimental units is $N = rg$.

Therefore in a BIBD, we must have

$$\boxed{\text{N} = \text{bk} = \text{rg}}.$$

# Second Balancing Condition of BIBD

Suppose each pair of treatments appears in the same block $\lambda$ times.

- Each of the $b$ blocks contains $\binom{k}{2}$ pairs of treatments. In total there are $b\binom{k}{2}$ pairs
- The $g$ treatments can form $\binom{g}{2}$ different pairs, and each pair appears in the same block $\lambda$ times. In total there are $\lambda\binom{g}{2}$ pairs.

The two ways to count the total number of treatment pairs appearing the same block must give the same answer. So

$$b\binom{k}{2} = \frac{\overbrace{bk}^{=rg}(k-1)}{2} = \lambda\binom{g}{2} = \frac{\lambda g(g-1)}{2} \Rightarrow \boxed{r(k-1) = \lambda(g-1)}$$

Remarks. In a BIBD, we must have $\lambda = \dfrac{r(k-1)}{g-1}$ being a **whole number**.

In the eyedrop example: $g = 3$, $k = 2$, $r = 2$.

$$\lambda = \frac{r(k-1)}{g-1} = \frac{2(2-1)}{3-1} = 1 \text{ is a whole number,}$$

and we need $b = rg/k = 3$ blocks.

# Example — A Marketing Psychology Experiment

- Goal: to compare the effects of 5 advertisements, $g = 5$
- Response: recall after being shown the ad.
- A subject is a block
- Block size: Subjects lose patience after being shown 3 ads, $k = 3$
- What is the smallest feasible BIBD? $b =?$ $r =?$

By $N = rg = kb$, and that $\frac{r(k-1)}{g-1}$ is a whole number, we need to make

$$r = \frac{3b}{5} \quad \text{and} \quad \frac{2r}{4} \text{ both being whole numbers}$$

So $r$ must be a multiple of 2 and 3, and $b$ be a multiple of 2 and 5.Thus the smallest BIBD is with $b = 10$, and $r = 6$.

The number of subjects (blocks) must be multiple of 10 (10, 20, 30,...).

The following design table is a BIBD with
$g = 5, k = 3, b = 10, r = 6, \lambda = 3$ (Oehlert Appendix C.2, p.609-615)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 5 | 1 | 3 | 4 | 2 | 5 | 4 | 3 |
| 2 | 4 | 1 | 3 | 1 | 5 | 3 | 2 | 5 | 4 |
| 3 | 1 | 2 | 4 | 5 | 1 | 4 | 3 | 2 | 5 |

- In the marketing psychology experiment, if we have $b = 20$ subjects (blocks), then we can do 2 repetitions of the above design.
- One obvious randomization is to randomize subjects to columns, then randomize the order of treatments in each block based on the above design.

# Still a "Balanced" design

In the marketing psychology example:

$$r = 6 \times 2 = 12\text{: Each treatment appears 12 times.}$$

The following table shows that, in the first 10 blocks, each treatment appears 6 times.

| | Block | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Treatment | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A | √ | √ | √ | √ | √ | √ | | | | |
| B | √ | √ | √ | | | | √ | √ | √ | |
| C | √ | | | √ | √ | | √ | √ | | √ |
| D | | √ | | √ | | √ | √ | | √ | √ |
| E | | | √ | | √ | √ | | √ | √ | √ |

$\lambda = 3 \times 2 = 6$: Each treatment pair appear 6 times.

The following table shows that, in the first 10 blocks, each treatment pair appear 3 times.

| | Block | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Treatment-pair | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| AB | $\surd$ | $\surd$ | $\surd$ | | | | | | | |
| AC | $\surd$ | | | $\surd$ | $\surd$ | | | | | |
| AD | | $\surd$ | | $\surd$ | | $\surd$ | | | | |
| AE | | | $\surd$ | | $\surd$ | $\surd$ | | | | |
| BC | $\surd$ | | | | | | $\surd$ | $\surd$ | | |
| BD | | $\surd$ | | | | | $\surd$ | | $\surd$ | |
| BE | | | $\surd$ | | | | | $\surd$ | $\surd$ | |
| CD | | | | $\surd$ | | | $\surd$ | | | $\surd$ |
| CE | | | | | $\surd$ | | | $\surd$ | | $\surd$ |
| DE | | | | | | $\surd$ | | | $\surd$ | $\surd$ |

# Models for BIBD

The model for BIBD looks familiar:

$$y_{ij} = \mu + \underset{\text{(treatment)}}{\alpha_i} + \underset{\text{(block)}}{\beta_j} + \underset{\text{(i.i.d. } N(0,\sigma^2))}{\varepsilon_{ij}}$$

for $i = 1, \ldots, g$, and $j = 1, \ldots, b$ with

$$\sum_{i=1}^{g} \alpha_i = \sum_{j=1}^{b} \beta_j = 0.$$

- Not all $y_{ij}$ are observed, as the case for Latin Square designs.
- There is nonorthogonality between treatments and blocks, just like *unbalanced* factorial designs.
  Thus, the Type I Sum of squares will change with the order of terms in the model.

# ANOVA for BIBD (Type I Sum of Squares!)

| Source | d.f. | SS | MS | $F$-value |
|---|---|---|---|---|
| Block | $b-1$ | $\text{SS}_{block}$ | $\text{MS}_{block}$ | $(\text{MS}_{block}/\text{MSE})$ |
| Treatment | $g-1$ | $\text{SS}_{Trt}$ | $\text{MS}_{Trt}$ | $\text{MS}_{trt}/\text{MSE}$ |
| Error | $N-g-b+1$ | SSE | MSE | |
| Total | $N-1$ | $SS_{total}$ | | |

Let $$I_{ij} = \begin{cases} 1, & \text{if treatment } i \text{ appears in block } j, \\ 0, & \text{otherwise.} \end{cases}$$

Then $$SS_{total} = \sum_{i=1}^{g} \sum_{j=1}^{b} I_{ij}(y_{ij} - \overline{y}_{\bullet\bullet})^2$$

$$SS_{block} = k \sum_{j=1}^{b} (\overline{y}_{\bullet j} - \overline{y}_{\bullet\bullet})^2$$

$$SS_{Trt} = \frac{rk}{\lambda g} \sum_{i=1}^{g} \sum_{j=1}^{b} I_{ij} \left(\overline{y}_{i\bullet} - \overline{y}_{\bullet j(i)}\right)^2 = \frac{rk}{\lambda g} \cdot r \cdot \sum_{i=1}^{g} \left(\overline{y}_{i\bullet} - \overline{y}_{\bullet j(i)}\right)^2$$

in which $\overline{y}_{\bullet j(i)} = \frac{1}{r} \sum_{j=1}^{b} I_{ij} \overline{y}_{\bullet j}$ (average of block means containing trt $i$)

# Example of BIBD — Problem 14.3 on. p.381-382

The State Board of Education has adopted basic skills tests for high school graduation. One of these is a writing test. The student writing samples are graded by professional graders, and the board is taking some care to be sure that the graders are grading to the same standard. We examine grader differences with the following experiment. We select 30 writing samples at random; each writing sample will be graded by 5 graders. Thus each grader will grade 6 samples.

**Data file**: `http://users.stat.umn.edu/~gary/book/fcdae.data/pr14.3`

**Questions of Interest**

- Do the 25 graders grade consistently?
- If graders don't grade consistently, how should we adjust the grades?
- If graders don't grade consistently, can we identify which graders fail to follow the grading standard?

| Exam | Grader | Score | Exam | Grader | Score |
|---|---|---|---|---|---|
| 1 | 1 2 3 4 5 | 60 59 51 64 53 | 16 | 1 9 12 20 23 | 61 67 69 68 65 |
| 2 | 6 7 8 9 10 | 64 69 63 63 71 | 17 | 2 10 13 16 24 | 78 75 76 75 72 |
| 3 | 11 12 13 14 15 | 84 85 86 85 83 | 18 | 3 6 14 17 25 | 67 72 72 75 76 |
| 4 | 16 17 18 19 20 | 72 76 77 74 77 | 19 | 4 7 15 18 21 | 84 81 76 79 77 |
| 5 | 21 22 23 24 25 | 65 73 70 71 70 | 20 | 5 8 11 19 22 | 81 84 85 84 81 |
| 6 | 1 6 11 16 21 | 52 54 62 54 55 | 21 | 1 8 15 17 24 | 70 65 61 66 66 |
| 7 | 2 7 12 17 22 | 56 51 52 57 51 | 22 | 2 9 11 18 25 | 84 82 86 85 86 |
| 8 | 3 8 13 18 23 | 55 60 59 60 61 | 23 | 3 10 12 19 21 | 72 85 77 82 79 |
| 9 | 4 9 14 19 24 | 88 76 77 77 74 | 24 | 4 6 13 20 22 | 85 75 78 82 83 |
| 10 | 5 10 15 20 25 | 65 68 72 74 77 | 25 | 5 7 14 16 23 | 58 64 58 57 58 |
| 11 | 1 10 14 18 22 | 79 77 77 77 79 | 26 | 1 7 13 19 25 | 66 71 73 70 70 |
| 12 | 2 6 15 19 23 | 70 66 63 62 66 | 27 | 2 8 14 20 21 | 73 67 63 70 66 |
| 13 | 3 7 11 20 24 | 48 49 51 48 50 | 28 | 3 9 15 16 22 | 58 70 69 61 71 |
| 14 | 4 8 12 16 25 | 75 64 75 68 65 | 29 | 4 10 11 17 23 | 95 84 88 88 87 |
| 15 | 5 9 13 17 21 | 79 77 81 79 83 | 30 | 5 6 12 18 24 | 47 47 51 49 56 |

Here a exam is a writing sample.

- Which factor is the treatment factor? Graders or writing samples?
- Which factor is the block factor? Graders or writing samples?
- Is this a BIBD?

$$Y_{ij} = \mu + \underset{\text{(grader)}}{\alpha_i} + \underset{\text{(exam)}}{\beta_j} + \varepsilon_{ij}$$

As writing samples differ in levels, we expect $\beta_j$ not all equal.

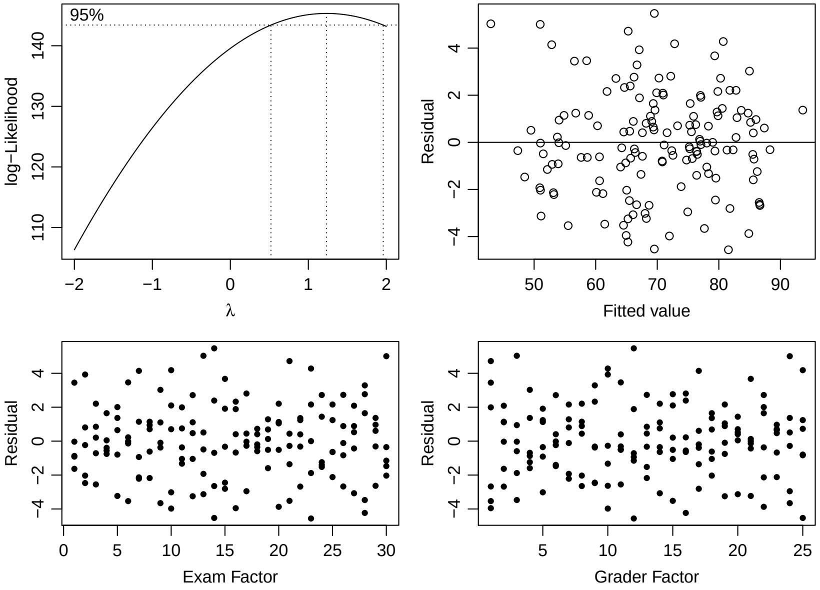
If the graders grade consistently, they should give the same score to a writing sample, i.e., $\alpha_1 = \alpha_2 = \cdots = \alpha_{25}$

```
> anova(lm(score ~ grader + exam))
          Df  Sum Sq Mean Sq F value    Pr(>F)
grader    24  4073.1  169.71  23.659 < 2.2e-16 ***
exam      29 13342.0  460.07  64.138 < 2.2e-16 ***
Residuals 96   688.6    7.17

> anova(lm(score ~ exam + grader))
          Df  Sum Sq Mean Sq F value    Pr(>F)
exam      29 16609.0  572.72 79.8424 < 2.2e-16 ***
grader    24   806.2   33.59  4.6828 2.694e-08 ***
Residuals 96   688.6    7.17
```

- Why are the two ANOVA tables different?
- Which one should we look at?

# Model diagnostic for $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$



As the 25 graders grade inconsistently, how to adjust the scores?

Based on the model $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$, the score of the $i$th writing sample should be $\mu + \beta_j$, and is estimated by $\widehat{\mu} + \widehat{\beta}_j$.

How to get $\widehat{\beta}_j$ in R? Recall that by default, R reports estimates of parameters using the baseline constraints $\alpha_1 = \beta_1 = 0$, not the sum-to-zero constraints $\sum_{i=1}^{g} \alpha_i = \sum_{j=1}^{b} \beta_j = 0$.

One can use `constrasts()` and `contr.sum()` to force R using the the sum-to-zero constraints.

```
> contrasts(exam) = contr.sum(30)
> contrasts(grader) = contr.sum(25)
> lm1 = lm(score ~ exam + grader); lm1$coef
(Intercept)       exam1       exam2   .... (omitted)
     69.960     -12.568      -3.368
     exam28      exam29     grader1   .... (omotted)   grader24
     -2.128      16.192      -0.840                       0.160
```

Why there are neither estimates for exam30, nor for grader 25?

```
> muhat = lm1$coef[1]
> betahat = vector("numeric",length=30)
> betahat[1:29] = lm1$coef[2:30]
> betahat[30] = -sum(betahat[1:29])
> adjustedscore = muhat + betahat; adjustedscore
 [1] 57.392 66.592 84.392 75.152 69.472 56.376 51.616 60.416
 [9] 77.496 71.496 77.848 65.648 49.328 68.208 80.568 65.792
[17] 74.792 73.952 78.112 83.352 66.120 83.440 80.240 78.760
[25] 60.240 69.512 67.672 67.832 86.152 50.832
> adjustedscore = array(adjustedscore,dim=c(1,30))
> adjustedscore = data.frame(adjustedscore)
> colnames(adjustedscore) = 1:30
> adjustedscore
       1      2      3      4      5      6      7      8      9     10
1 57.392 66.592 84.392 75.152 69.472 56.376 51.616 60.416 77.496 71.496
      11     12     13     14     15     16     17     18     19     20
1 77.848 65.648 49.328 68.208 80.568 65.792 74.792 73.952 78.112 83.352
     21    22    23    24    25     26     27     28     29     30
1 66.12 83.44 80.24 78.76 60.24 69.512 67.672 67.832 86.152 50.832
```

How to identify graders that grade inconsistently?

From the parameter estimates, it appears that most graders grade consistently except grader 2, 3, 4, and 5 (and possibly grader 6, 11, and 16 also).

```
> summary(lm(score ~ exam + grader))
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  69.9600     0.2187 319.919  < 2e-16 ***
exam1       -12.5680     1.2714  -9.885 2.62e-16 ***
exam2        -3.3680     1.2714  -2.649 0.009437 **
...(omitted)
exam29       16.1920     1.2714  12.736  < 2e-16 ***
grader1      -0.8400     1.1736  -0.716 0.475871
grader2       3.2400     1.1736   2.761 0.006908 **
grader3      -6.3600     1.1736  -5.419 4.44e-07 ***
grader4       7.4800     1.1736   6.374 6.41e-09 ***
grader5      -3.4800     1.1736  -2.965 0.003814 **
grader6      -2.3600     1.1736  -2.011 0.047133 *
grader7       1.6000     1.1736   1.363 0.175955
grader8      -1.5600     1.1736  -1.329 0.186904
grader9      -1.1200     1.1736  -0.954 0.342299
grader10      0.4800     1.1736   0.409 0.683443
grader11      2.1600     1.1736   1.841 0.068777 .
grader12      1.3200     1.1736   1.125 0.263486
grader13      0.7600     1.1736   0.648 0.518789
grader14     -1.6000     1.1736  -1.363 0.175955
grader15     -1.6000     1.1736  -1.363 0.175955
grader16     -2.6000     1.1736  -2.215 0.029091 *
grader17      1.2400     1.1736   1.057 0.293340
grader18      0.2000     1.1736   0.170 0.865037
grader19     -0.4000     1.1736  -0.341 0.733967
grader20      1.8000     1.1736   1.534 0.128370
grader21     -1.2400     1.1736  -1.057 0.293340
grader22      1.5200     1.1736   1.295 0.198357
grader23     -0.1200     1.1736  -0.102 0.918769
grader24      0.1600     1.1736   0.136 0.891840
```

We can test if grader 2, 3, 4, 5 are the only ones that grade inconsistently.

```
> newgrader = grader
> newgrader[as.numeric(grader) >= 6]=1
> newgrader = factor(newgrader, levels=1:5); newgrader
  [1] 1 2 3 4 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2
 [32] 1 1 1 1 3 1 1 1 1 4 1 1 1 1 5 1 1 1 1 1 1 1 1 1 2 1 1 1 1 3 1
 [63] 1 1 1 4 1 1 1 1 5 1 1 1 1 1 1 1 1 1 2 1 1 1 1 3 1 1 1 1 4 1 1
 [94] 1 1 5 1 1 1 1 1 1 1 1 1 2 1 1 1 1 3 1 1 1 1 4 1 1 1 1 5 1 1 1
[125] 1 1 1 1 1 1 2 1 1 1 1 3 1 1 1 1 4 1 1 1 1 5 1 1 1 1
Levels: 1 2 3 4 5
> anova(lm(score ~ exam + newgrader),lm(score ~ exam + grader))
Analysis of Variance Table

Model 1: score ~ exam + newgrader
Model 2: score ~ exam + grader
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    116 899.58
2     96 688.62 20    210.95 1.4704 0.1103
```

This confirms that $\alpha_1 = \alpha_6 = \alpha_7 = \alpha_8 = \cdots = \alpha_{25}$.

```
> contrasts(exam) = contr.treatment(30)
> contrasts(newgrader) = contr.treatment(5)
> summary(lm(score ~ exam + newgrader))
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   57.190      1.359  42.088  < 2e-16 ***
exam2          8.810      1.843   4.779 5.20e-06 ***
...(output omitted)...
exam30        -6.503      1.821  -3.572 0.000517 ***
newgrader2     3.282      1.275   2.575 0.011292 *
newgrader3    -6.318      1.275  -4.957 2.47e-06 ***
newgrader4     7.522      1.275   5.901 3.66e-08 ***
newgrader5    -3.438      1.275  -2.697 0.008035 **
```

This shows on average,

- grader 2 gives 3.28 points higher,
- grader 3 gives 6.32 points lower,
- grader 4 gives 7.52 points higher,
- and grader 5 gives 3.44 points lower.

# Problem 14.2 — Incomplete Block Design but Not BIBD

Milk can be strained through filter disks to remove dirt and debris. Filters are made by surface-bonding fiber webs to both sides of a disk.

- Goal of the experiment: to know how the construction of the filter affects the speed of milk flow through the filter
- Treatments: We have a $2^4$ factorial structure for the filters. The 4 factors are
  - fiber weight (normal or heavy),
  - loft (thickness of the filter, normal or low),
  - bonding solution on bottom surface (A or B), and
  - bonding solution on top surface (A or B).

  Treatments 1 through 16 are the factor-level combinations in standard order.

Response: filtration time when pouring a measured amount of milk through the disk

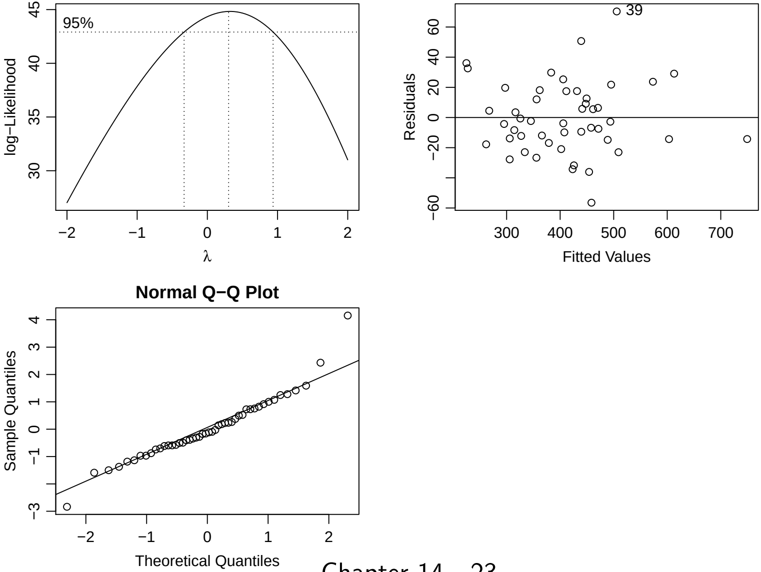
- considerable variation from farm to farm, so block on farm. 16 farms were selected
- considerable variation from milking to milking, so only one milking per farm.
- Only 3 filters can be used at a single milking.
  So blocks size $\leq 3$
- Is this a BIBD?

Treatments and Responses

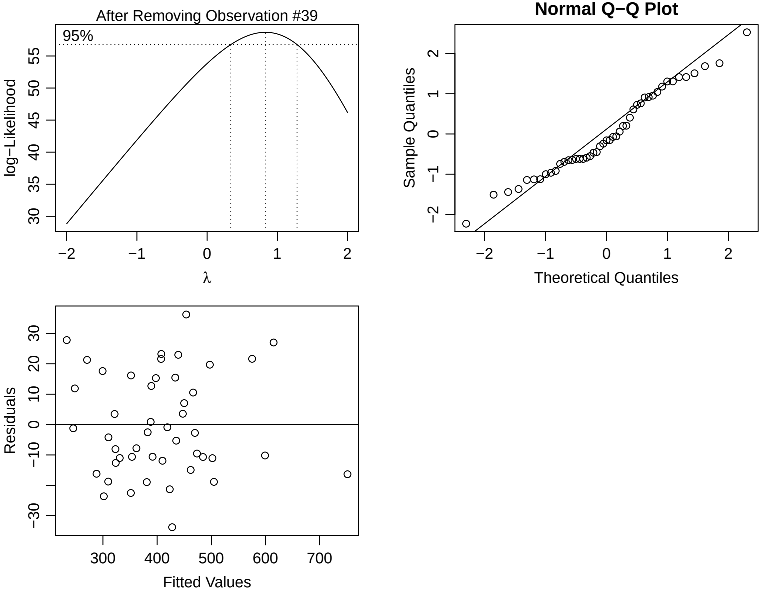
| | Filtration time | | | | | |
|---|---|---|---|---|---|---|
| Farm | First | | Second | | Third | |
| 1 | 10 | 451 | 7 | 457 | 16 | 343 |
| 2 | 11 | 260 | 8 | 418 | 13 | 320 |
| 3 | 12 | 464 | 5 | 317 | 14 | 315 |
| 4 | 9 | 306 | 6 | 462 | 15 | 291 |
| 5 | 13 | 381 | 4 | 597 | 6 | 491 |
| 6 | 14 | 362 | 1 | 325 | 7 | 449 |
| 7 | 15 | 292 | 2 | 402 | 8 | 576 |
| 8 | 16 | 431 | 3 | 477 | 5 | 394 |
| 9 | 7 | 329 | 9 | 261 | 4 | 430 |
| 10 | 8 | 389 | 10 | 413 | 1 | 272 |
| 11 | 5 | 368 | 11 | 244 | 2 | 447 |
| 12 | 6 | 398 | 12 | 517 | 3 | 354 |
| 13 | 2 | 490 | 16 | 311 | 9 | 278 |
| 14 | 3 | 467 | 13 | 429 | 10 | 486 |
| 15 | 4 | 735 | 14 | 642 | 11 | 474 |
| 16 | 1 | 402 | 15 | 380 | 12 | 589 |

# Model diagnostic for

$$Y_{ij} = \mu + \underset{\text{(trt)}}{\alpha_i} + \underset{\text{(farm)}}{\beta_j} + \varepsilon_{ij}$$



After removing observation # 39,



```
          Df Sum Sq Mean Sq F value    Pr(>F)    
farm      15 248469 16564.6  20.477 1.197e-07 ***
trt       15 231587 15439.1  19.085 2.000e-07 ***
Residuals 16  12943   808.9                      
```

In a BIBD, $Var(\widehat{\alpha}_{i_1} - \widehat{\alpha}_{i_2})$ will be the same for all $i_1$, $i_2$ since all pairs of treatments appear in one block the same number of times. If not a BIBD, then $Var(\widehat{\alpha}_{i_1} - \widehat{\alpha}_{i_2})$ will depend on the number of times $i_1$, $i_2$ appear in the same block.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  337.925     28.754  11.752 2.78e-09 ***
fm[-39]2     -18.954     30.753  -0.616 0.546346
fm[-39]3     -74.269     30.204  -2.459 0.025710 *
fm[-39]4     -21.402     30.498  -0.702 0.492926
fm[-39]5      41.625     30.167   1.380 0.186624
fm[-39]6     -16.406     27.539  -0.596 0.559676
fm[-39]7     -60.462     34.351  -1.760 0.097482 .
fm[-39]8      54.137     27.579   1.963 0.067278 .
fm[-39]9     -98.426     27.579  -3.569 0.002563 **
fm[-39]10    -49.735     28.179  -1.765 0.096647 .
fm[-39]11    -21.820     30.315  -0.720 0.482045
fm[-39]12    -50.519     30.204  -1.673 0.113845
fm[-39]13    -29.995     27.901  -1.075 0.298289
fm[-39]14     57.426     27.579   2.082 0.053734 .
fm[-39]15    217.625     30.167   7.214 2.07e-06 ***
fm[-39]16     51.368     30.315   1.694 0.109547
trt[-39]2    145.835     31.470   4.634 0.000276 ***
trt[-39]3     74.394     30.204   2.463 0.025498 *
trt[-39]4    195.801     30.204   6.483 7.56e-06 ***
trt[-39]5     35.726     30.315   1.179 0.255826
trt[-39]6    122.507     30.315   4.041 0.000946 ***
trt[-39]7    112.019     27.579   4.062 0.000907 ***
trt[-39]8     99.920     30.644   3.261 0.004910 **
trt[-39]9     -6.317     30.498  -0.207 0.838521
trt[-39]10   109.512     27.579   3.971 0.001097 **
trt[-39]11   -70.875     30.167  -2.349 0.031974 *
trt[-39]12   209.882     27.700   7.577 1.11e-06 ***
trt[-39]13    12.043     30.204   0.399 0.695366
trt[-39]14    59.426     27.579   2.155 0.046765 *
trt[-39]15    -6.759     28.960  -0.233 0.818413
trt[-39]16    15.695     30.315   0.518 0.611731
```

```
> lm3 = lm(time[-39] ~ fm[-39]+wt[-39]*lt[-39]*bn[-39]*tp[-39])
> anova(lm3)
Analysis of Variance Table

Response: time[-39]
                                Df Sum Sq Mean Sq  F value    Pr(>F)
fm[-39]                         15 248469   16565  20.4768 1.197e-07 ***
wt[-39]                          1 108563  108563 134.2033 3.422e-09 ***
lt[-39]                          1   4146    4146   5.1249  0.037837 *
bn[-39]                          1   6777    6777   8.3776  0.010563 *
tp[-39]                          1  37105   37105  45.8687 4.479e-06 ***
wt[-39]:lt[-39]                  1    300     300   0.3712  0.550892
wt[-39]:bn[-39]                  1  43458   43458  53.7224 1.693e-06 ***
lt[-39]:bn[-39]                  1   2380    2380   2.9422  0.105591
wt[-39]:tp[-39]                  1   1292    1292   1.5968  0.224465
lt[-39]:tp[-39]                  1   7312    7312   9.0385  0.008367 **
bn[-39]:tp[-39]                  1   2847    2847   3.5196  0.079011 .
wt[-39]:lt[-39]:bn[-39]          1   5187    5187   6.4115  0.022192 *
wt[-39]:lt[-39]:tp[-39]          1   9192    9192  11.3628  0.003892 **
wt[-39]:bn[-39]:tp[-39]          1   1470    1470   1.8174  0.196401
lt[-39]:bn[-39]:tp[-39]          1    107     107   0.1317  0.721461
wt[-39]:lt[-39]:bn[-39]:tp[-39]  1   1451    1451   1.7941  0.199143
Residuals                       16  12943     809
```

```
> Anova(lm3, type=2)
Anova Table (Type II tests)

Response: time[-39]
                             Sum Sq Df  F value    Pr(>F)
fm[-39]                      199743 15  16.4612 5.820e-07 ***
wt[-39]                      115273  1 142.4984 2.220e-09 ***
lt[-39]                        3799  1   4.6966  0.045656 *
bn[-39]                        5765  1   7.1261  0.016788 *
tp[-39]                       33501  1  41.4126 8.242e-06 ***
wt[-39]:lt[-39]                 407  1   0.5036  0.488127
wt[-39]:bn[-39]               39070  1  48.2980 3.273e-06 ***
lt[-39]:bn[-39]                2380  1   2.9422  0.105591
wt[-39]:tp[-39]                1292  1   1.5968  0.224465
lt[-39]:tp[-39]                7703  1   9.5224  0.007087 **
bn[-39]:tp[-39]                1869  1   2.3105  0.148016
wt[-39]:lt[-39]:bn[-39]        5321  1   6.5774  0.020779 *
wt[-39]:lt[-39]:tp[-39]        6914  1   8.5471  0.009943 **
wt[-39]:bn[-39]:tp[-39]        1514  1   1.8718  0.190180
lt[-39]:bn[-39]:tp[-39]         107  1   0.1317  0.721461
wt[-39]:lt[-39]:bn[-39]:tp[-39] 1451  1   1.7941  0.199143
Residuals                     12943 16
```