The Analytic Hierarchy Process (AHP) and the Analytic Network Process (ANP) for Decision Making

Decision Making involves setting priorities and the AHP/ANP is the methodology for doing that.

The book MEASUREMENTS, NUMBERS, AND SIZES ENCOUNTERED IN EVERYDAY LIFE (Viking Penguin, 1989) by Mary Blocksma acclaimed at the time by Scientific American as one of the best science books of the year, lists slightly over 100 variables for which there is some kind of measurement, and only a few of these are legitimate for strictly mathematical representation.

Age, Extraordinary (carbon dating); Alcohol; Alcohol, Blood Levels; Annual Percentage Rate; Bar Codes- Barometric Pressure; Binoculars; Blood Pressure; Calcium; Calendar; Cans; Checks, Bank; Cholesterol; Cigarettes; Circles; Clothing Sizes; Comfort Index (Weather); Commercial Items; Compass; Computers;Consumer Price Index (CPI); Copyright Page; Crash Test;Rating Index (CTRI); Currency (Notes); Distance, Nautical; Dow Jones Industrial Averages; Dwellings (Size); Earthquakes; Electricity; Engines (Horsepower);Exponents; Fabric Care; Fabric Widths; Fertilizers; Financial Indexes; Firewood; Food (Energy Value); Food Grading; Gas; Gasoline; Gold; Greenwich Mean Time; Gross National Product (GNP); Hats; Heart Rate (Pulse); Highways; Humidity; Insulation; ISBN Numbers; Land Measures; Latitude and Longitude; Length (Common Short); Light Beer; Light Bulbs; Lumber; The Metric System; Microwave Ovens; Military Time; Motorcycles; Nails; Oil (Engine); Paper; Paper Clips; Pencils; Ph; Pins; Plywood; Points, Mortgage; Postal Rates; Precious Stones; Prefixes, Astronomical; Prefixes for the Minuscule; Prime Rate; Produce; Property (Legal Description); Radio Waves; Rain; Roman Numerals; Rubber Bands; Sandpaper; Screws and Bolts; Ships; Shoes; Snow; Social Security Numbers; Socks; Sodium (Salt); Soil, Garden; Sound; Staples; Steel Wool; Street Addresses; Sunscreen Lotion; Temperature; Tide Tables; Time; Time Units; Time Zones; Tires; Type; Universe (Distances); Vision; Vitamins and Minerals; Volume; Week; Weight; Wind; ZIP Codes.

In his book The Process of cognition, Prentice hall, 1977,Arthur Blumenthal (Blumenthal 1977) writes that there are two types of judgment: "*Comparative judgment* which is the identification of some relation between two stimuli both present to the observer, and *absolute judgment* which involves the relation between a single stimulus and some information held in short term memory about some former comparison stimuli or about some previously experienced measurement scale using which the observer rates the single stimulus."

Henri Lebesgue, who was concerned with questions of measure theory and measurement, wrote in his book Lecons sur l'integration, 2nd ed., Gauthier-Villars, (1928) Paris:

"It would seem that the principle of economy would always require that we evaluate ratios directly and not as ratios of measurements. However, in practice, all lengths are measured in meters, all angles in degrees, etc.; that is we employ auxiliary units and, as it seems, with only the disadvantage of having two measurements to make instead of one. Sometimes, this is because of experimental difficulties or impossibilities that prevent the direct comparison of lengths or angles. But there is also another reason.

In geometrical problems, one needs to compare two lengths, for example, and only those two. It is quite different in practice when one encounters a hundred lengths and may expect to have to compare these lengths two at a time in all possible manners. Thus it is desirable and economical procedure to measure each new length. One single measurement for each length, made as precisely as possible, gives the ratio of the length in question to each other length. This explains the fact that in practice, comparisons are never, or almost never, made directly, but through comparisons with a standard scale."

For a very long time people believed and argued strongly that it is impossible to express the intensity of people's feelings with numbers. The epitome of such a belief was expressed by A.F. MacKay who writes in his book MacKay, A.F. Arrow's Theorem: The Paradox of Social Choice - A Case Study in the Philosophy of Economics. New Haven: Yale University Press, 1980. that pursuing the cardinal approaches is like chasing what cannot be caught. It was also expressed by Davis, P.J. and R. Hersh, "Descartes Dream", Harcourt Brace and Jovanovich, New York, 1986, "If you are more of a human being, you will be aware there are such things as emotions, beliefs, attitudes, dreams, intentions, jealousy, envy, yearning, regret, longing, anger, compassion and many others. These things- the inner world of human life- can never be mathematized."

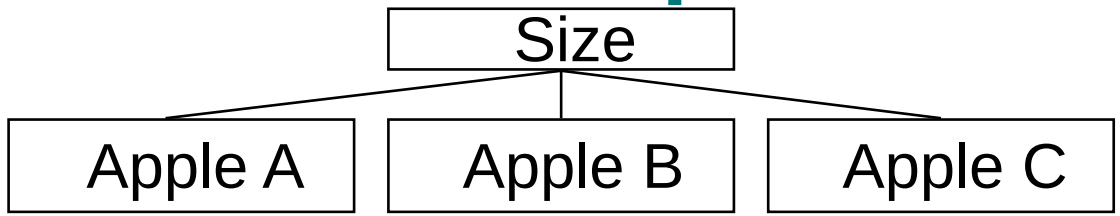
In their book Einstein's space and Van Gogh's sky: Physical reality and beyond, Macmillan, 1982, Lawrence LeShan and Henry Margenau write: We cannot as we have indicated before, quantify the observables in the domain of consciousness. There are no rules of correspondence possible that would enable us to quantify our feelings. We can make statements of the relative intensity of feelings, but we cannot go beyond this. I can say, "I feel angrier at him today than I did yesterday "We cannot, however, make meaningful statements such as, I feel three and one half times angrier than I did yesterday." The physicists' schema, so faithfully emulated by generations of psychologists, epistemologists and aestheticians, is probably blocking their progress, defeating possible insights by its prejudicial force. The schema is not false —it is perfectly reasonable—but it is bootless for the study of mental phenomena.

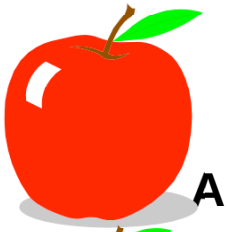
The Nobel Laureate, Henri Bergson in "The Intensity of Psychic States". Chapter 1 in *Time and Free Will: An Essay on the Immediate Data of Consciousness*, translated by F.L. Pogson, M.A. London: George Allen and Unwin (1910): 1-74, writes: But even the opponents of psychophysics do not see any harm in speaking of one sensation as being more intense than another, of one effort as being greater than another, and in thus setting up differences of quantity between purely internal states. Common sense, moreover, has not the slightest hesitation in giving its verdict on this point ; people say they are more or less warm, or more or less sad, and this distinction of more and less, even when it is carried over to the region of subjective facts and unextended objects, surprises nobody.

Social Choice and Individual Values, Kenneth Arrow derives a key result that: It is impossible to formulate a social preference ordering that satisfies all of the following conditions:

Unrestricted Domain: For each state X and Y, based on the social preference ordering, society prefers either state X to Y or Y to X. i.e. society can compare any pair of candidates (completeness).
Unanimity: If everyone in society prefers a to b, then society should prefer a to b.
Non-Dictatorship: Societal preferences cannot be based on the preferences of only one person regardless of the preferences of other agents and of that person.
Transitive Property: If society prefers (based on social rule aggregation of individual preferences) state X to Y and prefers Y to Z then society prefers X to Z.
Independence of Irrelevant Alternatives: If for some X, Y, and Z, X is preferred to Y, then changing the position in the ordering of Z does not affect the relative ordering of X and Y i.e. X is still preferred to Y. In other words, changing the position of Z in the preference ordering should not be allowed to "flip" the social choice between X and Y.
Universality: Any possible individual rankings of alternatives is permissible.

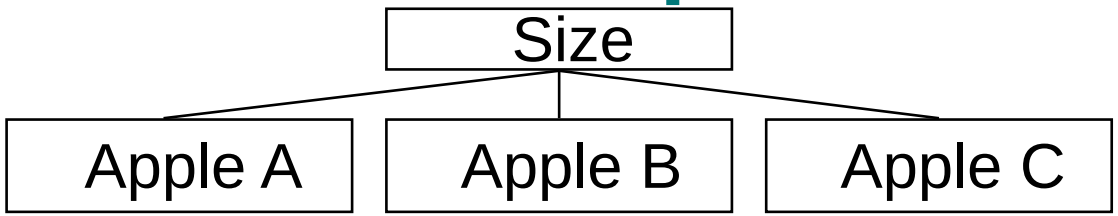
Pairwise Comparisons

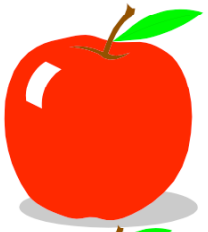


Size Comparison	Apple A 	Apple B 	Apple C 
Apple A 	S_1/S_1	S_1/S_2	S_1/S_3
Apple B 	S_2/S_1	S_2/S_2	S_2/S_3
Apple C 	S_3/S_1	S_3/S_2	S_3/S_3

We Assess The Relative Sizes of the Apples By Forming Ratios

Pairwise Comparisons



Size Comparison	Apple A 	Apple B 	Apple C 	Resulting Priority Eigenvector	Relative Size of Apple
 Apple A	1	2	6	6/10	A
 Apple B	1/2	1	3	3/10	B
 Apple C	1/6	1/3	1	1/10	C

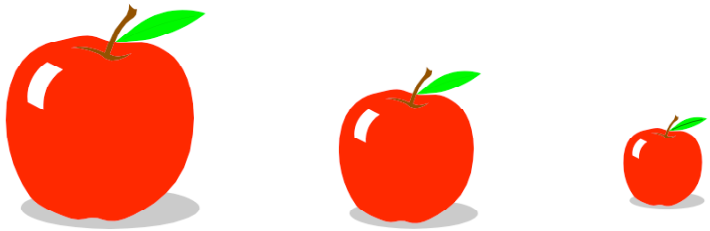
When the judgments are consistent, as they are here, any normalized column gives the priorities.

Consistency (cont.)

- Consistency itself is a necessary condition for a better understanding of relations in the world but it is not sufficient. For example we could judge all three of the apples to be the same size and we would be perfectly consistent, but very wrong.

- We also need to improve our validity by using redundant information.

- It is fortunate that the mind is not programmed to be always consistent. Otherwise, it could not integrate new information by changing old relations.



Pairwise Comparisons using Judgments and the Derived Priorities

Politician comparisons	**B. Clinton** 	**M. Tatcher** 	**G. Bush** 	Normalized	Total
 **B. Clinton**	1	3	7	0.6220	1
 **M. Tatcher**	1/3	1	5	0.2673	0.4297
 **G. Bush**	1/7	1/5	1	0.1107	0.1780

Scales (From Weakest to Strongest)

- **Nominal Scale** invariant under one to one correspondence where a number is assigned to each object; for example, handing out numbers for order of service to people in a queue.
- **Ordinal Scale** invariant under monotone transformations, where things are ordered by number but the magnitudes of the numbers only serve to designate order, increasing or decreasing; for example, assigning two numbers 1 and 2, to two people to indicate that one is taller than the other, without including any information about their actual heights. The smaller number may be assigned to the taller person and vice versa.
- **Interval Scale** invariant under a positive linear transformation; for example, the linear transformation F = (9/5) C + 32 for converting a Celsius to a Fahrenheit temperature reading. Note that one cannot add two readings and on an interval scale because then which is of the form and not of the form . However, one can take an average of such readings because dividing by 2 yields the correct form.
- **Ratio Scale** invariant under a similarity transformation; an example is converting weight measured in pounds to kilograms by using the similarity transformation $K = 2.2\ P$. The ratio of the weights of the two objects is the same regardless of whether the measurements are done in pounds or in kilograms. Zero is not the measurement of anything; it applies to objects that do not have the property and in addition one cannot divide by zero to preserve ratios in a meaningful way. Note that one can add two readings from a ratio scale, but not multiply them because does not have the form . The ratio of two readings from a ratio scale such as 6 kg/ 3 kg = 2 is a number that belongs to an absolute scale that says that the 6 kg object is twice heavier than the 3 kg object. The ratio 2 cannot be changed by some formula to another number. Thus we introduce the next scale.
- **Absolute Scale**: invariant under the identity transformation $x = x$; for example, numbers used in counting the people in a room.

How to go from $$Aw = nw$$

to $$Aw = cw$$

and then to $$Aw = \boxed{}_{max} w$$

Clearly in the first formula n is a simple eigenvalue and all other eigenvalues are equal to zero.

A forcing perurbation of eigenvalues theorem:

If $\boxed{}$ is a simple eigenvalue of A, then for small $\boxed{} > 0$, there is an eigenvalue $\boxed{}(\boxed{})$ of $A(\boxed{})$ with power series expansion in $\boxed{}$:

$$\boxed{}(\boxed{}) = \boxed{} + \boxed{}\ \boxed{}^{(1)} + \boxed{}^2\ \boxed{}^{(2)} + \ldots$$

and corresponding right and left eigenvectors w ($\boxed{}$) and v ($\boxed{}$) such that $$w(\boxed{}) = w + \boxed{}\ w^{(1)} + \boxed{}^2\ w^{(2)} + \ldots$$

$$v(\boxed{}) = v + \boxed{}\ v^{(1)} + \boxed{}^2\ v^{(2)} + \ldots$$

On the Measurement of Inconsistency

A positive reciprocal matrix *A* has $\lambda_{\max} \geq n$ with equality if and only if *A* is consistent. As our measure of deviation of *A* from consistency, we choose the *consistency index*

$$\mu \equiv \frac{\lambda_{\max} - n}{n - 1}.$$

Deriving the Fundamental Scale from First Principles

We obtain the necessary condition on the eigenfunction that it should always satisfy the fundamental functional equation

$$w(as)=bw(s)$$

$$\text{where } b=\boxed{\text{PRIVATE USE}}\,a.$$

This functional equation represents the obvious but powerful fact that response by the brain to a stimulus away from the origin of that stimulus is proportional to response to it at that origin

It took many months to develop the real and complex solutions of this equation by my friend Janos Aczel, the leading mathematician in the world in the field of functional equations. It has profound implications.

The second term on the right is a periodic function of period one.First we use the solution to derive our 1-9 fundamental scale of judgments. The first order terms in its series expansion is the well known Weber – Fechner law of stimulus response.

- The mathematician and cognitive neuropsychologist, Stanislas Dehaene (1997) writes in his book, The Number Sense “Introspection suggests that we can mentally represent the meaning of numbers 1 through 9 with actual acuity. Indeed, these symbols seem equivalent to us. They all seem equally easy to work with, and we feel that we can add or compare any two digits in a small and fixed amount of time like a computer

The Weber-Fechner Law

We take the ratios of the responses M_i to the first non-zero response M_1

$$M_0 = a \log s_0,\ M_1 = a \log \boxed{\text{PRIVATE USE}},$$
$$M_2 = 2a \log \boxed{\text{PRIVATE USE}}, \ldots,\ M_n = na \log \boxed{\text{PRIVATE USE}}.$$

We know that $\mu \geq 0$ and is zero if and only if *A* is consistent. Thus the numerator indicates departure from consistency. The term "*n*-1" in the denominator arises as follows: Since trace (*A*) = *n* is the sum of all the eigenvalues of *A*, if we denote the eigenvalues of *A* that are different from λ_{max} by $\lambda_2, \ldots, \lambda_{n-1}$, we see that ,

$$n = \lambda_{\max} + \sum_{i=2}^{n} \lambda_i$$

so $n - \lambda_{\max} = \sum_{i=2}^{n} \lambda_i$ and $-\mu = \frac{1}{n-1} \sum_{i=2}^{n} \lambda_i$ is the average of the non- principal eigenvalues of *A*.

The Continuous Case

$$K(s,t)\,K(t,s) = 1$$

$$K(s,t)\,K(t,u) = K(s,u),$$

for all s, t, and u

$$K(s,t) = k_1(s)\, k_2(t)$$

$$K(s,t) = k(s)/k(t)$$

$$K(as, at)=aK(s,t)=k(as)/k(at)$$
$$=a\ k(s)/k(t)$$

$$w(as)=bw(s)$$

where $b=$  a.

$$v(u)=C_1\, e^{-\frac{W}{\Omega}u}\, P(u)$$

The periodic function is bounded and the negative exponential gives rise to an alternating series. Thus, to a first order approximation this leads to the Weber-Fechner law:

The Weber-Fechner law: Deriving the Scale 1-9

$M_0 = a \log s_0$, $M_1 = a \log$ [PRIVATE USE E000 W F8FF], $M_2 = 2a \log$ [PRIVATE USE E000 W F8FF],... , $M_n = na \log$ [PRIVATE USE E000 W F8FF].

$M_i / M_1 = i$

Verbal Expressions for Making Pairwise Comparison Judgments

Equal importance

Moderate importance of one over another

Strong or essential importance

Very strong or demonstrated importance

Extreme importance

Total Dominance

- We have the dominance from the matrix itself by adding its rows and normalizing them. It is a vector each of whose entries is a priority number.
- We have the dominance from the square of the matrix to get dominance in two steps. Similarly for the cube and so on.
- This gives an infinite number of vectors of domincnce.
- Their average is the same as we get by raising the matrix to large powers adding its rows and normalizing them.
- This result coincides with the principal eigenvector obtained in the previous way.

Fundamental Scale of Absolute Numbers Corresponding to Verbal Comparisons

1Equal importance

3Moderate importance of one over another

5Strong or essential importance

7Very strong or demonstrated importance

9Extreme importance

2,4,6,8 Intermediate values

Use Reciprocals for Inverse Comparisons

Which Drink is Consumed More in the U.S.?

An Example of Estimation Using Judgments

Drink Consumption in the U.S.	Coffee	Wine	Tea	Beer	Sodas	Milk	Water
Coffee	1	9	5	2	1	1	1/2
Wine	1/9	1	1/3	1/9	1/9	1/9	1/9
Tea	1/5	2	1	1/3	1/4	1/3	1/9
Beer	1/2	9	3	1	1/2	1	1/3
Sodas	1	9	4	2	1	2	1/2
Milk	1	9	3	1	1/2	1	1/3
Water	2	9	9	3	2	3	1

The derived scale based on the judgments in the matrix is:

Coffee	Wine	Tea	Beer	Sodas	Milk	Water
.177	.019	.042	.116	.190	.129	.327

with a consistency ratio of .022.

The actual consumption (from statistical sources) is:

.180	.010	.040	.120	.180	.140	.330

Estimating which Food has more Protein

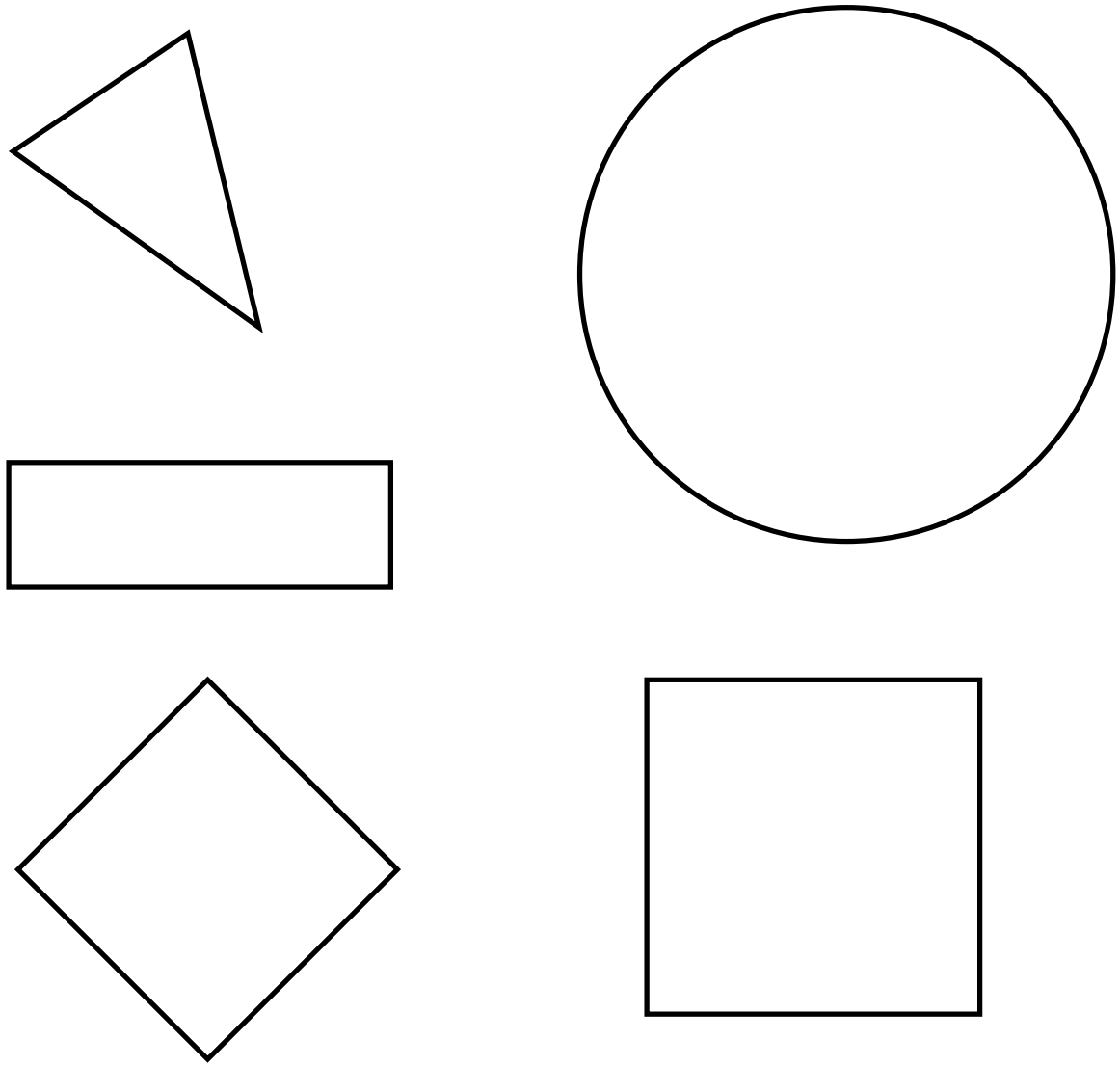
Food Consumption in the U.S.	**A**	**B**	**C**	**D**	**E**	**F**	**G**
A: Steak	1	9	9	6	4	5	1
B: Potatoes		1	1	1/2	1/4	1/3	1/4
C: Apples			1	1/3	1/3	1/5	1/9
D: Soybean				1	1/2	1	1/6
E: Whole Wheat Bread					1	3	1/3
F: Tasty Cake						1	1/5
G: Fish							1

(Reciprocals)

The resulting derived scale and the actual values are shown below:

	Steak	Potatoes	Apples	Soybean	W. Bread	T. Cake	Fish
Derived	.345	.031	.030	.065	.124	.078	.328
Actual	.370	.040	.000	.070	.110	.090	.320

(Derived scale has a consistency ratio of .028.)



WEIGHT COMPARISONS

Weight	Radio	Typewriter	Large Attache Case	Projector	Small Attache	Eigenvector	Actual Relative Weights
Radio	1	1/5	1/3	1/4	4	0.09	0.10
Typewriter	5	1	2	2	8	0.40	0.39
Large Attache Case	3	1/2	1	1/2	4	0.18	0.20
Projector	4	1/2	2	1	7	0.29	0.27
Small Attache Case	1/4	1/8	1/4	1/7	1	0.04	0.04

Nonlinearity of the Priorities

RELATIVE VISUAL BRIGHTNESS-I

	C_1	C_2	C_3	C_4
C_1	1	5	6	7
C_2	1/5	1	4	6
C_3	1/6	1/4	1	4
C_4	1/7	1/6	1/4	1

RELATIVE VISUAL BRIGHTNESS -II

	C_1	C_2	C_3	C_4
C_1	1	4	6	7
C_2	1/4	1	3	4
C_3	1/6	1/3	1	2
C_4	1/7	1/4	1/2	1

RELATIVE BRIGHTNESS EIGENVECTOR
The Inverse Square Law of Optics

	I	II
C_1	.62	.63
C_2	.23	.22
C_3	.10	.09
C_4	.05	.06

Distance	Normalized distance	Square of normalized distance	Reciprocal of previous column	Normalized reciprocal
9	0.123	0.015	67	0.61
15	0.205	0.042	24	0.22
21	0.288	0.083	12	0.11
28	0.384	0.148	7	0.06

Relative Electricity Consumption (Kilowatt Hours) of Household Appliances

Annual Electric Consumption	*Elec. Range*	*Refrig*	*TV*	*Dish Wash*	*Iron*	*Radio*	*Hair Dryer*	*Eigen-vector*	*Actual Relative Weights*
Electric Range	1	2	5	8	7	9	9	.393	.392
Refrig-erator	1/2	1	4	5	5	7	9	.261	.242
TV	1/5	1/4	1	2	5	6	8	.131	.167
Dish-washer	1/8	1/5	1/2	1	4	9	9	.110	.120
Iron	1/7	1/5	1/5	1/4	1	5	9	.061	.047
Radio	1/9	1/7	1/6	1/9	1/5	1	5	.028	.028
Hair-dryer	1/9	1/9	1/8	1/9	1/9	1/5	1	.016	.003

Relative coin sizes

.

	Dime	*Quarter*	*5Cents*	*Priorities*	*Size in mm^2*	*Actual relative size*
Cent	1.1	2	1.5	0.182	283.38	0.212
Dime		2.1	1.6	0.171	254.34	0.190
Quarter			1.6	0.382	452.16	0.338
5Cent				0.263	346.18	0.259

Percentage of Individuals that use different search engines	Google	Yahoo	MSN	AOL	My Web	Priorities	Actual Percentage
Google	1.000	1.811	7.057	8.491	9.000	0.463	0.485
Yahoo	0.552	1.000	6.566	8.073	9.000	0.367	0.225
MSN	0.142	0.152	1.000	4.264	4.076	0.101	0.107
AOL	0.118	0.124	0.235	1.000	2.943	0.048	0.066
My Web	0.111	0.111	0.245	0.340	1.000	0.030	0.027

Relative Distances from Pittsburgh

Distance From Pittsburgh	L.A.	New Orleans	St. Louis	Washington D.C.	Priorities	Actual Distance In Miles	Relative Values
L.A.	**1**	**3**	**5**	**7**	**.558**	**2446**	**.556**
New Orleans	**1/3**	**1**	**3**	**6**	**.267**	**1049**	**.249**
St. Louis	**1/5**	**1/3**	**1**	**4**	**.125**	**610**	**.139**
Washington D.C.	**1/7**	**1/6**	**1/4**	**1**	**.049**	**247**	**.056**

Model	Mercedez - E	BMW - 5	Acura - TL	Lexus - ES	Audi - A6	Priorities	Actual Cost	Relative Values
Mercedez - E	1	1	1.6	1.3	1.25	0.237128	52000	0.245283019
BMW - 5		1	1.5	1.25	1.2	0.230349	48000	0.226415094
Acura - TL			1	1.3	2	0.151647	30000	0.141509434
Lexus - ES				1	1	0.187402	40000	0.188679245
Audi - A6					1	0.193475	42000	0.198113208

Esther Judgments

	US	Japan	Germany	UK	France	Italy	Canada	Russia	Results	GDP Data
US	1	3	5	6	6	7	9	9	**0.4059**	**0.4425**
Japan		1	3	4	4	5	6	7	**0.2271**	**0.1573**
Germany			1	2	2	3	4	5	**0.1143**	**0.1040**
UK				1	1	3	4	5	**0.0846**	**0.0678**
France					1	2	3	4	**0.0732**	**0.0830**
Italy						1	1	2	**0.0392**	**0.0657**
Canada							1	1	**0.0302**	**0.0415**
Russia								1	**0.0253**	**0.0381**
								SUM	*0.9998*	
								Inconsistency	**.0334**	
								Inconsistency Index	**1.0879**	

Gershon Judgments

	US	Japan	Germany	UK	France	Italy	Canada	Russia	Results	GDP Data
US	1	3	5	5	6	7	9	9	**0.4203**	**0.4425**
Japan		1	2	2	2	5	2	6	**0.1710**	**0.1573**
Germany			1	1.2	1	2	3	4	**0.1007**	**0.1040**
UK				1	0.833333	1.8	3	4	**0.0933**	**0.0678**
France					1	1.4	1.2	3	**0.0810**	**0.0830**
Italy						1	2	3	**0.0592**	**0.0657**
Canada							1	2	**0.0478**	**0.0415**
Russia								1	**0.0267**	**0.0381**
									1	
								Inconsistency	**.0271**	
	*Actual judgment for (UK,France) is inverse: 1/1.2=.833						**Inconsistency Index**		**1.0346**	

GDP DATA for the Eight Countries

	Actual GDP	Fraction of *Total*	Gershon	Esther
1US	$14,256,000,000,000	0.442	0.4203	0.4059
2Japan	$5,068,000,000,000	0.157	0.171	0.2271
3Germany	$3,352,000,000,000	0.104	0.1006	0.1143
4UK	$2,183,000,000,000	0.068	0.0933	0.0846
5France	$2,675,000,000,000	0.083	0.081	0.0732
6Italy	$2,118,000,000,000	0.066	0.0592	0.0391
7Canada	$1,336,000,000,000	0.041	0.0478	0.0302
8Russia	$1,229,000,000,000	0.038	0.0267	0.0253
Total	*$32,217,000,000,000*		*Inconsistency .0271*	*0.0334*
		Compatibility Index 1.0346		***1.0879***

	Population	$/Person
1US	310,232,863	$45,953
2Japan	126,804,433	$39,967
3Germany	82,282,988	$40,737
4UK	62,348,447	$35,013
5France	64,768,389	$41,301
6Italy	58,090,681	$36,460
7Canada	33,759,742	$39,574
8Russia	139,390,205	$8,817

	Population	$/Person
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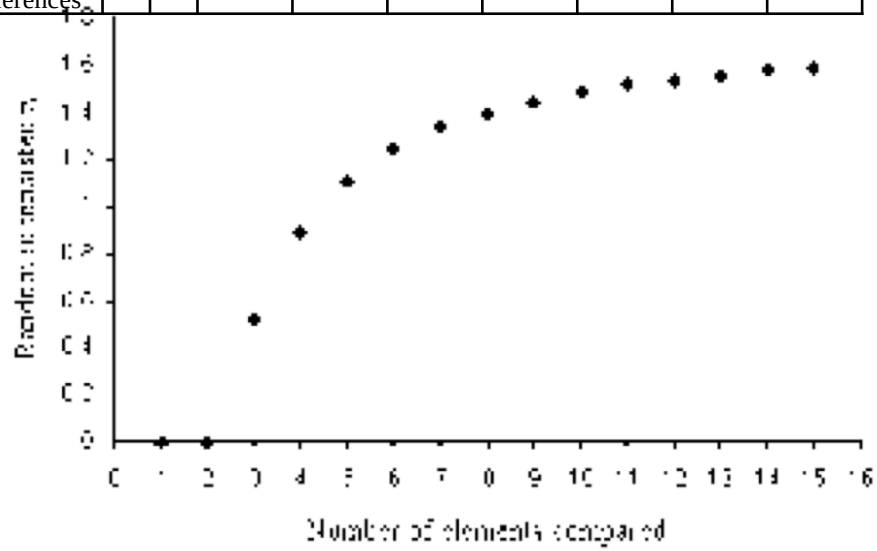
3Germany	82,282,988	$40,737
4UK	62,348,447	$35,013
5France	64,768,389	$41,301
6Italy	58,090,681	$36,460
7Canada	33,759,742	$39,574
8Russia	139,390,205	$8,817

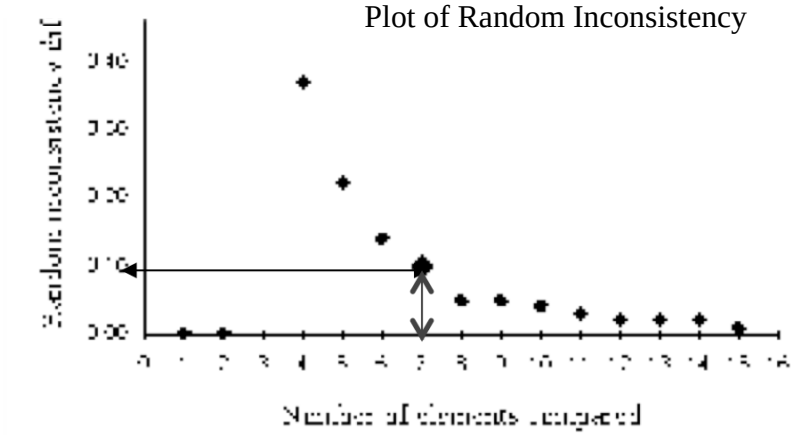
Comparing No More Than About Seven Elements

Table Random index

Order	1	2	3	4	5	6	7	8	9
R.I.	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45
First Order Differences		0	0.52	0.37	0.22	0.14	0.10	0.05	0.05



Plot of Random Inconsistency



Plot of First Differences in Random Inconsistency

.07

Unripe Cherry Tomato

.28

Small Green Tomato

.65

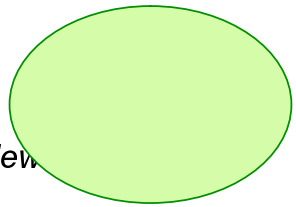
Lime

.08

Lime

.22

Grapefruit



.70

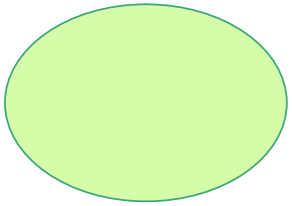
Honeydew

$= 1$

$\frac{.22}{.08} = 2.75$

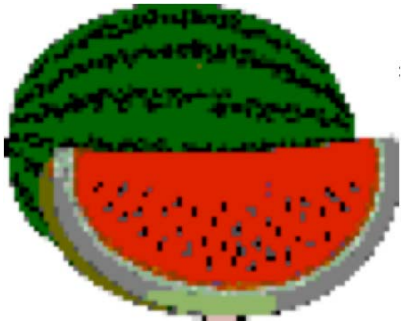
$\frac{.70}{\ } =$

$1x.65 = .65$



.10

Honeydew

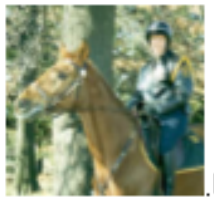
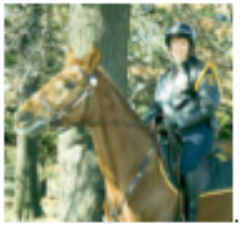


Sugar Baby Watermelon

Oblong Watermelon

This means that 34.14/.07 = 487.7 unripe cherry tomatoes are equal to the oblong watermelon

Comparing a Dog-Catcher w/ President

 .07 *Dog Catcher*	 .28 *Traffic Cop*	 .65 *Patrol Cop*
 .08 *Patrol Cop* $\frac{.08}{.08} = 1$ .65H1=.65	 .22 *Detective* $\frac{.22}{.08} = 2.75$ .65H2.75=1.79	 .70 *Chief of Police* $\frac{.70}{.08} = 8.75$ .65H8.75=5.69
 .10 *Chief of Police* $\frac{.10}{.10} = 1$ 5.69H1=5.69	 .30 *Mayor Council Member* $\frac{.30}{.10} = 3$ 5.69H3=17.07	 .60 *Mayor* $\frac{.60}{.10} = 6$ 5.69H6=34.14

Comparing a Dog-Catcher w/ President

.20

Mayor

$$\frac{.20}{.20} = 1$$

34.14H1=34.14

.35

Governor

$$\frac{.35}{.20} = 1.75$$

34.14H1.75=59.74

.45

Senator

$$\frac{.45}{.20} = 2.25$$

34.14H2.25=76.81

.15

Senator

$$\frac{.15}{.15} = 1$$

76.81H1=5.69

.35

Statesman

$$\frac{.35}{.15} = 2.33$$

76.81H2.33=177.5

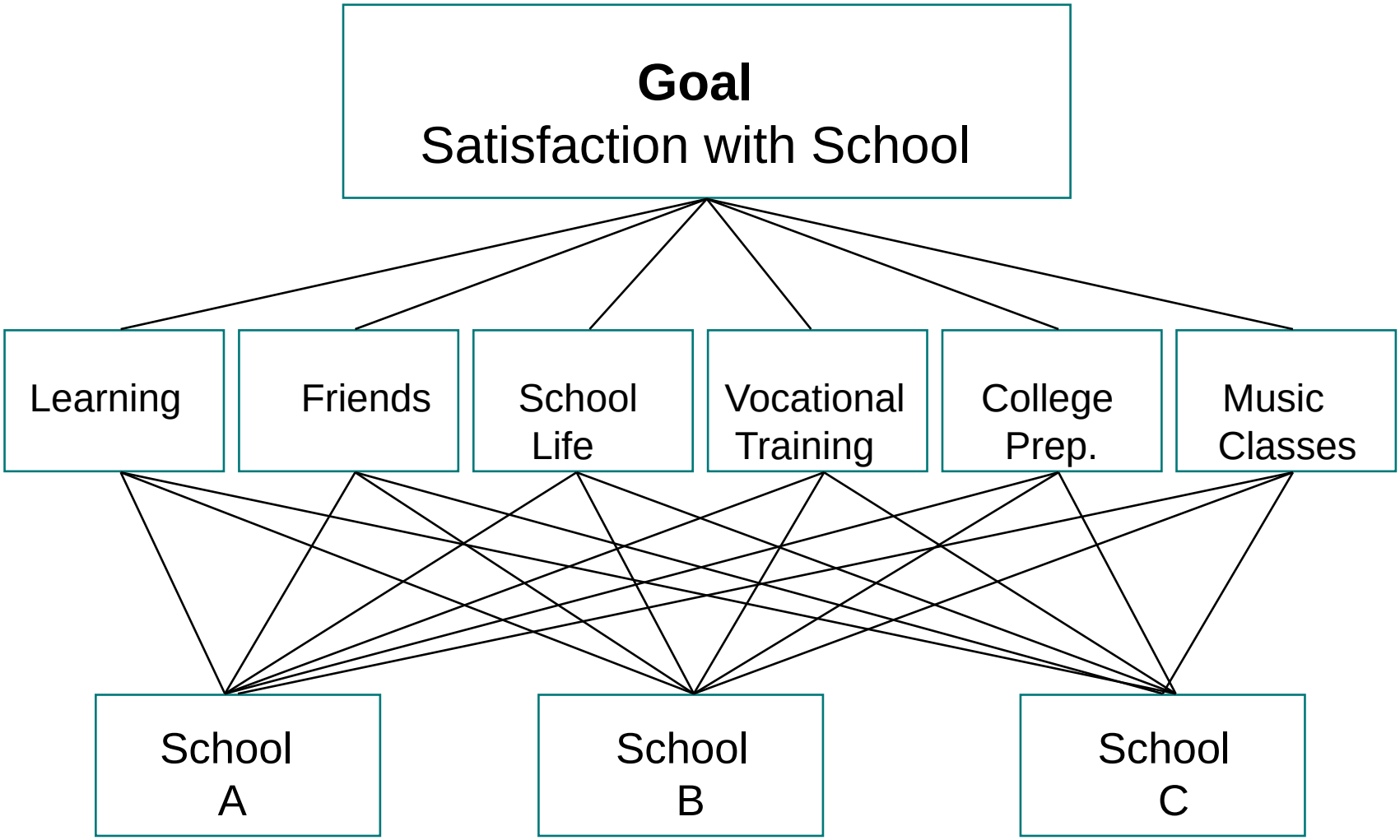
.50

President

$$\frac{.50}{.15} = 3.33$$

76.81H3.33=253.68

Figure 1: Comparisons According to Importance


Goal
Satisfaction with School
Learning
Friends
School Life
Vocational Training
College Prep.
Music Classes
School A
School B
School C

School Selection

	L	F	SL	VT	CP	MC	Weights
Learning	1	4	3	1	3	4	.32
Friends	1/4	1	7	3	1/5	1	.14
School Life	1/3	1/7	1	1/5	1/5	1/6	.03
Vocational Trng.	1	1/3	5	1	1	1/3	.13
College Prep.	1/3	5	5	1	1	3	.24
Music Classes	1/4	1	6	3	1/3	1	.14

Comparison of Schools with Respect to the Six Characteristics

	Learning A	B	C	Priorities
A	1	1/3	1/2	.16
B	3	1	3	.59
C	2	1/3	1	.25

	Friends A	B	C	Priorities
A	1	1	1	.33
B	1	1	1	.33
C	1	1	1	.33

	School Life A	B	C	Priorities
A	1	5	1	.45
B	1/5	1	1/5	.09
C	1	5	1	.46

	Vocational Trng. A	B	C	Priorities
A	1	9	7	.77
B	1/9	1	1/5	.05
C	1/7	5	1	.17

	College Prep. A	B	C	Priorities
A	1	1/2	1	.25
B	2	1	2	.50
C	1	1/2	1	.25

	Music Classes A	B	C	Priorities
A	1	6	4	.69
B	1/6	1	1/3	.09
C	1/4	3	1	.22

Composition and Synthesis

Impacts of School on Criteria

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	**Composite Impact of Schools**
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

The School Example Revisited Composition & Synthesis:

Impacts of Schools on Criteria

Distributive Mode

(Normalization: Dividing each entry by the total in its column)

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	Composite Impact of Schools
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

The Distributive mode is useful when the uniqueness of an alternative affects its rank. The number of copies of each alternative also affects the share each receives in allocating a resource. In planning, the scenarios considered must be comprehensive and hence their priorities depend on how many there are. This mode is essential for ranking criteria and sub-criteria, and when there is dependence.

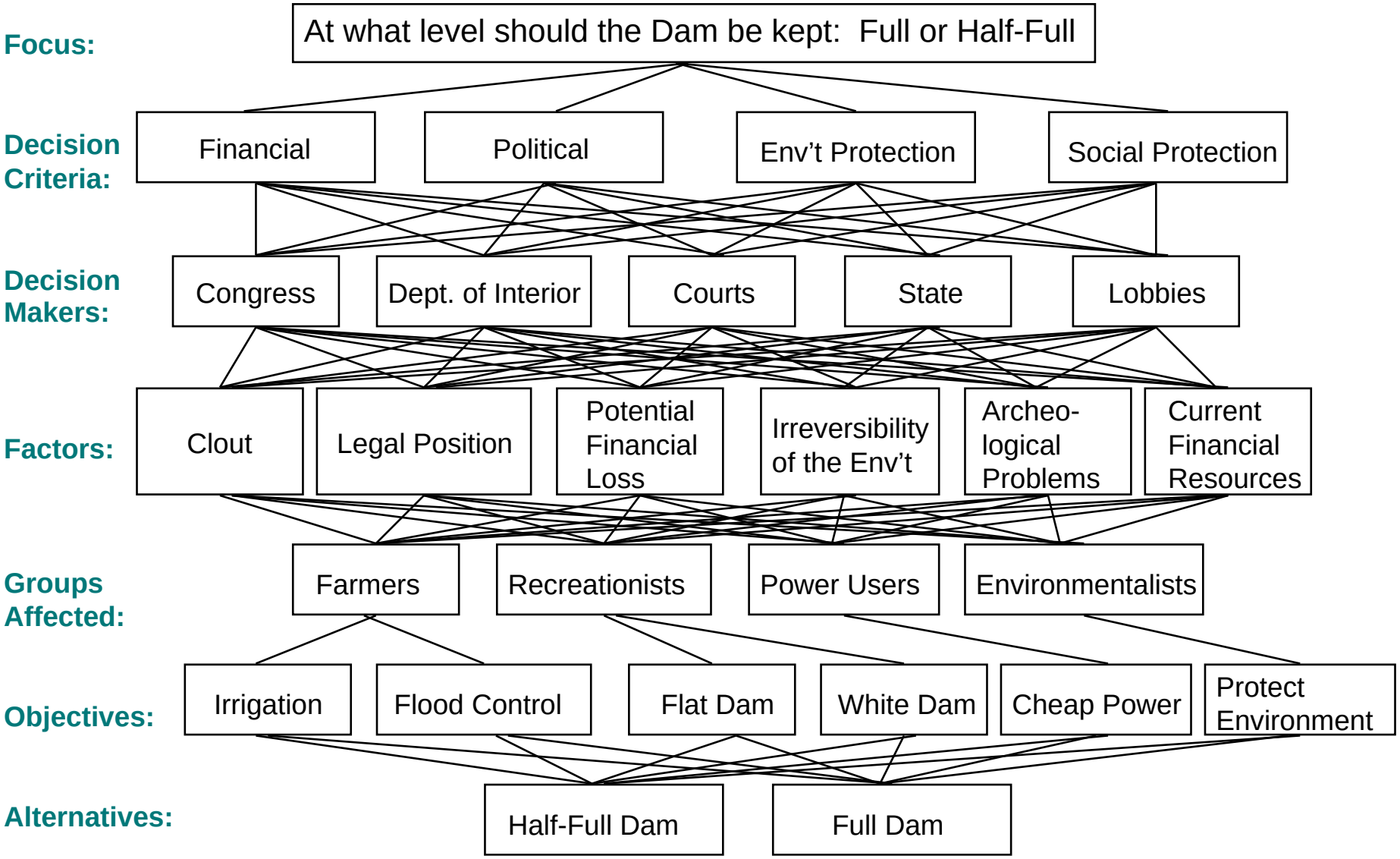
Ideal Mode

(Dividing each entry by the maximum value in its column)

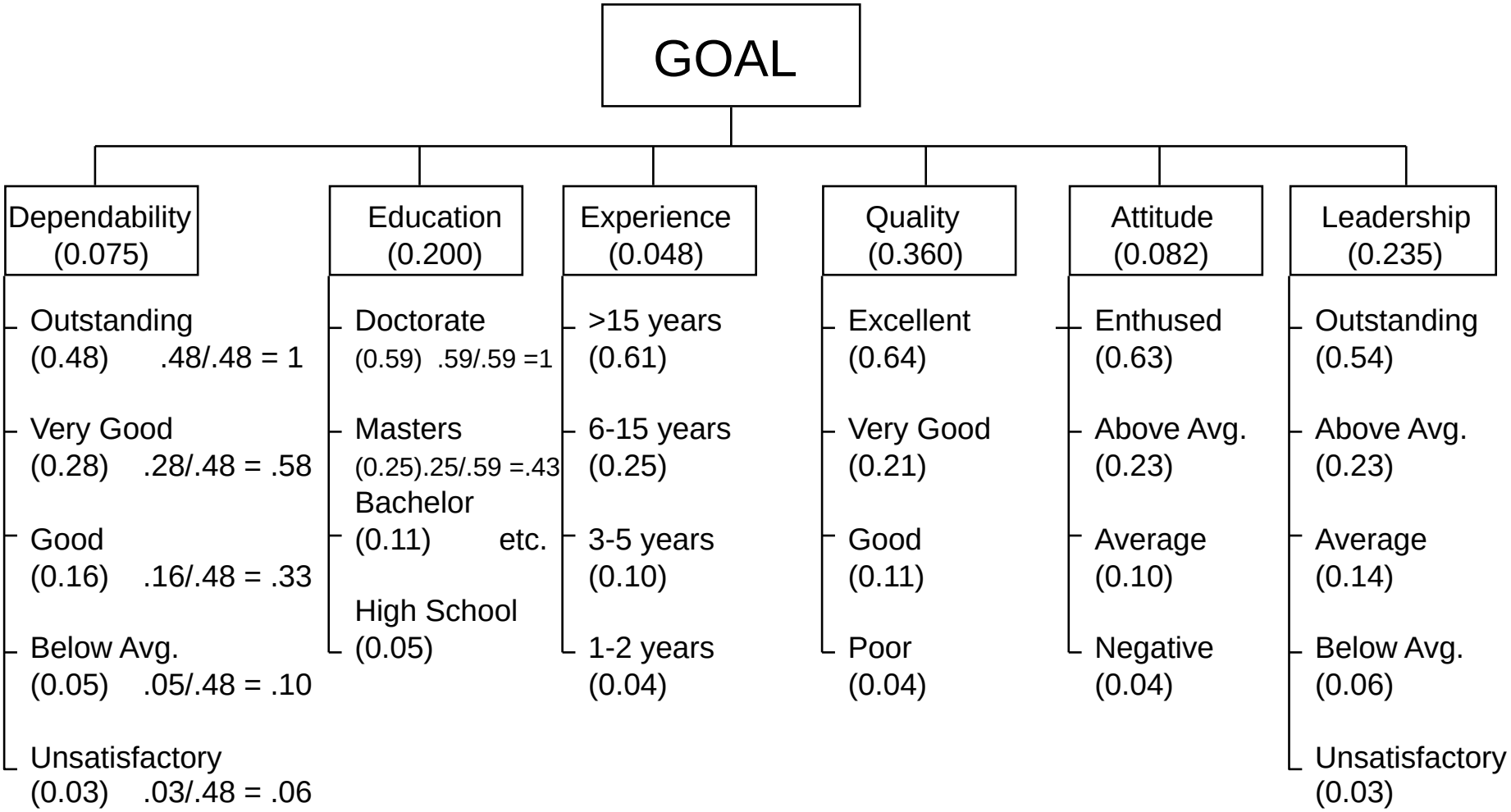
	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	Composite Impact of Schools	Normal-ized
A	.27	1	.98	1	.50	1	.65	.34
B	1	1	.20	.07	.50	.13	.73	.39
C	.42	1	1	.22	.50	.32	.50	.27

The Ideal mode is useful in choosing a best alternative regardless of how many other similar alternatives there are.

A Complete Hierarchy to Level of Objectives



Evaluating Employees for Raises



Final Step in Absolute Measurement

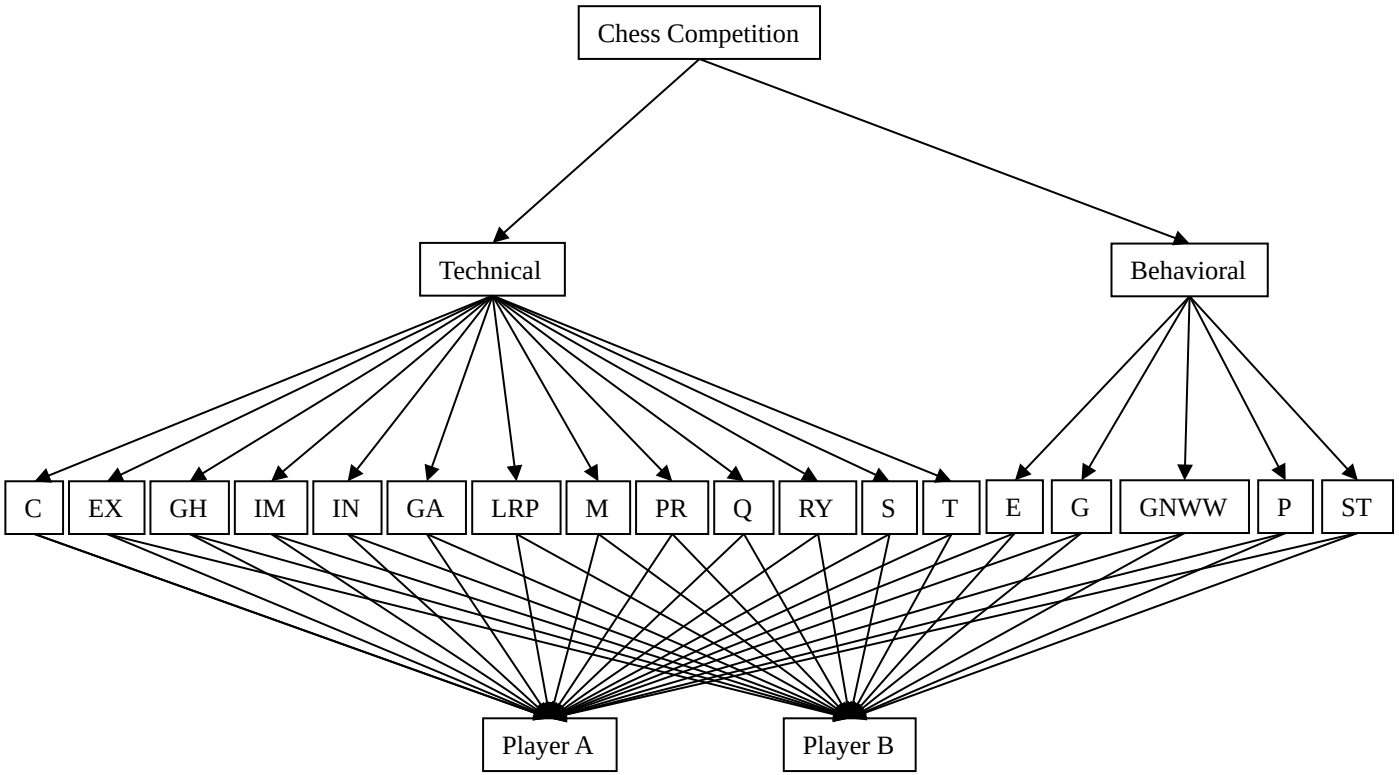
Rate each employee for dependability, education, experience, quality of work, attitude toward job, and leadership abilities.

	Dependability 0.0746	Education 0.2004	Experience 0.0482	Quality 0.3604	Attitude 0.0816	Leadership 0.2348	Total	Normalized
Esselman, T.	Outstand	Doctorate	>15 years	Excellent	Enthused	Outstand	1.000	0.153
Peters, T.	Outstand	Masters	>15 years	Excellent	Enthused	Abv. Avg.	0.752	0.115
Hayat, F.	Outstand	Masters	>15 years	V. Good	Enthused	Outstand	0.641	0.098
Becker, L.	Outstand	Bachelor	6-15 years	Excellent	Abv. Avg.	Average	0.580	0.089
Adams, V.	Good	Bachelor	1-2 years	Excellent	Enthused	Average	0.564	0.086
Kelly, S.	Good	Bachelor	3-5 years	Excellent	Average	Average	0.517	0.079
Joseph, M.	Blw Avg.	Hi School	3-5 years	Excellent	Average	Average	0.467	0.071
Tobias, K.	Outstand	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.466	0.071
Washington, S.	V. Good	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.435	0.066
O'Shea, K.	Outstand	Hi School	>15 years	V. Good	Enthused	Average	0.397	0.061
Williams, E.	Outstand	Masters	1-2 years	V. Good	Abv. Avg.	Average	0.368	0.056
Golden, B.	V. Good	Bachelor	.15 years	V. Good	Average	Abv. Avg.	0.354	0.054

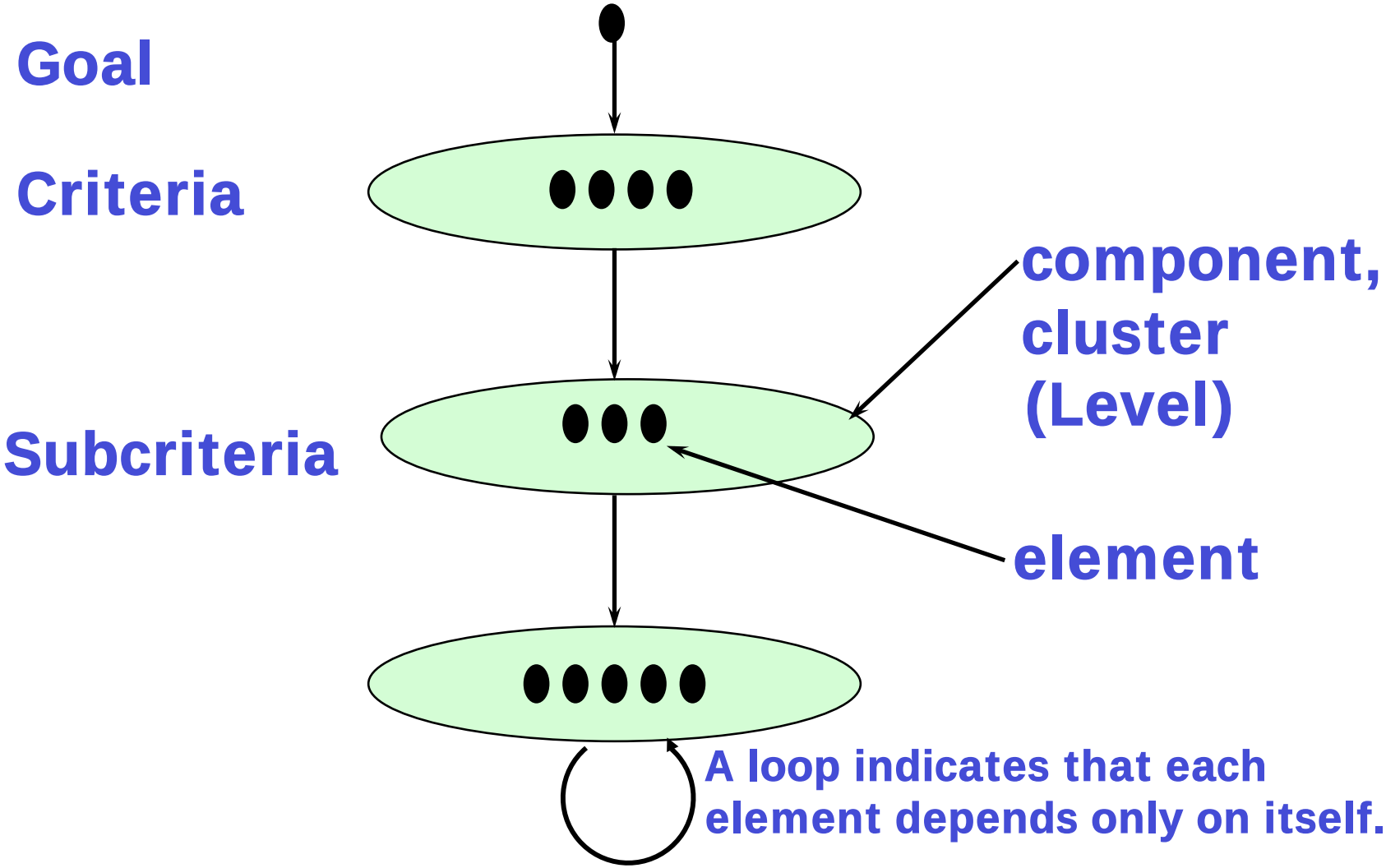
The total score is the sum of the weighted scores of the ratings. The money for raises is allocated according to the normalized total score. In practice different jobs need different hierarchies.

Chess Factors

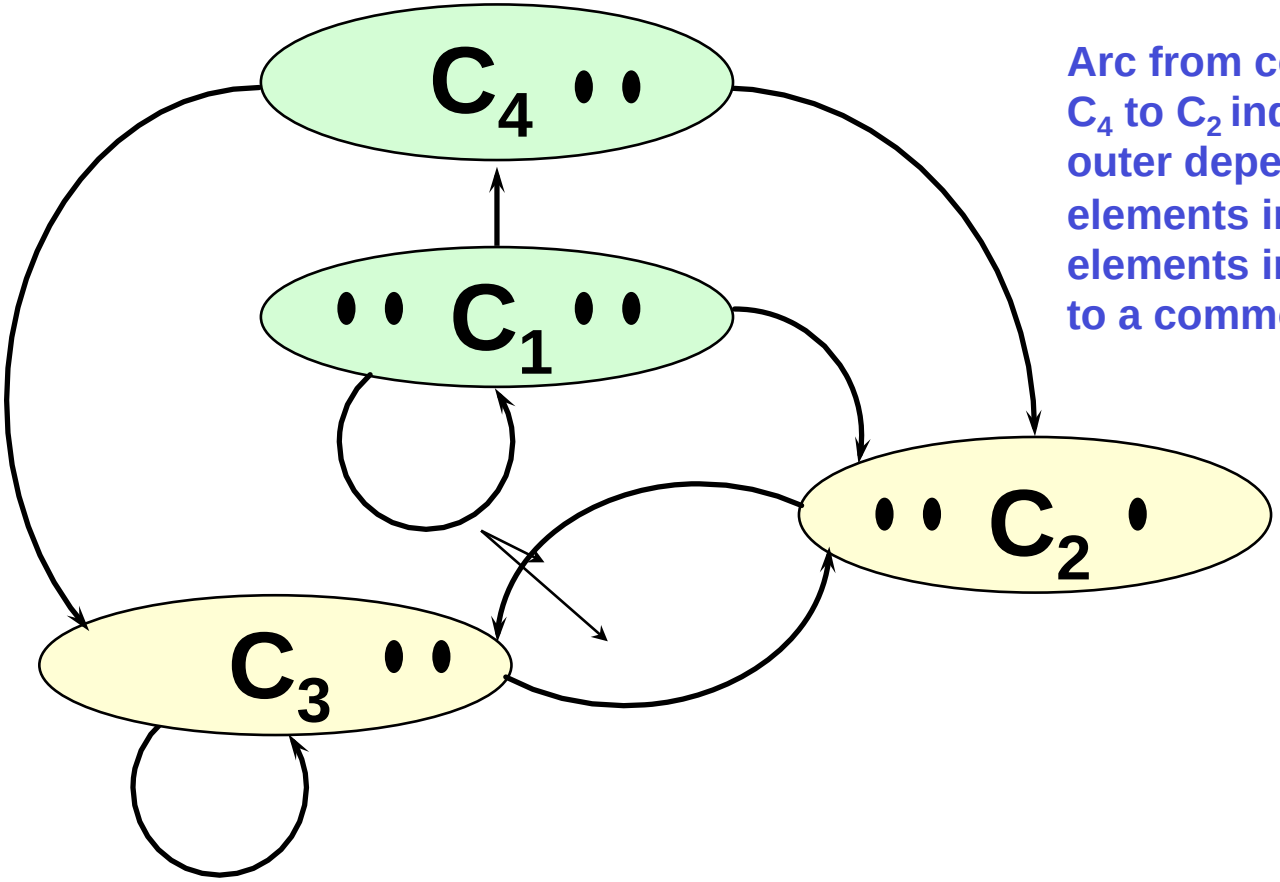
- T (1) Calculation (Q): The ability of a player to evaluate different alternatives or strategies in light of prevailing situations.
- B (2) Ego (E): The image a player has of himself as to his general abilities and qualification and his desire to win.
- T (3) Experience (EX): A composite of the versatility of opponents faced before, the strength of the tournaments participated in, and the time of exposure to a rich variety of chess players.
- B (4) Gamesmanship (G): The capability of a player to influence his opponent's game by destroying his concentration and self-confidence.
- T (5) Good Health (GH): Physical and mental strength to withstand pressure and provide endurance.
- B (6) Good Nerves and Will to Win (GN): The attitude of steadfastness that ensures a player's health perspective while the going gets tough. He keeps in mind that the situation involves two people and that if he holds out the tide may go in his favor.
- T (7) Imagination (IM): Ability to perceive and improvise good tactics and strategies.
- T (8) Intuition (IN): Ability to guess the opponent's intentions.
- T (9) Game Aggressiveness (GA): The ability to exploit the opponent's weaknesses and mistakes to one's advantage. Occasionally referred to as "killer instinct."
- T (10) Long Range Planning (LRP): The ability of a player to foresee the outcome of a certain move, set up desired situations that are more favorable, and work to alter the outcome.
- T (11) Memory (M): Ability to remember previous games.
- B (12) Personality (P): Manners and emotional strength, and their effects on the opponent in playing the game and on the player in keeping his wits.
- T (13) Preparation (PR): Study and review of previous games and ideas.
- T (14) Quickness (Q): The ability of a player to see clearly the heart of a complex problem.
- T (15) Relative Youth (RY): The vigor, aggressiveness, and daring to try new ideas and situations, a quality usually attributed to young age.
- T (16) Seconds (S): The ability of other experts to help one to analyze strategies between games.
- B (17) Stamina (ST): Physical and psychological ability of a player to endure fatigue and pressure.
- T (18) Technique (M): Ability to use and respond to different openings, improvise middle game tactics, and steer the game to a familiar ground to one's advantage.


Chess Competition
Technical
Behavioral
C
EX
GH
IM
IN
GA
LRP
M
PR
Q
RY
S
T
E
G
GNWW
P
ST
Player A
Player B

Linear Hierarchy



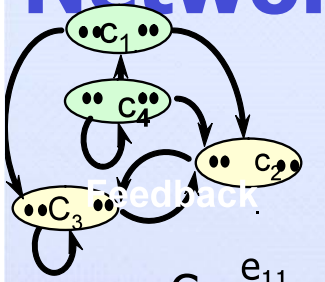
Feedback Network with components having Inner and Outer Dependence among Their Elements



Arc from component C_4 to C_2 indicates the outer dependence of the elements in C_2 on the elements in C_4 with respect to a common property.

Loop in a component indicates inner dependence of the elements in that component with respect to a common property.

Networks and the Supermatrix



$$
W = \begin{array}{c|c}
 & \begin{array}{ccccccccc} C_1 & & & & C_2 & & \cdots & C_N \\ e_{11} e_{12} & \cdots & e_{1n_1} & & e_{21} e_{22} & \cdots & e_{2n_2} & & e_{N1} e_{N2} \cdots e_{Nn_N} \end{array} \\
\begin{array}{c} C_1 \\ \\ \\ C_2 \\ \vdots \\ C_N \end{array} & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \\ e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \\ e_{N1} \\ e_{N2} \\ \vdots \\ e_{Nn_N} \end{array}
\begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix}
\end{array}
$$

where

$$w_{ij} = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_2)} & \cdots & W_{i1}^{(jn_j)} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_2)} & \cdots & W_{i2}^{(jn_j)} \\ \vdots & \vdots & & \vdots \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_2)} & \cdots & W_{in_i}^{(jn_j)} \end{bmatrix}$$

Supermatrix of a Hierarchy

$$
W = \begin{array}{c|c}
 & \begin{array}{cccccc} C_1 & C_2 & \cdots & C_{N-2} & C_{N-1} & C_N \end{array} \\
 & \begin{array}{cccccc} e_{11} \cdots e_{1n_1} & e_{21} \cdots e_{2n_2} & \cdots & e_{(N-2)1} \cdots e_{(N-2)\,n_{N-2}} & e_{(N-1)1} \cdots e_{(N-1)\,n_{N-1}} & e_{N1} \cdots e_{Nn_N} \end{array} \\
\begin{array}{c} C_1 \\ C_2 \\ \vdots \\ C_N \end{array} & \begin{bmatrix}
0 & 0 & \cdots & 0 & 0 & 0 \\
W_{21} & 0 & \cdots & 0 & 0 & 0 \\
0 & W_{32} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & W_{n-1,\,n-2} & 0 & 0 \\
0 & 0 & \cdots & 0 & W_{n,\,n-1} & I
\end{bmatrix}
\end{array}
$$

$$W^k = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ W_{n,n-1} & W_{n-1,n-2} & \cdots & W_{32} & W_{21} & W_{n,n-1} & W_{n-1,n-2} & \cdots & W_{32} & W_{n,n-1} & W_{n-1,n-2} & W_{n,n-1} & I \end{bmatrix}$$

for $k>n-1$

The School Hierarchy as Supermatrix

	Goal	Learning	Friends	School life	Vocational training	College preparation	Music classes	A	B	C
Goal	0	0	0	0	0	0	0	0	0	0
Learning	0.32	0	0	0	0	0	0	0	0	0
Friends	0.14	0	0	0	0	0	0	0	0	0
School life	0.03	0	0	0	0	0	0	0	0	0
Vocational training	0.13	0	0	0	0	0	0	0	0	0
College preparation	0.24	0	0	0	0	0	0	0	0	0
Music classes	0.14	0	0	0	0	0	0	0	0	0
Alternative A	0	0.16	0.33	0.45	0.77	0.25	0.69	1	0	0
Alternative B	0	0.59	0.33	0.09	0.06	0.5	0.09	0	1	0
Alternative C	0	0.25	0.34	0.46	0.17	0.25	0.22	0	0	1

Limiting Supermatrix & Hierarchic Composition

	Goal	Learning	Friends	School life	Vocational training	College preparation	Music classes	A	B	C
Goal	0	0	0	0	0	0	0	0	0	0
Learning	0	0	0	0	0	0	0	0	0	0
Friends	0	0	0	0	0	0	0	0	0	0
School life	0	0	0	0	0	0	0	0	0	0
Vocational training	0	0	0	0	0	0	0	0	0	0
College preparation	0	0	0	0	0	0	0	0	0	0
Music classes	0	0	0	0	0	0	0	0	0	0
Alternative A	0.3676	0.16	0.33	0.45	0.77	0.25	0.69	1	0	0
Alternative B	0.3781	0.59	0.33	0.09	0.06	0.5	0.09	0	1	0
Alternative C	0.2543	0.25	0.34	0.46	0.17	0.25	0.22	0	0	1

Date and Strength of Recovery of U.S. Economy

The U.S. Holarchy of Factors for Forecasting Turnaround in Economic Stagnation

Table 1: Matrices for subfactor importance relative to primary factors influencing the Timing of Recovery

Panel A: Which subfactor has the greater potential to influence Conventional Adjustment and how strongly?

		C	E	I	K	F	M	Vector Weights
Consumption	(C)	1	♪7	♪5	♪1/5	♪1/2	♪1/5	♪0.118♪
Exports	(E)	1/7	♪1	♪1/5	♪1/5	♪1/5	♪1/7	♪0.029♪
Investment	(I)	1/5	♪5	♪1	♪1/5	♪1/3	♪1/5	♪0.058♪
Confidence	(K)	5	♪5	♪5	♪1	♪5	♪1	♪0.334♪
Fiscal Policy	(F)	2	♪5	♪3	♪1/5	♪1	♪1/5	♪0.118♪
Monetary Policy	(M)	5	♪7	♪5	♪1	♪5	♪1	♪0.343♪

Panel B: Which subfactor has the greater potential to influence Economic Restructuring and how strongly?

		FS		DP		GC		Vector Weights
Financial Sector	(FS)	1	♪	♪3	♪	♪3	♪	♪0.584♪
Defense Posture	(DS)	♪ 1/3	♪	♪1	♪	♪3	♪	♪0.281♪
Global Competition	(GC)	♪ 1/3	♪	♪1/3	♪	♪1	♪	♪0.135♪

Table 2: Matrices for relative influence of subfactors on periods of adjustment (months) (Conventional Adjustment)

For each panel below, which time period is more likely to indicate a turnaround if the relevant factor is the sole driving force?

Panel A: Relative importance of targeted time periods for consumption to drive a turnaround

	3	6	12	24	Vec. Wts.
3 months	1	1/5	1/7	1/7	.043
6 months	5	1	1/5	1/5	.113
12 months	7	5	1	1/3	.310
24 months	7	5	3	1	.534

Panel B: Relative importance of targeted time periods for exports to drive a turnaround

	3	6	12	24	Vec. Wts.
3 months	1	1	1/5	1/5	.083
6 months	1	1	1/5	1/5	.083
12 months	5	5	1	1	.417
24 months	5	5	1	1	.417

Panel C: Relative importance of targeted time periods for investment to drive a turnaround

	3	6	12	24	Vec. Wts.
3 months	1	1	1/5	1/5	.078
6 months	1	1	1/5	1/5	.078
12 months	5	5	1	1/3	.305
24 months	5	5	3	1	.538

Panel D: Relative importance of targeted time periods for fiscal policy to drive a turnaround

	3	6	12	24	Vec. Wts.
3 months	1	1	1/3	1/5	.099
6 months	1	1	1/5	1/5	.087
12 months	3	5	1	1	.382
24 months	5	5	1	1	.432

Panel E: Relative importance of targeted time periods for monetary policy to drive a turnaround

	3	6	12	24	Vec. Wts.
3 months	1	5	7	7	.605
6 months	1/5	1	5	7	.262
12 months	1/7	1/5	1	1/5	.042
24 months	1/7	1/7	5	1	.091

Panel F: Expected time for a change of confidence indicators of consumer and investor activity to support a turnaround in the economy

	3	6	12	24	Vec. Wts.
3 months	1	3	5	5	.517
6 months	1/3	1	5	5	.305
12 months	1/5	1/5	1	5	.124
24 months	1/5	1/5	1/5	1	.054

Table 3: Matrices for relative influence of subfactors on periods of adjustment (months)
(Economic Restructuring)

For each panel below, which time period is more likely to indicate a turnaround if the relevant factor is the sole driving force?

Panel A: Financial system restructuring time

	3	6	12	24	Vec. Wts.
3 months	1	1/3	1/5	1/7	.049
6 months	3	1	1/5	1/7	.085
12 months	5	5	1	1/5	.236
24 months	7	7	5	1	.630

Panel B: Defense readjustment time

	3	6	12	24	Vec. Wts.
3 months	1	1/3	1/5	1/7	.049
6 months	3	1	1/5	1/7	.085
12 months	5	5	1	1/5	.236
24 months	7	7	5	1	.630

Panel C: Global competition adjustment time

	3	6	12	24	Vec. Wts.
3 months	1	1	1/5	1/5	.078
6 months	1	1	1/5	1/5	.078
12 months	5	5	1	1/3	.305
24 months	5	5	3	1	.538

Table 4: Most likely factor to dominate during a specified time period

Which factor is more likely to produce a turnaround during the specified time period?

Conventional Adjustment → CA
Restructuring → R

Panel A: 3 Months

	CA	R	Vec. Wts.
CA	1	5	.833
R	1/5	1	.167

Panel B: 6 Months

	CA	R	Vec. Wts.
CA	1	5	.833
R	1/5	1	.167

Panel C: 1 Year

	CA	R	Vec. Wts.
CA	1	1	.500♪
R	1	1	.500♪

Panel D: 2 Years

	CA	R	Vec. Wts.
CA	1	1/5	.167
R	5	1	.833

Table 5: The Completed Supermatrix

	Conven. Adjust	Economic. Restruc.	Consum.	Exports	Invest.	Confid.	Fiscal Policy	Monet. Policy	Financ. Sector	Defense Posture	Global Compet.	3 mo.	6 mo.	1 yr.	≥ 2 years
Conven. Adjust	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.833	0.833	0.500	0.167
Economic. Restru.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.167	0.167	0.500	0.833
Consum.	0.118	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Exports	0.029	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Invest.	0.058	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Confid.	0.334	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Fiscal Policy	0.118	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Monetary Policy	0.343	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Financ. Sector	0.0	0.584	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Defense Posture	0.0	0.281	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Global Compet.	0.0	0.135	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3 months	0.0	0.0	0.043	0.083	0.078	0.517	0.099	0.605	0.049	0.049	0.089	0.0	0.0	0.0	0.0
6 months	0.0	0.0	0.113	0.083	0.078	0.305	0.086	0.262	0.085	0.085	0.089	0.0	0.0	0.0	0.0
1 year	0.0	0.0	0.310	0.417	0.305	0.124	0.383	0.042	0.236	0.236	0.209	0.0	0.0	0.0	0.0
≥ 2 years	0.0	0.0	0.534	0.417	0.539	0.054	0.432	0.091	0.630	0.630	0.613	0.0	0.0	0.0	0.0

Table 6: The Limiting Supermatrix

	Conven. Adjust	Economic. Restruc.	Consum.	Exports	Invest.	Confid.	Fiscal Policy	Monet. Policy	Financ. Sector	Defense Posture	Global Compet.	3 mo.	6 mo.	1 yr.	≥ 2 years
Conven. Adjust	0.0	0.0	0.484	0.484	0.484	0.484	0.484	0.484	0.484	0.484	0.484	0.0	0.0	0.0	0.0
Economic Restru.	0.0	0.0	0.516	0.516	0.516	0.516	0.516	0.516	0.516	0.516	0.516	0.0	0.0	0.0	0.0
Consum.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.057	0.057	0.057	0.057
Exports	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.014	0.014	0.014	0.014
Invest.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.028	0.028	0.028	0.028
Confid.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.162	0.162	0.162	0.162
Fiscal Policy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.057	0.057	0.057	0.057
Monetary Policy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.166	0.166	0.166	0.166
Financ. Sector	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.301	0.301	0.301	0.301
Defense Posture	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.145	0.145	0.145	0.145
Global Compet.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.070	0.070	0.070	0.070
3 months	0.224	0.224	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6 months	0.151	0.151	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1 year	0.201	0.201	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
≥ 2 years	0.424	0.424	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Synthesis / Results

When the judgments were made, the AHP framework was used to perform a synthesis that produced the following results. First a meaningful turnaround in the economy would likely require an additional ten to eleven months, occurring during the fourth quarter of 1992. This forecast is derived from weights generated in the first column of the limiting matrix in Table 6, coupled with the mid-points of the alternate time periods (so as to provide unbiased estimates:

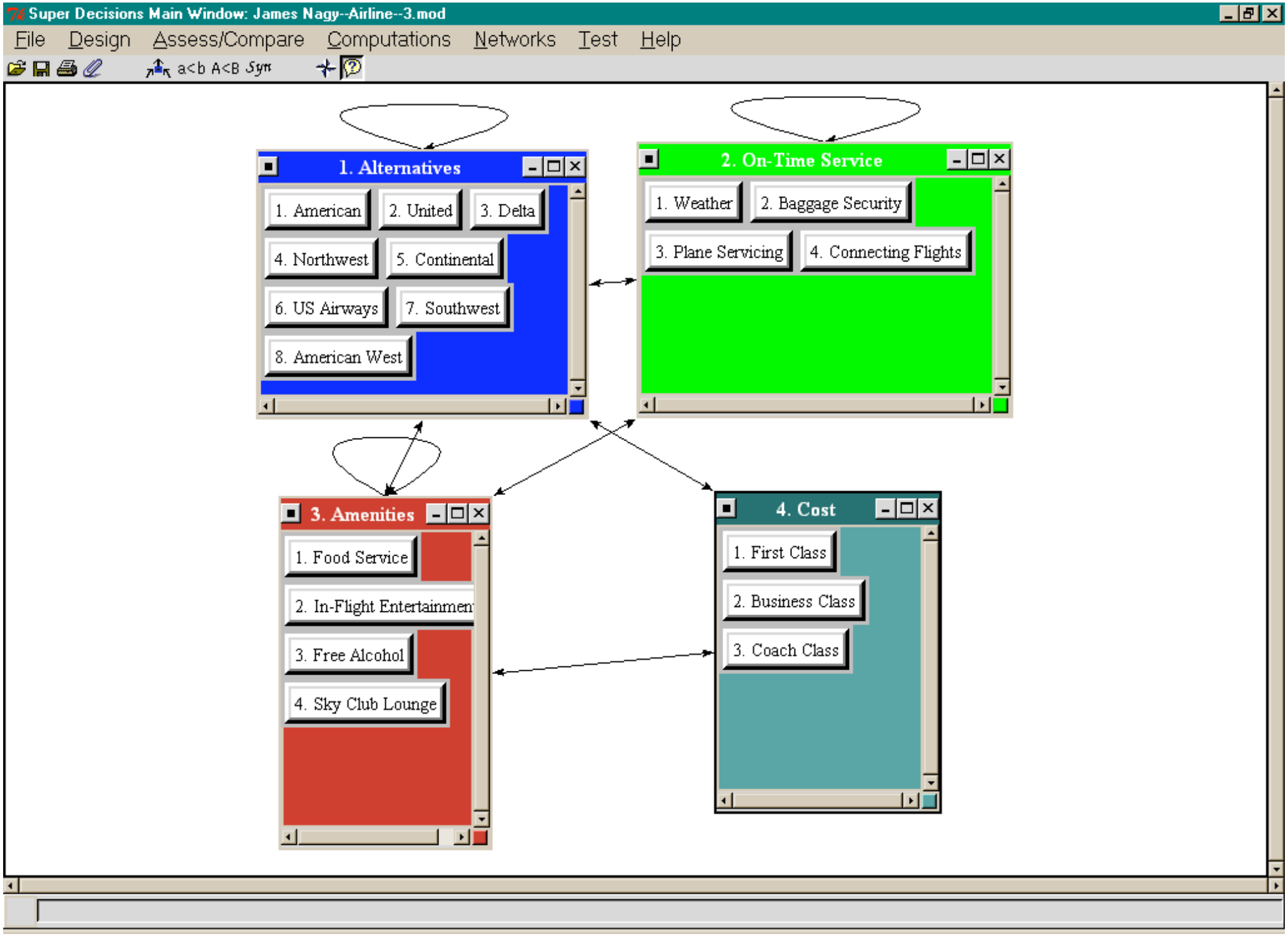
$$.224 \times 1.5 + .151 \times 4.5 + .201 \times 9 + .424 \times 18 =$$

10.45 months from late December 1991/early January 1992

Two Important Observations

For Costs and Risks we Must Ask,
Which is More Costly and Which is More Risky

We Must Also compare the Clusters Themselves as to Their Influence on Each Other Cluster in Order to Render the Super Matrix Stochastic. We do it in Several Separate Matrices of Paired Comparisons


Super Decisions Main Window: James Nagy--Airline--3.mod
File
Design
Assess/Compare
Computations
Networks
Test
Help
a<b A<B Sym
1. Alternatives
1. American
2. United
3. Delta
4. Northwest
5. Continental
6. US Airways
7. Southwest
8. American West
2. On-Time Service
1. Weather
2. Baggage Security
3. Plane Servicing
4. Connecting Flights
3. Amenities
1. Food Service
2. In-Flight Entertainmen
3. Free Alcohol
4. Sky Club Lounge
4. Cost
1. First Class
2. Business Class
3. Coach Class

Airlines' Market Shares

	Model Results	Actual (yr 2000)
American	23.9	24.0
United	18.7	19.7
Delta	18.0	18.0
Northwest	11.4	12.4
Continental	9.3	10.0
US Airways	7.5	7.1
Southwest	5.9	6.4
American West	4.4	2.9

Amit Prashar (February, 2005 class)upon seeing his market share results wrote:

	Actual Market Values	Super Decision Output
Dell	**64.75%**	**67.83%**
IBM	**22.60%**	**23.56%**
Toshiba	**12.64%**	**8.61%**

Saaty Compatibility Index 1.041 which is much less than 1.10 recommended

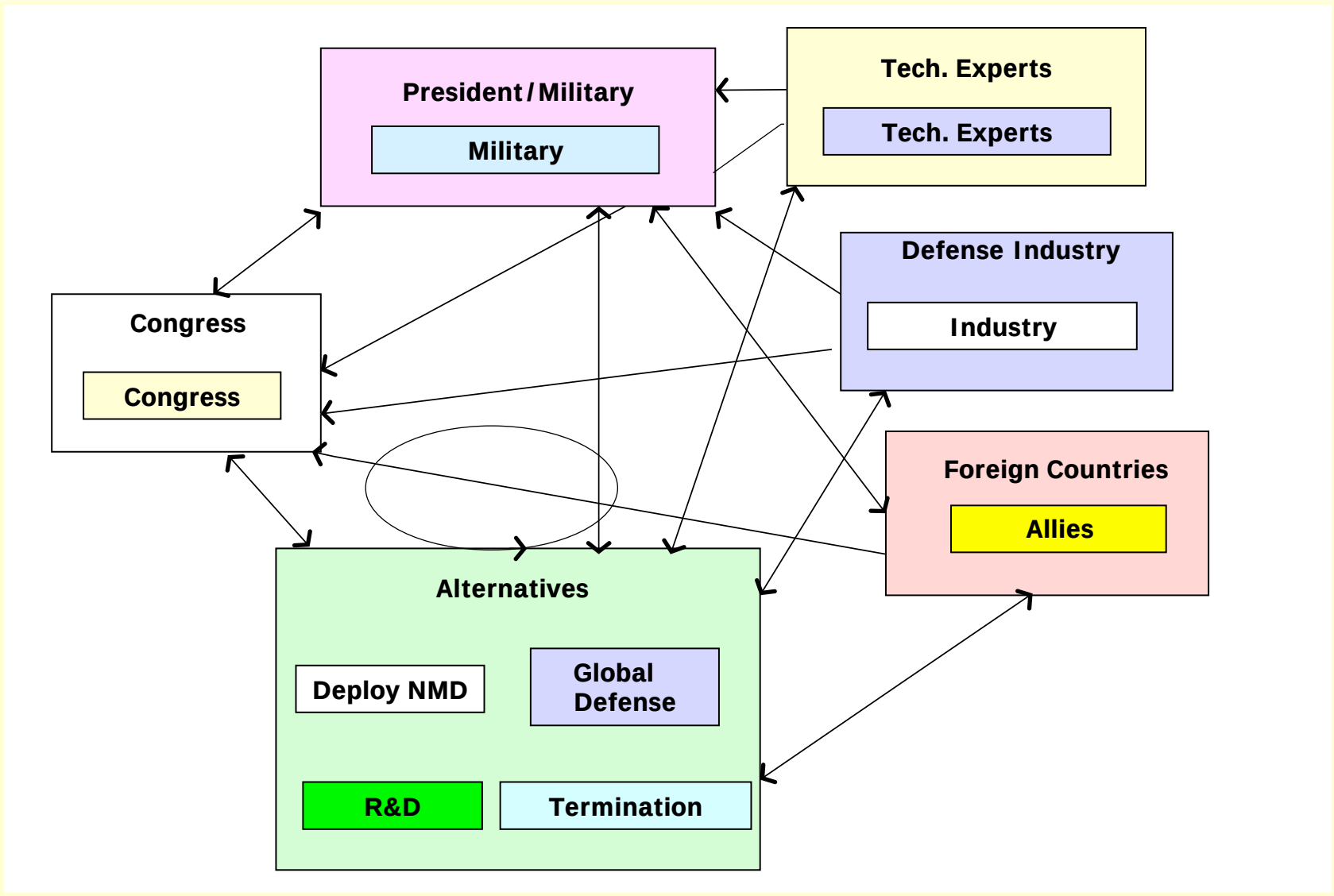
The results are very close (**I WAS REALLY STUNNED…..Geez..- UNBELIEVABLE**)

National Missile Defense (NMD)

Prioritization of national US criteria

Criteria and Their Priorities

Merits	Criteria	Sub-criteria	Global Priorities (Normalized)
Benefits	Economic (0.157)	Local Economy (0.141)	0.022
		Defense Industry (0.859)	0.014
	Political (0.074)	Bargaining Power (0.859)	0.064
		U.S. Military Leadership (0.141)	0.010
	Security (0.481)	Deterrence (0.267)	0.128
		Military Capability (0.590)	0.284
		Anti-terrorism (0.143)	0.069
	Technology (0.288)	Tech. Advancement (0.834)	0.240
		Tech. Leadership (0.166)	0.048
Opportunities	Arms Sales (0.520)		0.520
	Spin- off (0.326)		0.326
	Space Development (0.051)		0.051
	Protection of Allies (0.103)		0.103
Costs	Security Threat: Vulnerability to the security threat (0.687)		0.687
	Economic (0.228)	Sunk Cost (0.539)	0.123
		Further Investment (0.461)	0.103
	Political (0.085)	ABM Treaty (0.589)	0.050
		Foreign Relations (0.411)	0.035
Risks	Technical Failure (0.430)		0.430
	Arms Race (0.268)		0.268
	Increased Terrorism (0.052)		0.052
	Environmental Damage (0.080)		0.080
	U.S. Reputation (0.170)		0.170



Decision Network under Military Capability Control Subcriterion of Benefits

National Missile Defense (NMD)

Prioritization of national US criteria

Criteria and Their Priorities

Merits	Criteria	Sub-criteria	Global Priorities (Normalized)
Benefits	Economic (0.157)	Local Economy (0.141)	0.022
		Defense Industry (0.859)	0.014
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		U.S. Military Leadership (0.141)	0.010
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		Military Capability (0.590)	0.284
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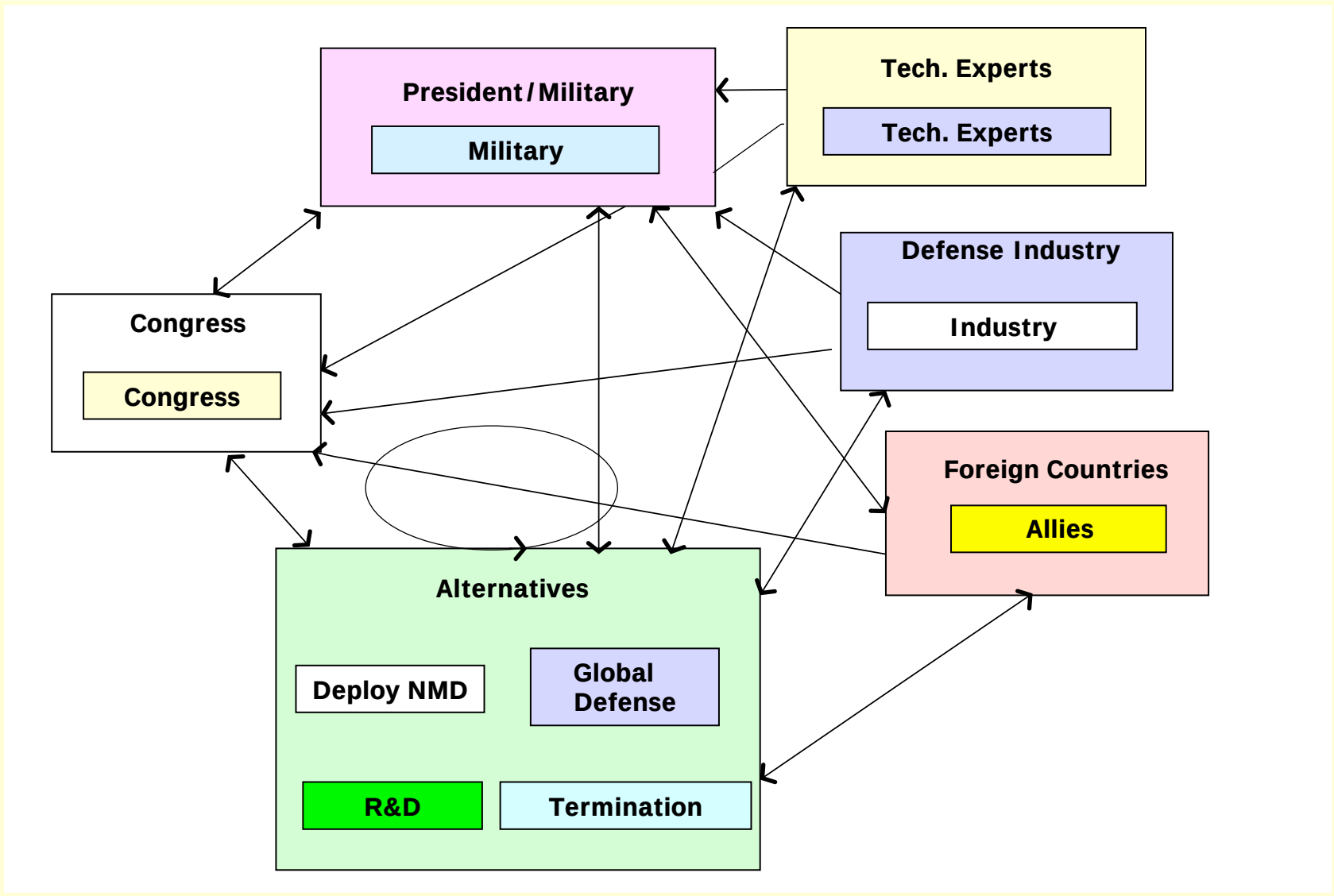
	Spin- off (0.326)		0.326
	Space Development (0.051)		0.051
	Protection of Allies (0.103)		0.103
Costs	Security Threat: Vulnerability to the security threat (0.687)		0.687
	Economic (0.228)	Sunk Cost (0.539)	0.123
		Further Investment (0.461)	0.103
	Political (0.085)	ABM Treaty (0.589)	0.050
		Foreign Relations (0.411)	0.035
Risks	Technical Failure (0.430)		0.430
	Arms Race (0.268)		0.268
	Increased Terrorism (0.052)		0.052
	Environmental Damage (0.080)		0.080
	U.S. Reputation (0.170)		0.170



Decision Network under Military Capability Control Subcriterion of Benefits

Pairwise comparing components with respect to the Congress component

Q: Which of a pair of components is influenced more by the Congress component with respect to Military Capability?

	Altern~	Pres~	Prior.
Altern~	1.0000	0.5638	0.3605
Pres~	1.7736	1.0000	0.6395

Pairwise comparing components with respect to the Foreign Countries component

Q: Which of a pair of components is influenced more by the Foreign Countries component with respect to Military Capability?

	Altern~	Cong~	Pres~	Prior.
Altern~	1.0000	2.5379	2.5379	0.5593
Congr~	0.3940	1.0000	1.0000	0.2204
Pres~	0.3940	1.0000	1.0000	0.2204

Pairwise comparing components with respect to the Technical Experts component

Q: Which of a pair of components is influenced more by the Technical Experts component with respect to Military Capability?

	Altern~	Cong~	Pres~	Prior.
Altern~	1.0000	0.5556	0.3259	0.1671
Congr~	1.8000	1.0000	0.4632	0.2781
Pres~	3.0682	2.1591	1.0000	0.5548

Pairwise comparing components with respect to the Defense Industry component

Q: Which of a pair of components is influenced more by the Defense Industry component with respect to Military Capability?

	Altern~	Cong~	Pres~	Prior.
Altern~	1.0000	0.6769	0.5388	0.2292
Congr~	1.4773	1.0000	0.6600	0.3181
Pres~	1.8561	1.5152	1.0000	0.4528

Pairwise comparing components with respect to the Presidnet/Military component

Q: Which of a pair of components is influenced more by the President/ Military component with respect to Military Capability?

	Altern~	Cong~	For~	Prior.
Altern~	1.0000	2.1887	3.6604	0.5735
Congr~	0.4569	1.0000	2.0377	0.2799
For~	0.2732	0.4907	1.0000	0.1467

Priorities Matrix of Eigenvectors

How much components are influenced by each component; imported from the matrices of the table above

♪

Clusters	*Altern~*	*Cong~*	*Def. Ind~*	*For~*	*Pres~*	*Tech~*
Altern~	0.0486	0.3605	0.2292	0.1671	0.5735	0.5593
Cong~	0.2889	0.0000	0.3181	0.2780	0.2799	0.2204
Def. Ind~	0.1653	0.0000	0.0000	0.0000	0.0000	0.0000
For~	0.0425	0.0000	0.0000	0.0000	0.1467	0.0000
Pres~	0.3742	0.6395	0.4528	0.5548	0.0000	0.2204
Tech~	0.0805	0.0000	0.0000	0.0000	0.0000	0.0000

The Weighted Supermatrix

Priorities from the above table are used to weight the corresponding blocks of the unweighted supermatrix

♪

MilCap		*Altern~*				*Cong~*	*Def. Ind~*	*For~*	*Pre/Mil~*	*Tech~*
Weighted		NMD	Glob~	R & D	Term~	Cong~	Industry	Allies	Military	Tech~
Altern~	NMD	0.0000	0.0280	0.0000	0.0000	0.1824	0.1280	0.0000	0.2958	0.1610
	Glob~	0.0000	0.0000	0.0000	0.0000	0.1042	0.0590	0.1671	0.1680	0.1467
	R & D	0.0000	0.0206	0.0000	0.0000	0.0471	0.0317	0.0000	0.0784	0.1325
	Term~	0.0000	0.0000	0.0000	0.0000	0.0268	0.0105	0.0000	0.0313	0.1191
Cong~	Cong~	0.3037	0.2889	0.3037	0.0000	0.0000	0.3181	0.2780	0.2799	0.2204
Def ense Ind	Industry	0.1737	0.1653	0.1737	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
For~	Allies	0.0446	0.0425	0.0446	0.0000	0.0000	0.0000	0.0000	0.1467	0.0000
Pre/Mil~	Military	0.3933	0.3742	0.3933	0.0000	0.6395	0.4528	0.5548	0.0000	0.2204
Tech~	Tech~	0.0846	0.0805	0.0846	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The Limit Supermatrix

The weighted supermatrix raised to sufficiently large powers to stabilize within rounded off four place decimals

MilCap		*Altern~*				*Cong~*	*Def. Ind~*	*For~*	*Pre/Mil~*	*Tech~*
Limited		NMD	Glob~	R & D	Term~	Cong~	Industry	Allies	Military	Tech~
Altern~	NMD	0.1532	0.1532	0.1532	0.0000	0.1532	0.1532	0.1532	0.1532	0.1532
	Glob~	0.0968	0.0968	0.0968	0.0000	0.0968	0.0968	0.0968	0.0968	0.0968
	R & D	0.0438	0.0438	0.0438	0.0000	0.0438	0.0438	0.0438	0.0438	0.0438
	Term~	0.0201	0.0201	0.0201	0.0000	0.0201	0.0201	0.0201	0.0201	0.0201
Cong~	Cong~	0.2224	0.2224	0.2224	0.0000	0.2224	0.2224	0.2224	0.2224	0.2224
Def ense Ind	Industry	0.0513	0.0513	0.0513	0.0000	0.0513	0.0513	0.0513	0.0513	0.0513
For~	Allies	0.0619	0.0619	0.0619	0.0000	0.0619	0.0619	0.0619	0.0619	0.0619
Pre/Mil~	Military	0.3255	0.3255	0.3255	0.0000	0.3255	0.3255	0.3255	0.3255	0.3255
Tech~	Tech~	0.0250	0.0250	0.0250	0.0000	0.0250	0.0250	0.0250	0.0250	0.0250

IDEALIZED DECISION NETWORK VECTORS times NORMALIZED CONTROL CRITERIA

Benefits	Military Capability		Technical Advancement		SUM of
Control Criterion wt. (CC)	0.075		0.063		wtd Alts
Normalized CC	*0.542*	*Col. 1*	*0.458*	*Col. 2*	*Col 1 + Col 2*
Alternatives	*Idealized*	*(CC x Ideal.)*	*Idealized*	*(CC x Ideal.)*	*SUM*
Deploy	1.000	0.542	0.928	0.425	**0.967**
Global	0.623	0.338	1.000	0.458	**0.796**
R&D	0.282	0.153	0.448	0.205	**0.358**
Terminate	0.129	0.070	0.085	0.039	**0.109**

Opportunities	Arms Sales		Spinoff		SUM of
Control Criteria (CC)	0.096		0.06		wtd Alts
Normalized CC	*0.614*	*Col. 1*	*0.386*	*Col. 2*	*Col 1 + Col 2*
Alternatives	*Idealized*	*(CC x Ideal.)*	*Idealized*	*(CC x Ideal.)*	*SUM*
Deploy	1.000	0.614	1.000	0.386	**1.000**
Global	0.674	0.414	0.521	0.201	**0.614**
R&D	0.341	0.209	0.288	0.111	**0.321**
Terminate	0.190	0.117	0.166	0.064	**0.181**

Costs	Sec. Threat		Sunk Cost		Further Inv.		**Costs**
Control Criteria (CC)	0.687		0.123		0.105		*Sum of*
Normalized CC	*0.751*	*Col. 1*	*0.134*	*Col. 2*	*0.115*	*Col. 3*	*Col's 1+2+3*
Alternatives	*Idealized*	*(CC x Ideal.)*	*Idealized*	*(CC x Ideal.)*	*Idealized*	(CC x Ideal.)	*SUM*
Deploy	0.183	0.137	1.000	0.134	1.000	0.115	**0.386**
Global	0.344	0.259	0.574	0.077	0.496	0.057	**0.393**
R&D	0.579	0.435	0.332	0.044	0.279	0.032	**0.512**
Terminate	1.000	0.751	0.193	0.026	0.147	0.017	**0.794**

Risks	Tech Failure		Arms Race		**Risks**
Control Criteria (CC)	0.43		0.268		*Sum of*
Normalized CC	*0.616*	*Col. 1*	*0.384*	*Col. 2*	*Col's 1 + 2*
Alternatives	*Idealized*	*(CC x Ideal.)*	*Idealized*	*(CC x Ideal.)*	SUM
Deploy	1.000	0.616	1.000	0.384	**1.000**
Global	0.621	0.382	0.693	0.266	**0.648**
R&D	0.375	0.231	0.441	0.169	**0.401**
Terminate	0.262	0.161	0.302	0.116	**0.277**

Priority Ratings for the Merits: Benefits, Opportunities, Costs and Risks

Very High (0.419), High (0.263), Medium (0.160), Low (0.097), Very Low (0.061)
* Idealized: Very High (1.000), High (0.619), Medium (0.381), Low (0.238), Very Low (0.143)

♪		Benefits	Opportunities	Costs	Risks ♪
World Peace .650	Adversary Countries .154	Very High	Medium	High	Very Low
	Security Dilemma .290	Very Low	Very Low	Very High	Very Low
	Terrorism .203	Medium	Very Low	High	High ♪
Human Well-Being .120 ♪	Technological Advancement.081	High	High ♪	Low	Very Low ♪
	Market Creation .041	Medium	High	Very Low	Very Low ♪
International Politics .230	Military Relations .138	High	High	Medium	Very Low ♪
	Diplomatic Relations .092	Low	Low ♪	Low	Very High
Ratings Total Normalized		**0.446** 0.264♪	**0.380** 0.184♪	**0.611** 0.363♪	**0.318** 0.188♪

Sum of the BOCR merit priorities times the “Totals“ for their control criteria

	Benefits		Opportunities		Costs		Risks	
	0.264		0.184		0.363		0.188	
Alts	*Sum(from above)*	*(Sum x .264)*	*Sum(from above)*	*(Sum x .184)*	*Sum(from above)*	*(Sum x .363)*	*Sum(from above)*	(Sum x .188)
Deploy	0.967	**0.255**	1.000	**0.184**	0.386	**0.140**	1.000	**0.188**
Global	0.796	**0.210**	0.614	**0.113**	0.393	**0.142**	0.648	**0.122**
R&D	0.358	**0.094**	0.321	**0.059**	0.512	**0.186**	0.401	**0.075**
Terminate	0.109	**0.029**	0.181	**0.033**	0.794	**0.288**	0.277	**0.052**
	*If a sum column is not ideal, that is, the largest value not 1.0, idealize by dividing by largest value in the column							

Synthesis of the Alternatives in Two Ways

	BO/CR		**bB+oO-cC-rR**		
	(from unwtd columns		(from weighted col's	(Unitized by dividing by number	
Alternatives	in table above)	Normalized	in table above)	with smallest absolute value)	
Deploy	2.504	0.493	0.111	**1.891**	
Global	1.921	0.379	0.059	**1.000**	
R&D	0.560	0.110	-0.108	**-1.831**	
Terminate	0.090	0.018	-0.278	**-4.736**	

ANWR

Arctic National Wildlife Refuge

ANWR Situation

- ANWR- Arctic National Wildlife Refuge
- Size- 19 Million Acres; Area 1002-1.5 Million Acres of Coastal Plain
- Protected in 1960-Eisenhower
- Land Compromise Carter Admin.
- Efforts to Re-open for Exploration- Bush
- Counter efforts to stop Exploration

Introduction to the ANWR situation

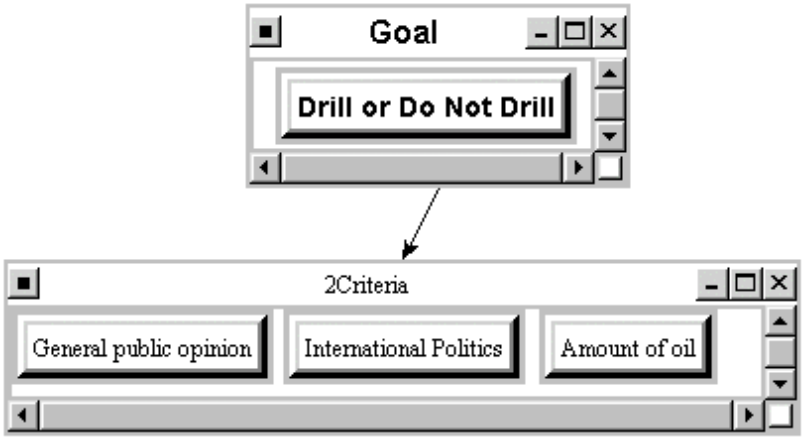
ANWR-Arctic National Wildlife Refuge covers 19 million acres on the Northern coast of Alaska. The entire refuge lies north of the Arctic Circle and 1,300 miles south of the North Pole. The Coastal Plain area comprising 1.5 million acres on the northern edge of ANWR, is bordered on the north by the Beaufort Sea, on the east by the U.S. Canadian border and on the west by the Canning River. The consensus of the geologic community is that the Coastal Plain of ANWR represents the highest petroleum potential onshore area yet to be explored in North America. If explored, it is estimated that it will take 15 years or more before oil and gas will reach the market.

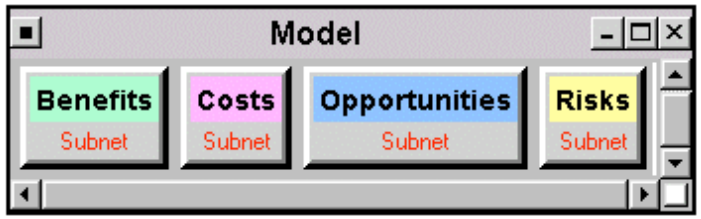
This coastal plain area, also known as area 1002, was originally protected in 1960 by President Eisenhower. Twenty years later President Carter signed the Alaska National Interest Conservation Act. This legislation was important as it created a majority of the National Parks in Alaska and expanded ANWR to its current size. A compromise was reached to pass the legislation, in return for designating a majority of the area-protected land; area 1002 was left unprotected and thus open for exploration.

Each administration since has had its own opinion regarding the land and what should be done with it. The Reagan Administration was ready to drill but was derailed by the Exxon Valdez catastrophe. Similarly the first Bush Administration was unsuccessful. The Clinton Administration designated the area for protection and it has been since. The second Bush Administration, in response to ongoing Middle East violence and 9/11 terrorist attacks, sees drilling in ANWR as vital not only for economic but also for national security reasons. Several environmental groups consider ANWR a great American natural treasure and one of the last places on earth where an intact expanse of arctic and sub-arctic lands remain protected. They feel that the habitat, the wildlife, and the culture need to be protected from the exploration of gas and oil.

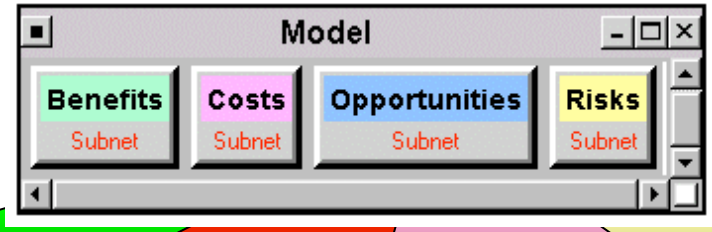
The following Super Decisions model was formed as a way to arrive at a decision regarding the use of this land. This model incorporates pair-wise comparisons of benefits, opportunities costs and risks associated with drilling or not drilling. By making these comparisons and choosing the answers that best represent the use of the land we are able to come to a plausible conclusion as to whether or not this area should be further explored.

Overview of Model



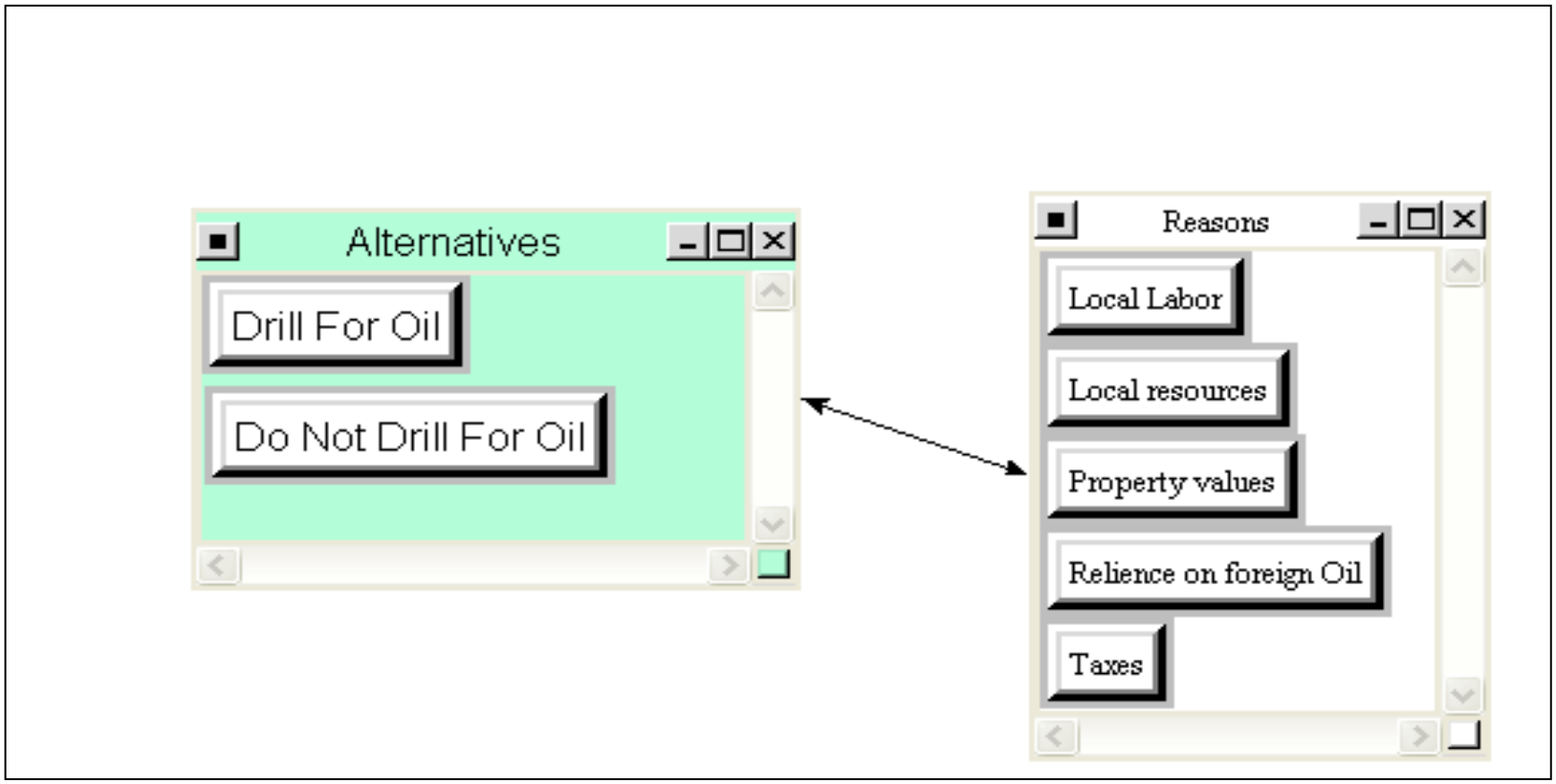


Cluster Matrix Overview



	Benefits	Costs	Opportunities	Risks
Economic	Local labor increase Less reliance on foreign oil. Local resources Property values Taxes	Labor Raw Materials Infrastructure Property Value Taxes	Local business ROI Exports Tax	Reliance on Foreign Oil Investments Other energy sources Jobs
Political	Taxes Less reliance on foreign oil. Elections Clout	Taxes Assessment Time Political Fallout World wide instability	Lobbying Clout National support Community support	Local image National image Elections
Social	Revitalization Development Public Programs Oil companies contributions Taxes to fund Ecological Programs	Environmental Crime Inconvenience Noise Cultural	Development Jobs	Health Environmental Jobs Cultural

Benefits -Economic Decision Subnet



.225

.098

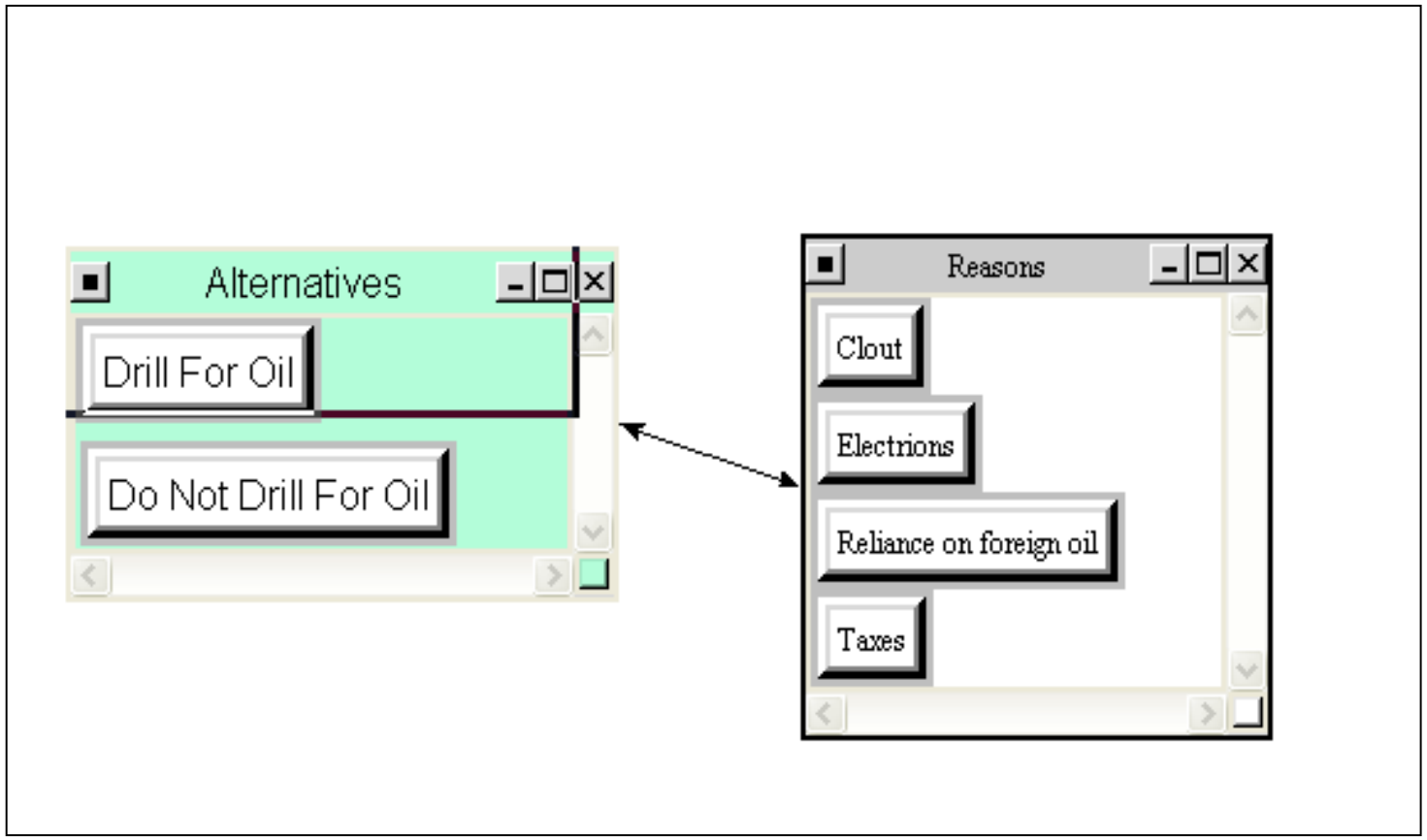
.178

.435

.063

Do Not Drill for Oil	.235
Drill for Oil	.765

Benefits -Political Decision Subnet



.105

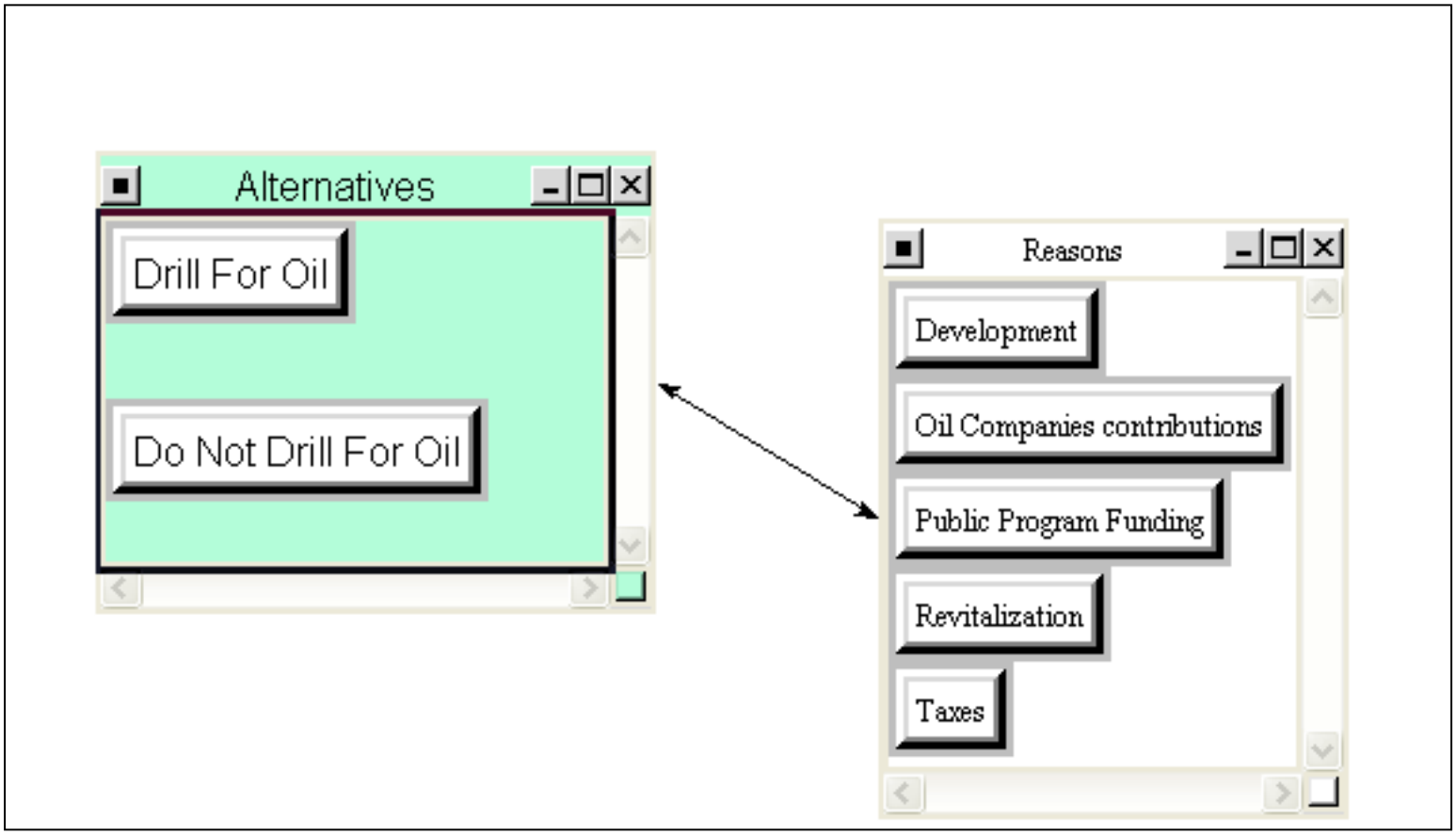
.326

.453

.116

Do Not Drill for Oil	.312
Drill for Oil	.688

Benefits -Social Decision Subnet



.301

.065

.106

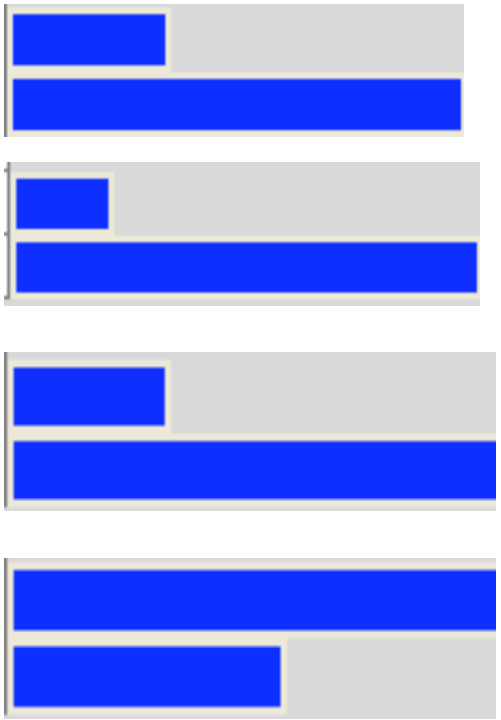
.340

.188

Do Not Drill for Oil	.225
Drill for Oil	.775

Overall Priorities

Benefits	Do Not Drill for Oil	.252
	Drill for Oil	.748
Opportunities	Do Not Drill for Oil	.165
	Drill for Oil	.835
Costs (most costly)	Do Not Drill for Oil	.238
	Drill for Oil	.762
Risks (most risky)	Do Not Drill for Oil	.644
	Drill for Oil	.356



Ratings for BOCR

Ratings

Ratings for Super Decisions Main Window: Cefalu_Emanuel Oil Drilling Policy in...

File Edit View Calculations Help

Super Decisions Ratings

	Priorities	Totals	General public opinion 0.100654	International Politics 0.225535	Amount of oil 0.673811
Benefits	0.425172	0.859486	High	Medium	High
Opportunities	0.380098	0.768368	Low	Medium	High
Costs	0.046866	0.094740	Low	Low	Low
Risks	0.147864	0.298907	Low	High	Low

Benefits	.425
Opportunities	.380
Costs	.047
Risks	.148

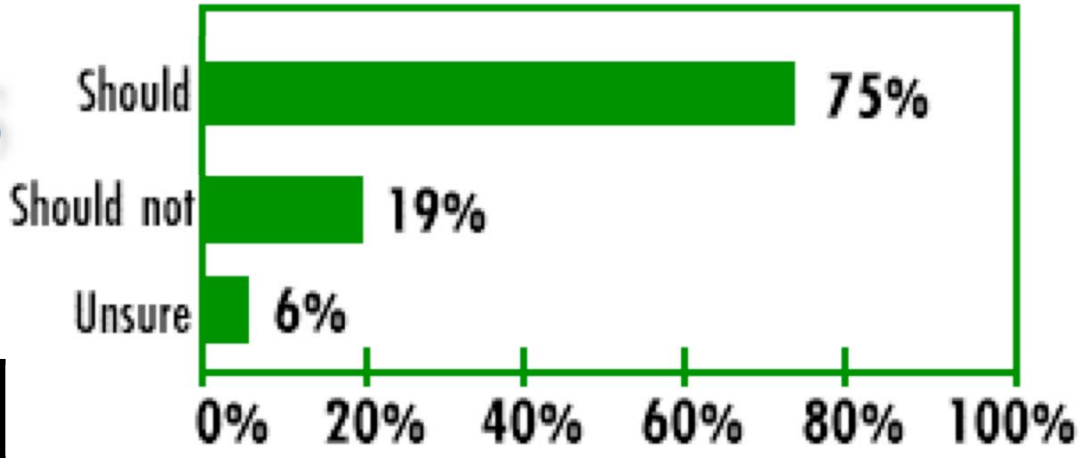
Priorities

Results

Our Results...

Should	77.7%
Should not	22.3%

Alaskan Poll Results



Splitting 6% Unsure Vote

Should	78%
Should Not	22%