STAT 234 Lecture 22
Analysis of Paired Data (Section 10.3)
Comparing Two Proportions (Section 10.4)

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Analysis of Paired Data (Section 10.3)

Example: Coffee & Blood Flow During Exercise

Doctors studying healthy men measured myocardial blood flow (MBF)[1] during bicycle exercise after giving the subjects a placebo or a dose of 200 mg of caffeine that was equivalent to drinking two cups of coffee[2].

There were 8 subjects, each was tested twice, 4 of them were randomly selected to receive caffeine in the first test and placebo in the second test; the other 4 received placebo first and caffeine second.

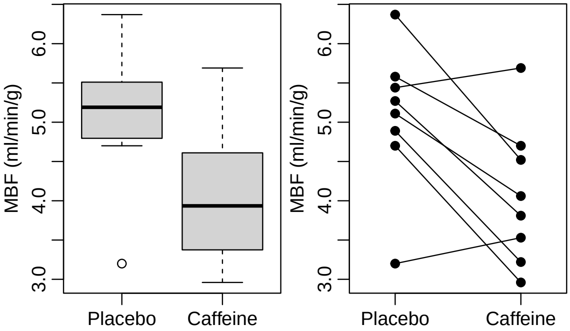
There was a 24-hour gap between the two tests (washout period).

[1] MBF was measured by taking positron emission tomography (PET) images after oxygen-15 labeled water was infused in the patients.

[2] Namdar et. al (2006). Caffeine decreases exercise-induced myocardial flow reserve. *Journal of the American College of Cardiology* **47**, 405-410.

Data for the Coffee & Blood Flow Experiment

	Myocardial Blood Flow (ml/min/g)									
Subject	1	2	3	4	5	6	7	8	Mean	SD
Placebo	6.37	5.44	5.58	5.27	5.11	4.89	4.70	3.20	5.07	0.91
Caffeine	4.52	5.69	4.70	3.81	4.06	3.22	2.96	3.53	4.06	0.89



Discussion

- Why did 4 subjects caffeine first and placebo second and the other 4 received placebo first and caffeine second?
- Why do we need a washout period (the 24 hour gap) between the two tests?
- Can we analyze the data of the experiment like two independent samples?

Hypothesis Tests for Paired Data

- Paired data cannot be analyzed like 2-sample data since the 2 measurements on the same subject are *dependent*.
- Nonetheless, if measurements on different pairs can be reasonably assumed independent, we can take differences of the two measurements within each pair and analyze the differences like **one-sample data**.

Subject	1	2	3	4	5	6	7	8	Mean	SD
Placebo	6.37	5.44	5.58	5.27	5.11	4.89	4.70	3.20	5.07	0.91
Caffeine	4.52	5.69	4.70	3.81	4.06	3.22	2.96	3.53	4.06	0.89
Diff (Placebo − Caffeine)	1.85	−0.25	0.88	1.46	1.05	1.67	1.74	−0.33	1.01	0.87

To test H_0: $\mu_1 - \mu_2 = \Delta_0$, the test statistic is

$$t = \frac{\bar{d} - \Delta_0}{s_d/\sqrt{n}} \sim t_{n-1}$$

where

$\bar{d}$ = sample mean of the diffs
s_d = sample SD of the diffs
n = # of **pairs.**

Example: Coffee & Blood Flow During Exercise

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In this example, $\bar{d} = 1.01$, $s_d = 0.87$. Please note that

$$\begin{array}{ccccc} \bar{d} & = & \bar{x}_{\text{placebo}} & - & \bar{x}_{\text{caffeine}} \\ 1.01 & = & 5.07 & - & 4.06 \end{array} \quad \text{but} \quad \begin{array}{ccccc} s_d & \neq & s_{\text{placebo}} & - & s_{\text{caffeine}} \\ 0.87 & \neq & 0.91 & - & 0.89 \end{array}$$

$s_d \approx 0.87$ is the sample SD of the 8 differences:

```
caffeine = c(4.52, 5.69, 4.7, 3.81, 4.06, 3.22, 2.96, 3.53)
placebo = c(6.37, 5.44, 5.58, 5.27, 5.11, 4.89, 4.7, 3.2)
diff = placebo - caffeine
sd(diff)
[1] 0.8683554
```

Example: Coffee & Blood Flow During Exercise

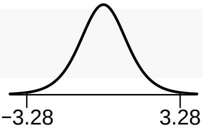
As $\bar{d} = 1.01$, $s_d = 0.87$, $n = 8$ pairs, the test statistic for H_0: $\mu_1 - \mu_2 = 0$ is

$$t = \frac{\bar{d} - \Delta_0}{s_d/\sqrt{n}} = \frac{1.01 - 0}{0.87/\sqrt{8}} \approx 3.28$$

with $n - 1 = 8 - 1 = 7$ degrees of freedom.

The two-sided P-value can be found in R to be ≈ 0.0135.

```
2 * pt(3.28, df = 7, lower.tail = F)
[1] 0.01348706
```



Or using the t-table, one can find the 2-sided P-value to be between $2(0.005) = 0.01$ and $2(0.01) = 0.02$.

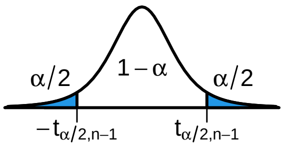
	α	0.1	0.05	0.025	*0.01*	*0.005*	0.001	0.0005
ν	7	1.415	1.895	2.365	*2.998*	*3.499*	4.785	5.408

Confidence Intervals for the Mean Difference in Paired Data

The $100(1-\alpha)\%$ confidence interval for the difference is

$$\bar{d} \pm t_{\alpha/2,n-1} \frac{s_d}{\sqrt{n}}$$

where $t_{\alpha/2,n-1}$ is the critical value for the t distribution with $n-1$ degrees of freedom as shown on the right.



For the coffee experiment, the critical value for a 95% CI is $t_{\alpha/2,n-1} = t_{0.05/2,8-1} \approx 2.365.$

```
qt(0.05/2, df = 7, lower.tail = F)
[1] 2.364624
```

α		0.1	0.05	0.025	0.01	0.005	0.001	0.0005
ν	7	1.415	1.895	2.365	2.998	3.499	4.785	5.408

The 95% CI for the mean difference is hence

$$\bar{d} \pm t_{\alpha/2,n-1} \frac{s_d}{\sqrt{n}} = 1.01 \pm 2.365 \times \frac{0.87}{\sqrt{8}} \approx 1.01 \pm 0.73 = (0.28, 1.74).$$

Tests/CIs for Paired Data in R

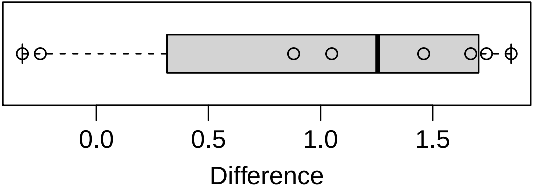
```
caffeine = c(4.52, 5.69, 4.7, 3.81, 4.06, 3.22, 2.96, 3.53)
placebo = c(6.37, 5.44, 5.58, 5.27, 5.11, 4.89, 4.7, 3.2)
t.test(placebo, caffeine, paired = T, conf.level = 0.95)

    Paired t-test

data:  placebo and caffeine
t = 3.2857, df = 7, p-value = 0.01338
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.2827867 1.7347133
sample estimates:
mean of the differences
                1.00875
```

Checking Conditions for Paired Data

As the inference for paired data is simply the inference for one-sample data on the differences with the pairs, just make sure that

- the differences are independent
- the distribution (histogram) of the differences is not too skewed and has no outlier

Whether the distributions of the two groups are skewed or have outlier(s) do not matter.



Example: Paired or Not

In each of the following scenarios, determine if the data are paired?

1. We would like to know if Intel's stock and Southwest Airlines' stock have similar rates of return. To find out, we take a random sample of 50 days, and record Intel's and Southwest's stock on those same days.
2. We randomly sample 50 items from Target stores and note the price for each. Then we visit Walmart and collect the price for each of those same 50 items.
3. A school board would like to determine whether there is a difference in average SAT scores for students at one high school versus another high school in the district. To check, they take a simple random sample of 100 students from each high school.

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2. We randomly sample 50 items from Target stores and note the price for each. Then we visit Walmart and collect the price for each of those same 50 items. ⇒ paired
3. A school board would like to determine whether there is a difference in average SAT scores for students at one high school versus another high school in the district. To check, they take a simple random sample of 100 students from each high school. ⇒ not paired

If Paired Data Were Analyzed Like 2-sample Data

Subject	1	2	3	4	5	6	7	8	Mean	SD
Placebo	6.37	5.44	5.58	5.27	5.11	4.89	4.70	3.20	5.07	0.91
Caffeine	4.52	5.69	4.70	3.81	4.06	3.22	2.96	3.53	4.06	0.89
Diff (Placebo − Caffeine)	1.85	−0.25	0.88	1.46	1.05	1.67	1.74	−0.33	1.01	0.87

If we ignore pairing, and analyze the data as 2-sample data, the two-sample t-statistic

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}} = \frac{5.07 - 4.06}{\sqrt{\frac{0.91^2}{8} + \frac{0.89^2}{8}}} \approx 2.244$$

would be less than the paired t-statistic $\dfrac{\bar{d}}{s_d/\sqrt{n}} = \dfrac{1.01}{0.87/\sqrt{8}} \approx 3.28.$

The p-value (6%) given by a two-sample t-test is larger than the one given by a paired t-test (1.3%), less significant.

95% two-sample CI: $5.07 - 4.06 \pm 2.144\sqrt{\frac{0.91^2}{8} + \frac{0.89^2}{8}} \approx 1.01 \pm 0.96$

95% paired CI: $1.01 \pm 2.365 \times 0.87/\sqrt{8} \approx 1.01 \pm 0.73$ (shorter)

Two-Sample Data v.s. Paired Data

Suppose the two samples are both of size n, the SEs for two-sample data and paired data would be respectively

$$\text{SE} = \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n}} \qquad \text{(two-sample)}$$

$$\text{SE} = \sigma_D / \sqrt{n} \qquad \text{(paired)}$$

where

$$\sigma_1^2 = \text{Var}(X_i), \qquad \sigma_2^2 = \text{Var}(Y_i),$$

$$\begin{aligned}
\sigma_D^2 = \text{Var}(X_i - Y_i) &= \text{Var}(X_i) + \text{Var}(Y_i) - 2\,\text{Cov}(X_i, Y_i) \\
&= \sigma_1^2 + \sigma_2^2 - 2\,\text{Cov}(X_i, Y_i) \\
&\le \sigma_1^2 + \sigma_2^2 \quad \text{if } \text{Cov}(X_i, Y_i) > 0
\end{aligned}$$

Observations within a pair are usually positively correlated. ⇒ Paired CIs are usually shorter and Paired tests usually have **smaller P-values**.

Comparing Two Proportions (Section 10.4)

Comparing Two Proportions

Choose an SRS of size n_1 from a large population having proportion p_1 of successes and an independent SRS of size n_2 from another population having proportion p_2 of successes.

Population	Population Proportion	Sample Size	Count of Successes	Sample Proportion
1	p_1	n_1	X_1	$\widehat{p_1} = X_1/n_1$
2	p_2	n_2	X_2	$\widehat{p_2} = X_2/n_2$

Large Sample Confidence Intervals for $p_1 - p_2$

When n_1 and n_2 are both large,

$$\hat{p}_1 - \hat{p}_2 \mathrel{\dot\sim} N\left(p_1 - p_2,\, \sigma^2 = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$$

An approximate $(1-\alpha)100\%$ CI for $p_1 - p_2$ is

$$\text{estimate} \pm z_{\alpha/2}\text{SE}$$

where

$$\text{estimate} = \hat{p}_1 - \hat{p}_2, \quad \text{SE} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Use this method only when the number of successes and the number of failures in both samples are at least 10, i.e.,

$$n_1\hat{p}_1, \quad n_1(1-\hat{p}_1), \quad n_2\hat{p}_2, \quad n_2(1-\hat{p}_2) \text{ all } \geq 10.$$

Example: Aspirin and Heart Attacks (1)

The Physicians' Health Study was a 5-year randomized study published testing whether regular intake of aspirin reduces mortality from cardiovascular disease[3].

- Participants were male physicians 40-84 years old in 1982 with no prior history of heart attack, stroke, and cancer, no current liver or renal disease, no contraindication of aspirin, no current use of aspirin
- Every other day, the male physicians participating in the study took either one aspirin tablet or a placebo.
- Response: whether the participant had a heart attack (including fatal or non-fatal) during the 5 year period.

---

[3] *Source*: Preliminary Report: Findings from the Aspirin Component of the Ongoing Physicians' Health Study. *New Engl. J. Med.*, **318**: 262-64,1988.

Example: Aspirin and Heart Attacks (2)

Result:

	Heart Attack?		
Group	Yes	No	Sample Size
Placebo	189	10845	11034
Aspirin	104	10933	11037

$\Rightarrow \hat{p}_1 = \frac{189}{11034} \approx 0.0171$

$\Rightarrow \hat{p}_2 = \frac{104}{11037} \approx 0.0094$

The $z_{\alpha/2}$ for a 99% CI is $z_{0.01/2} \approx 2.576$, so the 99% CI for $p_1 - p_2$ is

$$\hat{p}_1 - \hat{p}_2 \pm z_{0.01/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$= 0.0171 - 0.0094 \pm 2.576 \sqrt{\frac{0.0171(1-0.0171)}{11034} + \frac{0.0094(1-0.0094)}{11037}}$$

$$= 0.0077 \pm 0.0040 = (0.0037, 0.0117)$$

Interpretation: As 99% confidence, the probability of heart attack in aspirin group is 0.0037 to 0.0117 lower than the corresponding probability in the placebo group

While we test

$$H_0: \; p_1 = p_2$$

the SE for $\hat{p}_1 - \hat{p}_2$ under H_0 is

$$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = \sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where p is the common value of p_1 and p_2.

How to estimate the common p?

Testing the Equality of Two Proportions (2)

When $p_1 = p_2 = p$, both $\hat{p}_1$ and $\hat{p}_2$ are unbiased estimates for the common p. we can combine the two samples, and get the *pooled estimate* for p:

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{n_1}{n_1 + n_2}\hat{p}_1 + \frac{n_2}{n_1 + n_2}\hat{p}_2$$

The SE for testing H$_0$: $p_1 = p_2$ is hence

$$\text{SE} = \sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

and the z-statistic for testing H$_0$: $p_1 = p_2$ is

$$z = \frac{\text{estimate}}{\text{SE}} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Under H_0, the z-statistic is approx. $N(0, 1)$ provided that

$$n_1\hat{p}, \quad n_1(1 - \hat{p}), \quad n_2\hat{p}, \quad n_2(1 - \hat{p}) \text{ all } \geq 10.$$

Example: Aspirin and Heart Attacks (4)

Group	Sample Size	Heart Attack
Placebo	11034	189
Aspirin	11037	104

For testing $H_0 : p_1 = p_2$,

$$\hat{p}_1 - \hat{p}_2 = \frac{189}{11034} - \frac{104}{11037} \approx 0.0077$$

Example: Aspirin and Heart Attacks (4)

Group	Sample Size	Heart Attack
Placebo	11034	189
Aspirin	11037	104

For testing $H_0 : p_1 = p_2$,

$$\begin{aligned}
\hat{p}_1 - \hat{p}_2 &= \frac{189}{11034} - \frac{104}{11037} \approx 0.0077 \\
\hat{p} &= \frac{189 + 104}{11034 + 11037} \approx 0.0132
\end{aligned}$$

Example: Aspirin and Heart Attacks (4)

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Placebo	11034	189
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For testing $H_0 : p_1 = p_2$,

$$\begin{aligned}
\hat{p}_1 - \hat{p}_2 &= \frac{189}{11034} - \frac{104}{11037} \approx 0.0077 \\
\hat{p} &= \frac{189 + 104}{11034 + 11037} \approx 0.0132 \\
\text{SE} &= \sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \\
&\approx \sqrt{0.0132(1 - 0.0132)\left(\frac{1}{11034} + \frac{1}{11037}\right)} \approx 0.00154
\end{aligned}$$

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Group	Sample Size	Heart Attack
Placebo	11034	189
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For testing $\mathrm{H}_0 : p_1 = p_2$,

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\hat{p}_1 - \hat{p}_2 &= \frac{189}{11034} - \frac{104}{11037} \approx 0.0077 \\
\hat{p} &= \frac{189 + 104}{11034 + 11037} \approx 0.0132 \\
\text{SE} &= \sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \\
&\approx \sqrt{0.0132(1 - 0.0132)\left(\frac{1}{11034} + \frac{1}{11037}\right)} \approx 0.00154 \\
z\text{-statistic} &= \frac{\hat{p}_1 - \hat{p}_2}{\text{SE}} \approx \frac{0.0077}{0.00154} \approx 5.001
\end{aligned}
$$

Example: Aspirin and Heart Attacks (4)

The 2-sided p-value is 0.00000057 by R.

```
2 * pnorm(5.001, lower.tail = FALSE)
[1] 5.703371e-07
```

Not surprisingly, we are getting strong evidence that the two probabilities are different.

Example: Partisanship 2015

A Gallop poll in 2015 based on a random sample of 12137 adults in U.S. (aged ≥ 18), found that 29% self-identified as Democrats, 26% as Republicans, and 45% as independent or other. *True or False and explain*: a 95% confidence interval for the difference of proportions of American adults self-identified as Democrats and Republicans $p_D - p_R$ is

$$0.29 - 0.26 \pm 1.96\sqrt{\frac{0.29(1-0.29)}{12137} + \frac{0.26(1-0.26)}{12137}} = (0.019, 0.041)$$

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- How many samples are there? One or two?

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- How many samples are there? One or two?
- The two sample percentages, 29% and 26%, are calculated based on the same sample. They were not independent, but negatively correlated. The more people identified as Democrats, the fewer identified as Republicans. One cannot use a two-sample CI here.

http://http://news.gallup.com/poll/188096/democratic-republican-identification-near-historical-lows.aspx

Example: Partisanship 2015 v.s. 2011

Continue the previous example. Another survey of 15,000 American adults in 2011 found that 35.3% identified as Democrats, 34.0% as Republicans, and 30.7% as independent or other. Assume both surveys in 2011 and 2015 were both based on simple random samples. Can we test whether there were more American adults self-identified as independent or other in 2015 than in 2011 using a two-sample z-test for proportions?

Yes, the percentages identified as independent or others in 2011 and in 2015 were based on two independent samples.

Example: Partisanship 2015 v.s. 2011

For testing $H_0 : p_{2011} = p_{2015}$,

$$\hat{p}_{2015} - \hat{p}_{2011} = 0.450 - 0.307 \approx 0.143$$

Example: Partisanship 2015 v.s. 2011

For testing $H_0 : p_{2011} = p_{2015}$,

$$\hat{p}_{2015} - \hat{p}_{2011} = 0.450 - 0.307 \approx 0.143$$

$$\hat{p} = \frac{0.450 \times 12137 + 0.307 \times 15000}{12137 + 15000} \approx 0.371$$

Example: Partisanship 2015 v.s. 2011

For testing $\mathrm{H}_0 : p_{2011} = p_{2015}$,

$$
\begin{aligned}
\hat{p}_{2015} - \hat{p}_{2011} &= 0.450 - 0.307 \approx 0.143 \\
\hat{p} &= \frac{0.450 \times 12137 + 0.307 \times 15000}{12137 + 15000} \approx 0.371 \\
\mathrm{SE} &= \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \\
&\approx \sqrt{0.371(1-0.371)\left(\frac{1}{12137} + \frac{1}{15000}\right)} \approx 0.00590
\end{aligned}
$$

Example: Partisanship 2015 v.s. 2011

For testing $\text{H}_0 : p_{2011} = p_{2015}$,

$$
\begin{aligned}
\hat{p}_{2015} - \hat{p}_{2011} &= 0.450 - 0.307 \approx 0.143 \\
\hat{p} &= \frac{0.450 \times 12137 + 0.307 \times 15000}{12137 + 15000} \approx 0.371 \\
\text{SE} &= \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \\
&\approx \sqrt{0.371(1-0.371)\left(\frac{1}{12137} + \frac{1}{15000}\right)} \approx 0.00590 \\
z\text{-statistic} &= \frac{\hat{p}_{2015} - \hat{p}_{2011}}{\text{SE}} \approx \frac{0.143}{0.00590} \approx 24.2
\end{aligned}
$$

As the z-statistic is huge, there is super strong evidence that there were a higher percentages of American adults self-identified as independent or other in 2015 than in 2011.

Summary of CIs and Test Statistics

	Confidence Interval	H_0	Test Statistic
(1-sample mean)	$\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$	$\mu = \mu_0$	$t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$
(1-sample proportion)	$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$	$p = p_0$	$z = \frac{\widehat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$
paired	$\bar{d} \pm t_{\alpha/2,n-1}\frac{s_d}{\sqrt{n}}$ where $d_i = x_{1i} - x_{2i}$	$\mu_1 - \mu_2 = \Delta_0$	$t = \frac{\bar{d}-\Delta_0}{s_d/\sqrt{n}}$ n = # of pairs
(2-sample mean)	$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2,\nu}$SE where SE = $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ if $\sigma_1 \neq \sigma_2$ or SE = $s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ where $s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ if $\sigma_1 = \sigma_2$	$\mu_1 - \mu_2 = \Delta_0$	$t = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\text{SE}}$
(2-sample proportions)	CI for $p_1 - p_2$: $(\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2} \times \sqrt{\frac{\widehat{p}_1(1-\widehat{p}_1)}{n_1}+\frac{\widehat{p}_2(1-\widehat{p}_2)}{n_2}}$ H_0: $p_1 = p_2$, $z = \frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $\widehat{p} = \frac{x_1+x_2}{n_1+n_2}$		

A Larger Population Does NOT Require a Larger Sample

SE are all proportional to $\frac{1}{\sqrt{\text{Sample Size}}}$.

- All the SEs depend on the sample size only, not the population size.
- The relative size of a sample to the population size doesn't matter. It is the absolute size of a sample that matters.
- A larger population does NOT require a larger sample!