

INTRODUCTION

- The chance of something happening gives the percentage of time it is expected to happen, when the basic process is done over and over again, independently and under the same conditions.
 - Examples:
 - Tossing a coin.
 - Chance of getting heads is 1 in 2, or 50%.
 - Rolling a die.
 - Chance of rolling a 3 spot is 1 in 6, or $16\frac{2}{3}\%$.
 - Shuffling a deck of cards and dealing out the top 5.
 - Chance of getting a full house (one pair and three of a kind) is ???
 - Chances are between 0% and 100%.
 - Something that's impossible happens _____% of the time.
 - Something that's sure to happen occurs _____% of the time.
 - Chances can be expressed in various ways. If in the long run some thing happens 3 times out of 4, it's chances are:
 - 3 out of 4, or
 - 3 in 4, or
 - $3/4$, or
 - 0.75, or
 - 75%.

- In poker (five-card draw), the chance of being dealt a full house (one pair and three of a kind) is about 0.14%.
 - 0.14% is the same as $0.\underline{\quad}$, or _____ in 10,000.
 - If 10,000 poker hands are dealt, about _____ will be a full house.
 - If 10,000 poker hands are dealt, about _____ will *not* be a full house.
 - The chance of not being dealt a full house is _____ in 10,000, or _____%.
- The chance of something happening equals 100% minus the chance of the opposite thing.
 - This is called the *complementation* rule.
 - The complementation rule is especially useful when the opposite thing has a simple description (so it's chance is easy to compute directly), while the thing itself has a complex description (so it's chance is hard to compute directly).

USING THE LONG-RUN ARGUMENT

- A deck of cards is shuffled and the top card is placed face down on a table. What is the chance that the card is a king?
 - Imagine making many such deals, say 52,000.
 - You would expect each one of the 52 cards in the deck to be dealt about _____ times.
 - In particular, you would expect the king of spades to be dealt about _____ times.
 - Ditto, for the king of hearts, diamonds, and clubs.
 - Out of the 52,000 deals, you would expect about _____ kings.
 - The chance that the card is a king equals _____ in 52, or _____ in 13, or 7.7%.
- If a thing can happen in some ways and fail to happen in some other ways, and if all those ways are equally likely, then the chance that the thing happens equals the ratio
$$\frac{\text{number of ways for the thing to happen}}{\text{total number of ways for the thing to happen or fail}}$$
.
- A deck of cards is shuffled and the two top cards are placed face down on a table. What is the chance that the second card is a king?
 - There are _____ possibilities for the second card, and they are all equally likely.
 - _____ of those possibilities are kings.
 - The chance that the second card is a king equals _____ in 52, or _____ in 13, or _____%.

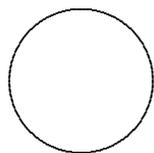
CONDITIONAL PROBABILITIES

- A deck of cards is shuffled and the two top cards are placed face down on a table. The first card is turned over. It is a king. What is the chance that the second card is a king?
 - Given that the first card is a king:
 - There are _____ possibilities for the second card, all of which are equally likely.
 - _____ of those possibilities are kings.
 - The chance that the second card is a king equals _____ in _____ .
 - The *conditional* chance that the second card is a king, *given* that the first card is a king, equals _____ in _____ , or 5.9%.
- A deck of cards is shuffled and the two top cards are placed face down on a table. The first card is turned over. It is *not* a king. Now what is the chance that the second card is a king?
 - Given that the first card isn't a king:
 - There are _____ equally likely possibilities for the second card.
 - _____ of those possibilities are kings.
 - The conditional chance that the second card is a king, given that the first card is not, equals _____ in _____ , or 7.8%

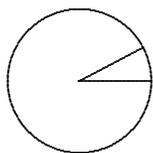
THE MULTIPLICATION RULE

- A deck of cards is shuffled and the two top cards are placed face down on a table. What is the chance that both cards are kings?

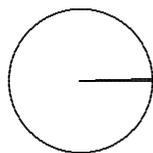
- Imagine making many such deals.
 - The first card will be a king about _____ of the time.
 - Among the deals where the first card is king, the second card will be a king about _____ of the time.
 - So both cards will be kings about _____ of _____ of the time.



All deals



First card
a king



Both cards
kings

- The chance that both cards are kings equals:
 - $\frac{3}{51} \times \frac{1}{13}$, or
 - $\frac{1}{13} \times \frac{3}{51}$, or
 - the *unconditional* chance that the first card is a king, multiplied by the *conditional* chance that the second card is a king *given* that the first card is a king, or
 - 0.5% .

- What is the *multiplication rule* for finding the chance that two things will both happen?

- The chance that two things will both happen equals the unconditional chance that the first will happen, multiplied by the conditional chance that the second will happen given that the first has happened.

- A deck of cards is shuffled and the two top cards are placed face down on a table. What is the chance that both cards are not kings?

- The chance that the first card is not a king equals _____.
- Given that the first card is not a king, the conditional chance that the second card is not a king equals _____.
- So the chance that both cards are not kings equals _____ \times _____, or 85.1%.

- A deck of cards is shuffled and the two top cards are placed face down on a table. What is the chance that at least one card is a king?

- The chance of the opposite thing — that neither card is a king — equals _____.
- By the complementation rule, the chance that at least one card is a king equals _____, or _____.
- This is not equal to the chance that first card is a king, plus the chance that the second card is a king!

- A die is rolled twice. What is the chance of getting two aces? (An ace is a 1-spot.)

- The chance of an ace on the first roll equals _____ .
- Given an ace on the first roll, the conditional chance of an ace on the second roll equals _____ .
- So the chance of getting two aces equals _____ × _____, or _____ .

- When multiplying chances, it is safest to work with fractions or decimals, rather than percents. For example, if the first chance is 1 in 2, and the second is 3 in 5, then the product is

$$\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5} = \frac{3}{10}, \quad \text{or} \quad 0.5 \times 0.6 = 0.30.$$

The equivalent calculation with percents is

$$50\% \text{ of } 60\% = 30\% \quad (\text{or } 60\% \text{ of } 50\% = 30\%),$$

not

$$50\% \times 60\% = 3000\%^2$$

which is nonsense.

- Is there a multiplication rule for finding the chance that three things will all happen?

- Yes. Start with the unconditional chance that the first thing will happen.
- Multiply that by the conditional chance that the second thing will happen given that the first thing has happened.
- Multiply again, by the conditional chance that the third thing will happen given that the first two things have happened.

- In (five-card draw) poker, what's the chance of being dealt, in order, a pair followed by three of a kind?

- 5 things must happen:
 - The first card can be anything.
 - The second card must have the same rank as the first card.
 - The third card must have a different rank than the first card.
 - The fourth card must have the same rank as the third card.
 - And so must the fifth card.

- The corresponding conditional and unconditional probabilities are:

- _____ in _____ , or _____ .

- By the multiplication rule, the sought after chance equals $\frac{1}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{4}{49} \times \frac{3}{48}$, or 0.000144 .

INDEPENDENCE

- Two things are *independent* if the chances for the second one given the first are the same, no matter how the first one turns out. Otherwise, they are *dependent*.
- A deck of cards is shuffled and the two top cards are placed face down on a table.
 - Think about these two events:
 - Event 1: the first card is a king.
 - Event 2: the second card is a king.
 - Are these two events independent, or dependent?
 - Given that the first card is a king, the chance that the second card is a king equals _____ /51.
 - Given that the first card is not a king, the chance that the second card is a king equals _____ /51.
 - The chances for the second event change, depending on how the first event turns out. So the two events are _____.
- Someone is going to roll a die twice. Are the two rolls independent, or dependent?
 - No matter how the first roll turns out, the second roll will give 1, 2, 3, 4, 5, or 6, with equal chances. So the two rolls are _____.

- Suppose two events are independent, so that the conditional chance of the second event stays the same, no matter how the first event comes out. What is the unconditional chance of the second event?
 - The same as its conditional chances.
- What is the *multiplication rule for independent events*?
 - If two events are independent, the chance that both of them will happen equals the product of their unconditional chances.
- Every day you buy a lottery ticket that offers 1 chance in 1000 of winning. What is the chance that you never win in 1000 plays?
 - The question asks for the chance of losing on each play.
 - The plays are independent.
 - Your chance of losing on any particular play equals _____ in _____, or _____.
 - Your chance of losing on all 1000 plays equals _____, or 0.368.
 - The chance that you win at least once in 1000 plays equals _____, or 0.632.
 - This would be hard to compute directly.
 - 1000 days is _____ years and _____ months.

- A computer is about to select a person at random from the U.S. population. It could happen that:
 - The person will turn out to be over 65.
 - The person will turn out to be female.
- Are these two eventualities independent, or dependent?
 - The conditional chance that the person is female, given that the person is over 65, equals the _____ of people over 65 that are female.
 - The conditional chance that the person is female, given that the person is under 65, equals the _____ of people under 65 that are female.
 - The first percentage is _____ than the second percentage, because women tend to live longer than men. So age and sex are _____.
- True or false, and explain. The percentage of people over 65 that are women equals the percentage of people over 65, multiplied by the percentage of people that are female.
 - _____. You can't multiply the unconditional probabilities, because the events are dependent.
- Blindly multiplying unconditional probabilities can give the wrong answer.

SUMMARY

- The *frequency theory* of chance applies most directly to chance processes which can be repeated over and over again, independently and under the same conditions.
- The chance of something says about what percentage of the time it is expected to happen, when the basic process is repeated over and over again.
- Chances are between 0% and 100%. Impossibility is represented by 0%, certainty by 100%.
- If a chance process is sure to turn out in some one of several equally likely ways, and if a thing of interest happens in some of those ways (the “favorable” possibilities) but fails in the other ways (the “unfavorable” possibilities), then the chance that the thing happens equals the ratio

$$\frac{\text{number of favorable possibilities}}{\text{total number of possibilities}}.$$
- The chance of something equals 100% minus the chance of its opposite.
- The chance that two things will both happen equals the (*unconditional*) chance that the first one will happen, multiplied by the *conditional* chance that the second will happen given that the first has happened.
- Two things are *independent* if the chances for the second one stay the same no matter how the first turns out.
- If two things are independent, the chance that both will happen equals the product of their unconditional probabilities.
- The mathematical theory of chance applies in some situations. Using it elsewhere can lead to ridiculous results.