

Measuring Causality between Volatility and Returns with High-Frequency Data*

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ABSTRACT

In this paper we measure the relationship between volatility and return with high-frequency equity returns. The leverage hypothesis claims that return shocks lead to changes in conditional volatility. The feedback effect theory, based on the existence of a time-varying risk premium, implies that return shocks are caused by changes in conditional volatility. Within the framework of a vector autoregressive linear model of returns and realized volatility, we quantify these effects by applying short-run and long-run causality measures proposed by Dufour and Taamouti (2005). These causality measures go beyond simple correlation measures used recently by Bollerslev, Litvinova, and Tauchen (2006). Using 5-minute observations on S&P 500 Index futures contracts, we measure a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility feedback effect is found to be insignificant at all horizons. We also use these causality measures to quantify and test statistically the dynamic impact of good and bad news on volatility. We measure a much stronger impact for bad news at several horizons.

Keywords: Volatility asymmetry, Leverage effect, Volatility Feedback Effect, Multi-Horizon Causality, Causality Measure, High-frequency Data, Realized volatility, bipower variation.

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1 Introduction

One of the many stylized facts about equity returns is an asymmetric relationship between returns and volatility. In the literature there are two explanations for volatility asymmetry. The first is the leverage effect. A decrease in the price of an asset increases financial leverage and the probability of bankruptcy, making the asset riskier, hence an increase in volatility [see Black (1976) and Christie (1982)]. When applied to an equity index, this original idea translates into a dynamic leverage effect¹. The second explanation or volatility feedback effect is related to the time-varying risk premium theory: if volatility is priced, an anticipated increase in volatility would raise the rate of return, requiring an immediate stock price decline in order to allow for higher future returns [see Pindyck (1984), French, Schwert and Stambaugh (1987), Campbell and Hentschel (1992), and Bekaert and Wu (2000)].

As mentioned by Bekaert and Wu (2000) and more recently by Bollerslev et al. (2006), the difference between the leverage and volatility feedback explanations for volatility asymmetry is related to the issue of *causality*. The leverage effect explains why a negative return shock leads to higher subsequent volatility, while the volatility feedback effect justifies how an increase in volatility may result in a negative return. Thus, volatility asymmetry may result from various causality links: from returns to volatility, from volatility to returns, instantaneous causality, all of these causal effects, or just some of them.

Bollerslev et al. (2006) looked at these relationships using high frequency data and realized volatility measures. This strategy increases the chance to detect true causal links since aggregation may make the relationship between returns and volatility simultaneous. Using an observable approximation for volatility avoids the necessity to commit to a volatility model. Their empirical strategy is thus to use correlation between returns and realized volatility to measure and compare the magnitude of the leverage and volatility feedback effects. However, correlation is a measure of linear association but does not necessarily imply a causal relationship. In this paper, we propose an approach which consists in modelling at high frequency both returns and volatility as a vector autoregressive (*VAR*) model and using short and long run causality measures proposed by Taamouti and Dufour (2005) to

¹The concept of leverage effect, which means that negative returns today increases volatility of tomorrow, has been introduced in the context of the individual stocks (individual firms). However, this concept was also conserved and studied within the framework of the stock market indices by Bouchaud, Matacz, and Potters (2001), Jacquier, Polson, and Rossi (2004), Brandt and Kang (2004), Ludvigson and Ng (2005), Bollerslev et al. (2006), among others.

quantify and compare the strength of dynamic leverage and volatility feedback effects.

Studies focusing on the leverage hypothesis [see Christie (1982) and Schwert (1989)] conclude that it cannot completely account for changes in volatility. However, for the volatility feedback effect, there are conflicting empirical findings. French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Ghysels, Santa-Clara, and Valkanov (2005), find the relation between volatility and expected returns to be positive, while Turner, Startz, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993), and Nelson (1991) find the relation to be negative. Often the coefficient linking volatility to returns is statistically insignificant. Ludvigson and Ng (2005), find a strong positive contemporaneous relation between the conditional mean and conditional volatility and a strong negative lag-volatility-in-mean effect. Guo and Savickas (2006), conclude that the stock market risk-return relation is positive, as stipulated by the CAPM; however, idiosyncratic volatility is negatively related to future stock market returns. For individual assets, Bekaert and Wu (2000) argue that the volatility feedback effect dominates the leverage effect empirically. With high-frequency data, Bollerslev et al. (2006) find an important negative correlation between volatility and current and lagged returns lasting for several days. However, correlations between returns and lagged volatility are all close to zero.

A second contribution of this paper is to show that the causality measures may help to quantify the dynamic impact of bad and good news on volatility². A common approach for empirically visualizing the relationship between news and volatility is provided by the news-impact curve originally studied by Pagan and Schwert (1990) and Engle and Ng (1993). To study the effect of current return shocks on future expected volatility, Engle and Ng (1993) introduced the News Impact Function (hereafter *NIF*). The basic idea of this function is to condition at time $t + 1$ on the information available at time t and earlier, and then consider the effect of the return shock at time t on volatility at time $t + 1$ in isolation [see Yu (2005)]. Engle and Ng (1993) explained that this curve, where all the lagged conditional variances are evaluated at the level of the asset return unconditional variance, relates past positive and negative returns to current volatility. In this paper, we propose a new curve

²In this study we mean by bad and good news negative and positive returns, respectively. In parallel, there is another literature about the impact of macroeconomic news announcements on financial markets (e.g. volatility), see for example Cutler, Poterba and Summers (1989), Schwert (1981), Pearce and Roley (1985), Hardouvelis (1987), Haugen, Talmor and Torous (1991), Jain (1988), McQueen and Roley (1993), Balduzzi, Elton, and Green (2001), Andersen, Bollerslev, Diebold, and Vega (2003), and Huang (2007), among others.

for the impact of news on volatility based on causality measures. In contrast to the *NIF* of Engle and Ng (1993), our curve can be constructed for parametric and stochastic volatility models and it allows one to consider all the past information about volatility and returns. Furthermore, we build confidence intervals using a bootstrap technique around our curve, which provides an improvement over current procedures in terms of statistical inference.

Using 5-minute observations on S&P 500 Index futures contracts, we measure a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility feedback effect is found to be negligible at all horizons. These findings are consistent with those in Bollerslev et al. (2006). We also use the causality measures to quantify and test statistically the dynamic impact of good and bad news on volatility. First, we assess by simulation the ability of causality measures to detect the differential effect of good and bad news in various parametric volatility models. Then, empirically, we measure a much stronger impact for bad news at several horizons. Statistically, the impact of bad news is found to be significant for the first four days, whereas the impact of good news is negligible at all horizons.

The plan of this paper is as follows. In section 2, we define volatility measures in high frequency data and we review the concept of causality at different horizons and its measures. In section 3, we propose and discuss VAR models that allow us to measure leverage and volatility feedback effects with high frequency data, as well as to quantify the dynamic impact of news on volatility. In section 4, we conduct a simulation study with several symmetric and asymmetric volatility models to assess if the proposed causality measures capture well the dynamic impact of news. Section 5 describes the high frequency data, the estimation procedure and the empirical findings. In section 6 we conclude by summarizing the main results.

2 Volatility and causality measures

Since we want to measure causality between volatility and returns at high frequency, we need to build measures for both volatility and causality. For volatility, we use various measures of realized volatility introduced by Andersen, Bollerslev, and Diebold (2003) [see also Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Labys (2001), and Barndorff-Nielsen and Shephard (2002a,b)]. For causality, we rely on the short and long run causality measures proposed by Taamouti and Dufour (2005).

We first set notations. We denote the time- t logarithmic price of the risky asset or portfolio by p_t and the continuously compounded returns from time t to $t + 1$ by $r_{t+1} = p_{t+1} - p_t$. We assume that the price process may exhibit both stochastic volatility and jumps. It could belong to the class of continuous-time jump diffusion processes,

$$dp_t = \mu_t dt + \sigma_t dW_t + \kappa_t dq_t, \quad 0 \leq t \leq T, \quad (1)$$

where μ_t is a continuous and locally bounded variation process, σ_t is the stochastic volatility process, W_t denotes a standard Brownian motion, dq_t is a counting process with $dq_t = 1$ corresponding to a jump at time t and $dq_t = 0$ otherwise, with jump intensity λ_t . The parameter κ_t refers to the size of the corresponding jumps. Thus, the quadratic variation of return from time t to $t + 1$ is given by:

$$[r, r]_{t+1} = \int_t^{t+1} \sigma_s^2 ds + \sum_{0 < s \leq t} \kappa_s^2,$$

where the first component, called integrated volatility, comes from the continuous component of (1), and the second term is the contribution from discrete jumps. In the absence of jumps, the second term on the right-hand-side disappears, and the quadratic variation is simply equal to the integrated volatility.

2.1 Volatility in high frequency data: realized volatility, bipower variation, and jumps

In this section, we define the various high-frequency measures that we will use to capture volatility. In what follows we normalize the daily time-interval to unity and we divide it into h periods. Each period has length $\Delta = 1/h$. Let the discretely sampled Δ -period returns be denoted by $r_{(t,\Delta)} = p_t - p_{t-\Delta}$ and the daily return is $r_{t+1} = \sum_{j=1}^h r_{(t+j\Delta,\Delta)}$. The daily realized volatility is defined as the summation of the corresponding h high-frequency intradaily squared returns,

$$RV_{t+1} \equiv \sum_{j=1}^h r_{(t+j\Delta,\Delta)}^2.$$

As noted by Andersen, Bollerslev, and Diebold (2003) [see also Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Labys (2001), Barndorff-Nielsen and Shephard (2002a,b) and Comte and Renault (1998)], the realized volatility satisfies

$$\lim_{\Delta \rightarrow 0} RV_{t+1} = \int_t^{t+1} \sigma_s^2 ds + \sum_{0 < s \leq t} \kappa_s^2 \quad (2)$$

and this means that RV_{t+1} is a consistent estimator of the sum of the integrated variance³ $\int_t^{t+1} \sigma_s^2 ds$ and the jump contribution. Similarly, a measure of standardized bipower variation is given by

$$BV_{t+1} \equiv \frac{\pi}{2} \sum_{j=2}^h |r_{(t+j\Delta, \Delta)}| |r_{(t+(j-1)\Delta, \Delta)}|.$$

Based on Barndorff-Nielsen and Shephard's (2003c) results [see also Barndorff-Nielsen, Graversen, Jacod, Podolskij, and Shephard (2005)], under reasonable assumptions about the dynamics of (1), the bipower variation satisfies,

$$\lim_{\Delta \rightarrow 0} BV_{t+1} = \int_t^{t+1} \sigma_s^2 ds. \quad (3)$$

Again, equation (3) means that BV_{t+1} provides a consistent estimator of the integrated variance unaffected by jumps.

Finally, as noted by Barndorff-Nielsen and Shephard (2003c), combining the results in equation (2) and (3), the contribution to the quadratic variation due to the discontinuities (jumps) in the underlying price process may be consistently estimated by

$$\lim_{\Delta \rightarrow 0} (RV_{t+1} - BV_{t+1}) = \sum_{0 < s \leq t} \kappa_s^2. \quad (4)$$

We can also define the relative measure

$$RJ_{t+1} = \frac{(RV_{t+1} - BV_{t+1})}{RV_{t+1}} \quad (5)$$

or the corresponding logarithmic ratio

$$J_{t+1} = \log(RV_{t+1}) - \log(BV_{t+1}).$$

Huang and Tauchen (2005) argue that these are more robust measures of the contribution of jumps to total price variation.

³See Meddahi (2002) for an interesting related literature about a theoretical comparison between integrated and realized volatility in the absence of jumps.

Since in practice J_{t+1} can be negative in a given sample, we impose a non-negativity truncation of the actual empirical jump measurements,

$$J_{t+1} \equiv \max[\log(RV_{t+1}) - \log(BV_{t+1}), 0].$$

as suggested by Barndorff-Nielsen and Shephard (2003c).⁴

2.2 Short-run and long-run causality measures

The concept of noncausality that we consider in this paper is defined in terms of orthogonality conditions between subspaces of a Hilbert space of random variables with finite second moments. To give a formal definition of noncausality at different horizons, we need to consider the following notations. We denote $\underline{r}(t) = \{r_{t+1-s}, s \geq 1\}$ and $\underline{\sigma}_t^2 = \{\sigma_{t+1-s}^2, s \geq 1\}$ the information sets which contain all the past and present values of returns and volatility, respectively. We denote by I_t the information set which contains $\underline{r}(t)$ and $\underline{\sigma}_t^2$. For any information set B_t ⁵, we denote by $Var[r_{t+h} | B_t]$ (respectively $Var[\sigma_{t+h}^2 | B_t]$) the variance of the forecast error of r_{t+h} (respectively σ_{t+h}^2) based on the information set B_t . Thus, we have the following definition of noncausality at different horizons [see Dufour and Renault (1998)].

Definition 1 For $h \geq 1$, where h is a positive integer,

(i) r does not cause σ^2 at horizon h given $\underline{\sigma}_t^2$, denoted $r \not\rightarrow_h \sigma^2 | \underline{\sigma}_t^2$, iff

$$Var(\sigma_{t+h}^2 | \underline{\sigma}_t^2) = Var(\sigma_{t+h}^2 | I_t),$$

(ii) r does not cause σ^2 up to horizon h given $\underline{\sigma}_t^2$, denoted $r \not\rightarrow_{(h)} \sigma^2 | \underline{\sigma}_t^2$, iff

$$r \not\rightarrow_k \sigma^2 | \underline{\sigma}_t^2 \text{ for } k = 1, 2, \dots, h,$$

(iii) r does not cause σ^2 at any horizon given $\underline{\sigma}_t^2$, denoted $r \not\rightarrow_{(\infty)} \sigma^2 | \underline{\sigma}_t^2$, iff

$$r \not\rightarrow_k \sigma^2 | \underline{\sigma}_t^2 \text{ for all } k = 1, 2, \dots$$

⁴See also Andersen, Bollerslev, and Diebold (2003).

⁵ B_t can be equal to I_t , $\underline{r}(t)$, or $\underline{\sigma}_t^2$.

Definition (1) corresponds to causality from r to σ^2 and means that r causes σ^2 at horizon h if the past of r improves the forecast of σ_{t+h}^2 given the information set $\underline{\sigma}_t^2$. We can similarly define noncausality at horizon h from σ^2 to r . This definition is a simplified version of the original definition given by Dufour and Renault (1998). Here we consider an information set I_t which contains only two variables of interest r and σ^2 . However, Dufour and Renault (1998) consider a third variable, called an auxiliary variable, which can transmit causality between r and σ^2 at horizon h strictly higher than one even if there is no causality between the two variables at horizon 1. In the absence of an auxiliary variable, Dufour and Renault (1998) show that noncausality at horizon 1 implies noncausality at any horizon h strictly higher than one. In other words, if we suppose that $I_t = \underline{r(t)} \cup \underline{\sigma}_t^2$, then we have:

$$\begin{aligned} r \underset{1}{\not\rightarrow} \sigma^2 \mid \underline{\sigma}_t^2 &\implies r \underset{(\infty)}{\not\rightarrow} \sigma^2 \mid \underline{\sigma}_t^2, \\ \sigma^2 \underset{1}{\not\rightarrow} r \mid \underline{r(t)} &\implies \sigma^2 \underset{(\infty)}{\not\rightarrow} r \mid \underline{r(t)}. \end{aligned}$$

For $h \geq 1$, where h is a positive integer, a measure of causality from r to σ^2 at horizon h , denoted $C(r \xrightarrow{h} \sigma^2)$, is given by following function [see Dufour and Taamouti (2005)]:

$$C(r \xrightarrow{h} \sigma^2) = \ln \left[\frac{\text{Var}[\sigma_{t+h}^2 \mid \underline{\sigma}_t^2]}{\text{Var}[\sigma_{t+h}^2 \mid I_t]} \right].$$

Similarly, a measure of causality from σ^2 to r at horizon h , denoted $C(\sigma^2 \xrightarrow{h} r)$, is given by:

$$C(\sigma^2 \xrightarrow{h} r) = \ln \left[\frac{\text{Var}[r_{t+h} \mid \underline{r(t)}]}{\text{Var}[r_{t+h} \mid I_t]} \right].$$

For example, $C(r \xrightarrow{h} \sigma^2)$ measure the causal effect from r to σ^2 at horizon h given the past of σ^2 . In terms of predictability, it measures the information given by the past of r that can improve the forecast of σ_{t+h}^2 . Since $\text{Var}[\sigma_{t+h}^2 \mid \underline{\sigma}_t^2] \geq \text{Var}[\sigma_{t+h}^2 \mid I_t]$, the function $C(r \xrightarrow{h} \sigma^2)$ is nonnegative, as any measure must be. Furthermore, it is zero when $\text{Var}[\sigma_{t+h}^2 \mid \underline{\sigma}_t^2] = \text{Var}[\sigma_{t+h}^2 \mid I_t]$, or when there is no causality. However, as soon as there is causality at horizon 1, causality measures at different horizons may considerably differ.

In Dufour and Taamouti (2005), a measure of instantaneous causality between r and σ^2 at horizon h is also proposed. It is given by the function

$$C(r \leftrightarrow \sigma^2)_h = \ln \left[\frac{\text{Var}[r_{t+h} \mid I_t] \text{Var}[\sigma_{t+h}^2 \mid I_t]}{\det \Sigma(r_{t+h}, \sigma_{t+h}^2 \mid I_t)} \right],$$

where $\det \Sigma(r_{t+h}, \sigma_{t+h}^2 | I_t)$ represents the determinant of the variance-covariance matrix, denoted $\Sigma(r_{t+h}, \sigma_{t+h}^2 | I_t)$, of the forecast error of the joint process $(r, \sigma^2)'$ at horizon h given the information set I_t . Finally, they propose a measure of dependence between r and σ^2 at horizon h which is given by the following formula:

$$C^{(h)}(r, \sigma^2) = \ln \left[\frac{\text{Var}[r_{t+h} | r(t)] \text{Var}[\sigma_{t+h}^2 | \sigma_t^2]}{\det \Sigma(r_{t+h}, \sigma_{t+h}^2 | I_t)} \right].$$

This last measure can be decomposed as follows:

$$C^{(h)}(r, \sigma^2) = C(r \xrightarrow{h} \sigma^2) + C(\sigma^2 \xrightarrow{h} r) + C(r \leftrightarrow_h \sigma^2). \quad (6)$$

3 Measuring causality in a VAR model

In this section, we first study the relationship between the return r_{t+1} and its volatility σ_{t+1}^2 . Our objective is to measure and compare the strength of *dynamic* leverage and volatility feedback effects in high-frequency equity data. These effects are quantified within the context of an autoregressive (*VAR*) linear model and by using short and long run causality measures proposed by Dufour and Taamouti (2005). Since the volatility asymmetry may be the result of causality from returns to volatility [leverage effect], from volatility to returns [volatility feedback effect], instantaneous causality, all of these causal effects, or some of them, this section aims at measuring all these effects and to compare them in order to determine the most important one. We also measure the dynamic impact of return news on volatility where we differentiate good and bad news.

3.1 Measuring the leverage and volatility feedback effects

We suppose that the joint process of returns and logarithmic volatility, $(r_{t+1}, \ln(\sigma_{t+1}^2))'$ follows an autoregressive linear model

$$\begin{pmatrix} r_{t+1} \\ \ln(\sigma_{t+1}^2) \end{pmatrix} = \mu + \sum_{j=1}^p \Phi_j \begin{pmatrix} r_{t+1-j} \\ \ln(\sigma_{t+1-j}^2) \end{pmatrix} + u_{t+1}, \quad (7)$$

where

$$\mu = \begin{pmatrix} \mu_r \\ \mu_{\sigma^2} \end{pmatrix}, \quad \Phi_j = \begin{bmatrix} \Phi_{11,j} & \Phi_{12,j} \\ \Phi_{21,j} & \Phi_{22,j} \end{bmatrix} \text{ for } j = 1, \dots, p, \quad u_{t+1} = \begin{pmatrix} u_{t+1}^r \\ u_{t+1}^{\sigma^2} \end{pmatrix},$$

$$E[u_t] = 0 \text{ and } E[u_t u_t'] = \begin{cases} \Sigma_u & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases} .$$

In the empirical application σ_{t+1}^2 will be replaced by the realized volatility RV_{t+1} or the bipower variation BV_{t+1} . The disturbance u_{t+1}^r is the one-step-ahead error when r_{t+1} is forecast from its own past and the past of $\ln(\sigma_{t+1}^2)$, and similarly $u_{t+1}^{\sigma^2}$ is the one-step-ahead error when $\ln(\sigma_{t+1}^2)$ is forecast from its own past and the past of r_{t+1} . We suppose that these disturbances are each serially uncorrelated, but may be correlated with each other contemporaneously and at various leads and lags. Since u_{t+1}^r is uncorrelated with I_t ⁶, the equation for r_{t+1} represents the linear projection of r_{t+1} on I_t . Likewise, the equation for $\ln(\sigma_{t+1}^2)$ represents the linear projection of $\ln(\sigma_{t+1}^2)$ on I_t .

Equation (7) allows one to model the first two conditional moments of the asset returns. We model conditional volatility as an exponential function process to guarantee that it is positive. The first equation of the $VAR(p)$ in (7) describes the dynamics of the return as

$$r_{t+1} = \mu_r + \sum_{j=1}^p \Phi_{11,j} r_{t+1-j} + \sum_{j=1}^p \Phi_{12,j} \ln(\sigma_{t+1-j}^2) + u_{t+1}^r. \quad (8)$$

This equation allows to capture the temporary component of Fama and French's (1988) permanent and temporary components model, in which stock prices are governed by a random walk and a stationary autoregressive process, respectively. For $\Phi_{12,j} = 0$, this model of the temporary component is the same as that of Lamoureux and Zhou (1996); see Brandt and Kang (2004), and Whitelaw (1994). The second equation of $VAR(p)$ describes the volatility dynamics as

$$\ln(\sigma_{t+1}^2) = \mu_{\sigma^2} + \sum_{j=1}^p \Phi_{21,j} r_{t+1-j} + \sum_{j=1}^p \Phi_{22,j} \ln(\sigma_{t+1-j}^2) + u_{t+1}^{\sigma^2}, \quad (9)$$

and it represents the standard stochastic volatility model. For $\Phi_{21,j} = 0$, equation (9) can be viewed as the stochastic volatility model estimated by Wiggins (1987), Andersen and Sorensen (1994), and many others. However, in this paper we consider that σ_{t+1}^2 is not a latent variable and it can be approximated by realized or bipower variations from high-frequency data.

⁶ $I_t = \{r_{t+1-s}, s \geq 1\} \cup \{\sigma_{t+1-s}^2, s \geq 1\}$.

We also note that the conditional mean equation includes the volatility-in-mean model used by French et al. (1987) and Glosten et al. (1993) to explore the contemporaneous relationship between the conditional mean and volatility [see Brandt and Kang (2004)]. To illustrate the connection to the volatility-in-mean model, we pre-multiply the system in (7) by the matrix

$$\begin{bmatrix} 1 & -\frac{Cov(r_{t+1}, \ln(\sigma_{t+1}^2))}{Var[\ln(\sigma_{t+1}^2)|I_t]} \\ -\frac{Cov(r_{t+1}, \ln(\sigma_{t+1}^2))}{Var[r_{t+1}|I_t]} & 1 \end{bmatrix}.$$

Then, the first equation of r_{t+1} is a linear function of $\underline{r(t)}$, $\underline{\ln(\sigma_{t+1}^2)}$, and the disturbance $u_{t+1}^r - \frac{Cov(r_{t+1}, \ln(\sigma_{t+1}^2))}{Var[\ln(\sigma_{t+1}^2)|I_t]} u_{t+1}^{\sigma^2}$.⁷ Since this disturbance is uncorrelated with $u_{t+1}^{\sigma^2}$, it is uncorrelated with $\underline{\ln(\sigma_{t+1}^2)}$ as well as with $\underline{r(t)}$ and $\underline{\ln(\sigma_t^2)}$. Hence the linear projection of r_{t+1} on $\underline{r(t)}$ and $\underline{\ln(\sigma_{t+1}^2)}$,

$$r_{t+1} = \nu_r + \sum_{j=1}^p \phi_{11,j} r_{t+1-j} + \sum_{j=0}^p \phi_{12,j} \ln(\sigma_{t+1-j}^2) + u_{t+1}^r \quad (10)$$

is provided by the first equation of the new system. The parameters ν_r , $\phi_{11,j}$, and $\phi_{12,j}$, for $j = 0, 1, \dots, p$, are functions of parameters in the vector μ and matrix Φ_j , for $j = 1, \dots, p$. Equation (10) is a generalized version of the usual volatility-in-mean model, in which the conditional mean depends contemporaneously on the conditional volatility. Similarly, the existence of the linear projection of $\ln(\sigma_{t+1}^2)$ on $\underline{r(t+1)}$ and $\underline{\ln(\sigma_t^2)}$,

$$\ln(\sigma_{t+1}^2) = \nu_{\sigma^2} + \sum_{j=0}^p \phi_{21,j} r_{t+1-j} + \sum_{j=1}^p \phi_{22,j} \ln(\sigma_{t+1-j}^2) + u_{t+1}^{\sigma^2} \quad (11)$$

follows from the second equation of the new system. The parameters ν_{σ^2} , $\phi_{21,j}$, and $\phi_{22,j}$, for $j = 1, \dots, p$, are functions of parameters in the vector μ and matrix Φ_j , for $j = 1, \dots, p$. The volatility model given by equation (11) captures the persistence of volatility through the terms $\phi_{22,j}$. In addition, it incorporates the effects of the mean on volatility, both at the contemporaneous and intertemporal levels through the coefficients $\phi_{21,j}$, for $j = 0, 1, \dots, p$.

Let us now consider the matrix

$$\Sigma_u = \begin{bmatrix} \Sigma_{u^r} & C \\ C & \Sigma_{u^{\sigma^2}} \end{bmatrix},$$

⁷ $\underline{\ln(\sigma_{t+1}^2)} = \{\ln(\sigma_{t+2-s}^2), s \geq 1\}$.

where Σ_{u^r} and $\Sigma_{u^{\sigma^2}}$ represent the variances of the one-step-ahead forecast errors of return and volatility, respectively. C represents the covariance between these errors. Based on equation (7), the forecast error of $(r_{t+h}, \ln(\sigma_{t+h}^2))'$ is given by:

$$e \left[(r_{t+h}, \ln(\sigma_{t+h}^2))' \right] = \sum_{i=0}^{h-1} \psi_i u_{t+h-i}, \quad (12)$$

where the coefficients ψ_i , for $i = 0, \dots, h-1$, represent the impulse response coefficients of the $MA(\infty)$ representation of model (7). These coefficients are given by the following equations

$$\begin{aligned} \psi_0 &= I, \\ \psi_1 &= \Phi_1 \psi_0 = \Phi_1, \\ \psi_2 &= \Phi_1 \psi_1 + \Phi_2 \psi_0 = \Phi_1^2 + \Phi_2, \\ \psi_3 &= \Phi_1 \psi_2 + \Phi_2 \psi_1 + \Phi_2 \psi_0 = \Phi_1^3 + \Phi_1 \Phi_2 + \Phi_2 \Phi_1 + \Phi_3, \\ &\dots \end{aligned} \quad (13)$$

where I is a 2×2 identity matrix and

$$\Phi_j = 0, \text{ for } j \geq p + 1.$$

The covariance matrix of the forecast error (12) is given by

$$Var[e[(r_{t+h}, \ln(\sigma_{t+h}^2))']] = \sum_{i=0}^{h-1} \psi_i \Sigma_u \psi_i'. \quad (14)$$

We also consider the following restricted model:

$$\begin{pmatrix} r_{t+1} \\ \ln(\sigma_{t+1}^2) \end{pmatrix} = \bar{\mu} + \sum_{j=1}^{\bar{p}} \bar{\Phi}_j \begin{pmatrix} r_{t+1-j} \\ \ln(\sigma_{t+1-j}^2) \end{pmatrix} + \bar{u}_{t+1} \quad (15)$$

where

$$\bar{\Phi}_j = \begin{bmatrix} \bar{\Phi}_{11,j} & 0 \\ 0 & \bar{\Phi}_{22,j} \end{bmatrix} \text{ for } j = 1, \dots, \bar{p}, \quad (16)$$

$$\bar{\mu} = \begin{pmatrix} \bar{\mu}_r \\ \bar{\mu}_{\sigma^2} \end{pmatrix}, \quad \bar{u}_{t+1} = \begin{pmatrix} \bar{u}_{t+1}^r \\ \bar{u}_{t+1}^{\sigma^2} \end{pmatrix},$$

$$E[\bar{u}_t] = 0 \text{ and } E[\bar{u}_t \bar{u}_t'] = \begin{cases} \bar{\Sigma}_u & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases},$$

$$\bar{\Sigma}_u = \begin{bmatrix} \Sigma_{\bar{u}r} & \bar{C} \\ \bar{C} & \Sigma_{\bar{u}\sigma^2} \end{bmatrix}.$$

Zero values in $\bar{\Phi}_j$ mean that there is noncausality at horizon 1 from returns to volatility and from volatility to returns. As mentioned in subsection 2.2, in a bivariate system, noncausality at horizon one implies noncausality at any horizon h strictly higher than one. This means that the absence of leverage effects at horizon one (respectively the absence of volatility feedback effects at horizon one) which corresponds to $\bar{\Phi}_{21,j} = 0$, for $j = 1, \dots, \bar{p}$, (respectively $\bar{\Phi}_{12,j} = 0$, for $j = 1, \dots, \bar{p}$,) is equivalent to the absence of leverage effects (respectively volatility feedback effects) at any horizon $h \geq 1$.

To compare the forecast error variance of model (7) with that of model (15), we assume that $p = \bar{p}$. Based on the restricted model (15), the covariance matrix of the forecast error of $(r_{t+h}, \ln(\sigma_{t+h}^2))'$ is given by:

$$\overline{Var}[t+h | t] = \sum_{i=0}^{h-1} \bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i', \quad (17)$$

where the coefficients $\bar{\psi}_i$, for $i = 0, \dots, h-1$, represent the impulse response coefficients of the $MA(\infty)$ representation of model (15). They can be calculated in the same way as in (13).

From the covariance matrices (14) and (17), we define the following measures of leverage and volatility feedback effects at any horizon h , where $h \geq 1$,

$$C(r \xrightarrow{h} \ln(\sigma^2)) = \ln \left[\frac{\sum_{i=0}^{h-1} e_2'(\bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i') e_2}{\sum_{i=0}^{h-1} e_2'(\psi_i \Sigma_u \psi_i') e_2} \right], \quad e_2 = (0, 1)', \quad (18)$$

$$C(\ln(\sigma^2) \xrightarrow{h} r) = \ln \left[\frac{\sum_{i=0}^{h-1} e_1'(\bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i') e_1}{\sum_{i=0}^{h-1} e_1'(\psi_i \Sigma_u \psi_i') e_1} \right], \quad e_1 = (1, 0)'. \quad (19)$$

The parametric measure of instantaneous causality at horizon h , where $h \geq 1$, is given by the following function

$$C(r \leftrightarrow \ln(\sigma^2)) = \ln \left[\frac{(\sum_{i=0}^{h-1} e_2'(\psi_i \Sigma_u \psi_i') e_2) (\sum_{i=0}^{h-1} e_1'(\bar{\psi}_i \bar{\Sigma}_{\bar{u}} \bar{\psi}_i') e_1)}{\det(\sum_{i=0}^{h-1} \psi_i \Sigma_u \psi_i')} \right].$$

Finally, the parametric measure of dependence at horizon h can be deduced from its decomposition given by equation (6).

3.2 Measuring the dynamic impact of news on volatility

In what follows we study the *dynamic* impact of bad news (negative innovations in returns) and good news (positive innovations in returns) on volatility. We quantify and compare the strength of these effects in order to determine the most important ones. To analyze the impact of news on volatility, we consider the following model,

$$\begin{aligned} \ln(\sigma_{t+1}^2) &= \mu_\sigma + \sum_{j=1}^p \varphi_j^\sigma \ln(\sigma_{t+1-j}^2) + \sum_{j=1}^p \varphi_j^- [r_{t+1-j} - E_{t-j}(r_{t+1-j})]^- \\ &\quad + \sum_{j=1}^p \varphi_j^+ [r_{t+1-j} - E_{t-j}(r_{t+1-j})]^+ + u_{t+1}^\sigma, \end{aligned} \quad (20)$$

where for $j = 1, \dots, p$,

$$[r_{t+1-j} - E_{t-j}(r_{t+1-j})]^- = \begin{cases} r_{t+1-j} - E_{t-j}(r_{t+1-j}), & \text{if } r_{t+1-j} - E_{t-j}(r_{t+1-j}) \leq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (21)$$

$$[r_{t+1-j} - E_{t-j}(r_{t+1-j})]^+ = \begin{cases} r_{t+1-j} - E_{t-j}(r_{t+1-j}), & \text{if } r_{t+1-j} - E_{t-j}(r_{t+1-j}) \geq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (22)$$

with

$$E[u_{t+1}^\sigma] = 0 \text{ and } E[(u_{t+1}^\sigma)^2] = \begin{cases} \Sigma_{u^\sigma} & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases}.$$

Equation (20) represents the linear projection of volatility on its own past and the past of centered negative and positive returns. This regression model allows one to capture the effect of centered negative or positive returns on volatility through the coefficients φ_j^- or φ_j^+ respectively, for $j = 1, \dots, p$. It also allows one to examine the different effects that large and small negative and/or positive information shocks have on volatility. This will provide a check on the results obtained in the literature on GARCH modeling, which has put forward overwhelming evidence on the effect of negative shocks on volatility.

Again, in our empirical applications, σ_{t+1}^2 will be replaced by realized volatility RV_{t+1} or bipower variation BV_{t+1} . Furthermore, the conditional mean return will be approximated by the following rolling-sample average:

$$\hat{E}_t(r_{t+1}) = \frac{1}{m} \sum_{j=1}^m r_{t+1-j}.$$

where we take an average around $m = 15, 30, 90, 120,$ and 240 days.

Now, let us consider the following restricted models:

$$\ln(\sigma_{t+1}^2) = \theta_\sigma + \sum_{i=1}^{\bar{p}} \bar{\varphi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\bar{p}} \bar{\varphi}_i^+ [r_{t+1-i} - E_{t-j}(r_{t+1-i})]^+ + e_{t+1}^\sigma \quad (23)$$

$$\ln(\sigma_{t+1}^2) = \bar{\theta}_\sigma + \sum_{i=1}^{\dot{p}} \dot{\varphi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\dot{p}} \dot{\varphi}_i^- [r_{t+1-j} - E_{t-j}(r_{t+1-j})]^- + v_{t+1}^\sigma. \quad (24)$$

Equation (23) represents the linear projection of volatility $\ln(\sigma_{t+1}^2)$ on its own past and the past of positive returns. Similarly, equation (24) represents the linear projection of volatility $\ln(\sigma_{t+1}^2)$ on its own past and the past of centred negative returns.

In our empirical application we also consider a model with non centered negative and positive returns:

$$\ln(\sigma_{t+1}^2) = \omega_\sigma + \sum_{j=1}^p \phi_j^\sigma \ln(\sigma_{t+1-j}^2) + \sum_{j=1}^p \phi_j^- r_{t+1-j}^- + \sum_{j=1}^p \phi_j^+ r_{t+1-j}^+ + \epsilon_{t+1}^\sigma,$$

where for $j = 1, \dots, p,$

$$r_{t+1-j}^- = \begin{cases} r_{t+1-j}, & \text{if } r_{t+1-j} \leq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$r_{t+1-j}^+ = \begin{cases} r_{t+1-j}, & \text{if } r_{t+1-j} \geq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$E[\epsilon_{t+1}^\sigma] = 0 \text{ and } E[(\epsilon_{t+1}^\sigma)^2] = \begin{cases} \Sigma_{e^\sigma} & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases},$$

and the corresponding restricted volatility models:

$$\ln(\sigma_{t+1}^2) = \lambda_\sigma + \sum_{i=1}^{\bar{p}} \bar{\phi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\bar{p}} \bar{\phi}_i^+ r_{t+1-i}^+ + v_{t+1}^\sigma \quad (25)$$

$$\ln(\sigma_{t+1}^2) = \bar{\lambda}_\sigma + \sum_{i=1}^{\dot{p}} \dot{\phi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\dot{p}} \dot{\phi}_i^- r_{t+1-j}^- + \varepsilon_{t+1}^\sigma. \quad (26)$$

To compare the forecast error variances of model (20) with those of models (23) and (24), we assume that $p = \bar{p} = \dot{p}$. Thus, a measure of the impact of bad news on volatility at horizon h , where $h \geq 1$, is given by the following equation:

$$C(r^- \xrightarrow{h} \ln(\sigma^2)) = \ln \left[\frac{\text{Var}[e_{t+h}^\sigma \mid \underline{\ln(\sigma_t^2)}, r^+(t)]}{\text{Var}[u_{t+h}^\sigma \mid J_t]} \right].$$

Similarly, a measure of the impact of good news on volatility at horizon h is given by:

$$C(r^+ \xrightarrow{h} \ln(\sigma^2)) = \ln \left[\frac{\text{Var}[v_{t+h}^\sigma \mid \underline{\ln(\sigma_t^2)}, r^-(t)]}{\text{Var}[u_{t+h}^\sigma \mid J_t]} \right]$$

where

$$\underline{r^-(t)} = \{[r_{t-s} - E_{t-1-s}(r_{t-s})]^- , s \geq 0\},$$

$$\underline{r^+(t)} = \{[r_{t-s} - E_{t-1-s}(r_{t-s})]^+ , s \geq 0\},$$

$$J_t = \underline{\ln(\sigma^2(t))} \cup \underline{r^-(t)} \cup \underline{r^+(t)}.$$

We also define a function which allows us to compare the impact of bad and good news on volatility. This function can be defined as follows:

$$C(r^-/r^+ \xrightarrow{h} \ln(\sigma^2)) = \ln \left[\frac{\text{Var}[e_{t+h}^\sigma \mid \underline{\ln(\sigma_t^2)}, r^+(t)]}{\text{Var}[v_{t+h}^\sigma \mid \underline{\ln(\sigma_t^2)}, r^-(t)]} \right].$$

When $C(r^-/r^+ \xrightarrow{h} \ln(\sigma^2)) \geq 0$, this means that bad news have more impact on volatility than good news. Otherwise, good news will have more impact on volatility than bad news.

4 A simulation study

In this section we verify with a thorough simulation study the ability of the causality measures to detect the well-documented asymmetry in the impact of bad and good news on

volatility [see Pagan and Schwert (1990), Gouriéroux and Monfort (1992), and Engle and Ng (1993)]. To assess the asymmetry in leverage effect, we consider the following structure. First, we suppose that returns are governed by the process

$$r_{t+1} = \sqrt{\sigma_t} \varepsilon_{t+1} \quad (27)$$

where $\varepsilon_{t+1} \sim N(0, 1)$ and σ_t represents the conditional volatility of return r_{t+1} . Since we are only interested in studying the asymmetry in leverage effect, equation (27) does not allow for a volatility feedback effect. Second, we assume that σ_t follows one of the following heteroskedastic forms:

1. *GARCH*(1, 1) model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\varepsilon_{t-1}^2; \quad (28)$$

2. *EGARCH*(1, 1) model:

$$\log(\sigma_t) = \omega + \beta \log(\sigma_{t-1}) + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}}} - \sqrt{2/\pi} \right]; \quad (29)$$

3. Nonlinear *NL-GARCH*(1, 1) model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha |\varepsilon_{t-1}|^\gamma; \quad (30)$$

4. *GJR-GARCH*(1, 1) model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1} \varepsilon_{t-1}^2 \quad (31)$$

where

$$I_{t-1} = \begin{cases} 1, & \text{if } \varepsilon_{t-1} \leq 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{for } t = 1, \dots, T;$$

5. Asymmetric *AGARCH*(1, 1) model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha(\varepsilon_{t-1} + \gamma)^2; \quad (32)$$

6. *VGARCH*(1, 1) model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha \left(\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} + \gamma \right)^2; \quad (33)$$

7. Nonlinear Asymmetric $GARCH(1, 1)$ model or $NGARCH(1, 1)$:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha(\varepsilon_{t-1} + \gamma\sqrt{\sigma_{t-1}})^2. \quad (34)$$

$GARCH$ and $NL-GARCH$ models are, by construction, symmetric. Thus, we expect that the curves of causality measures for bad and good news will be the same. Similarly, because $EGARCH$, $GJR-GARCH$, $AGARCH$, $VGARCH$, and $NGARCH$ are asymmetric we expect that these curves will be different.

Our simulation study consists in simulating returns from equation (27) and volatilities from one of the models given by equations (28)-(34). Once return and volatilities are simulated, we use the model described in subsection 3.2 to evaluate the causality measures of bad and good news for each of the above parametric models. All simulated samples are of size $n = 40,000$. We consider a large sample to eliminate the uncertainty in the estimated parameters. The parameter values for the different parametric models considered in our simulations, are reported in Table 1:⁸

In figures 1-9 we report the impact of bad and good news on volatility for the various volatility models, in the order shown above. For the $NL-GARCH(1, 1)$ model we select three values for γ : 0.5, 1.5, 2.5. Two main conclusions can be drawn from these figures. First, from figures 1, 4, 5, and 6 we see that $GARCH$ and $NL-GARCH$ are symmetric: the bad and good news have the same impact on volatility. Second, in figures 2, 3, 7, 8, and 9, we observe that $EGARCH$, $GJR-GARCH$, $AGARCH$, $VGARCH$, and $NGARCH$ are asymmetric: the bad and good news have different impact curves. More particularly, bad news have more impact on volatility than good news.

Considering the parameter values given by Table 1 in Appendix [see Engle and Ng (1993)]⁹, we found that the above parametric volatility models provide different responses to bad and good news. In the presence of bad news, Figure 10 shows that the magnitude of the volatility response is the most important in the $NGARCH$ model, followed by the $AGARCH$ and $GJR-GARCH$ models. The effect is negligible in $EGARCH$ and $VGARCH$ models. The impact of good news on volatility is more observable in $AGARCH$

⁸We also consider other parameter values from a paper by Engle and Ng (1993). The corresponding results are available upon request from the authors. These results are similar to those shown in this paper.

⁹These parameters are the results of an estimation of different parametric volatility models using the daily returns series of the Japanese TOPIX index from January 1, 1980 to December 31, 1988 [see Engle and Ng (1993) for more details].

and *NGARCH* models [see Figure 11]. Overall, we can conclude that the causality measures capture quite well the effects of returns on volatility both qualitatively and quantitatively. We now apply these measures to actual data. Instead of estimating a model for volatility as most of the previous studies have done [see for example Campbell and Hentschel (1992), Bekaert and Wu (2000), Glosten, Jagannathan, and Runkle (1993), and Nelson (1991)], we use a proxy measure given by realized volatility or bipower variation based on high-frequency data.

5 An empirical application

In this section, we first describe the data used to measure causality in the VAR models of the previous sections. Then we explain how to estimate confidence intervals of causality measures for leverage and volatility feedback effects. Finally, we discuss our findings.

5.1 Data

Our data consists of high-frequency tick-by-tick transaction prices for the S&P 500 Index futures contracts traded on the Chicago Mercantile Exchange, over the period January 1988 to December 2005 for a total of 4494 trading days. We eliminated a few days where trading was thin and the market was open for a shortened session. Due to the unusually high volatility at the opening, we also omit the first five minutes of each trading day [see Bollerslev et al. (2006)]. For reasons associated with microstructure effects we follow Bollerslev et al. (2006) and the literature in general and aggregate returns over five-minute intervals. We calculate the continuously compounded returns over each five-minute interval by taking the difference between the logarithm of the two tick prices immediately preceding each five-minute mark to obtain a total of 77 observations per day [see Dacorogna et al. (2001) and Bollerslev et al. (2006) for more details]. We also construct hourly and daily returns by summing 11 and 77 successive five-minute returns, respectively.

Summary statistics for the five-minute, hourly, and daily returns are given in Table 2. The daily returns are displayed in Figure 16. Looking at Table 2 and Figure 16 we can state three main stylized facts. First, the unconditional distributions of the five-minute, hourly, and daily returns show the expected excess kurtosis and negative skewness. The sample kurtosis is much greater than the normal value of three for all three series. Second, whereas the unconditional distribution of the hourly returns appears to be skewed to the left, the

sample skewness coefficients for the five-minute and daily returns are, loosely speaking, both close to zero.

We also compute various measures of return volatility, namely realized volatility and bipower variation, both in levels and in logarithms. The time series plots [see Figures 17, 18, 19, and 20] show clearly the familiar volatility clustering effect, along with a few occasional very large absolute returns. It also follows from Table 3 that the unconditional distributions of realized and bipower volatility measures are highly skewed and leptokurtic. However, the logarithmic transform renders both measures approximately normal [Andersen, Bollerslev, Diebold, and Ebens (2001)]. We also note that the descriptive statistics for the relative jump measure, J_{t+1} , clearly indicate a positively skewed and leptokurtic distribution.

One way to test if realized and bipower volatility measures are significantly different is to test for the presence of jumps in the data. We recall that,

$$\lim_{\Delta \rightarrow 0} (RV_{t+1}) = \int_t^{t+1} \sigma_s^2 ds + \sum_{0 < s \leq t} \kappa_s^2, \quad (35)$$

where $\sum_{0 < s \leq t} \kappa_s^2$ represents the contribution of jumps to total price variation. In the absence of jumps, the second term on the right-hand-side disappears, and the quadratic variation is simply equal to the integrated volatility: or asymptotically ($\Delta \rightarrow 0$) the realized variance is equal to the bipower variance.

Many statistics have been proposed to test for the presence of jumps in financial data [see for example Barndorff-Nielsen and Shephard (2003b), Andersen, Bollerslev, and Diebold (2003), Huang and Tauchen (2005), among others]. In this paper, we test for the presence of jumps in our data by considering the following test statistics:

$$z_{QP,l,t} = \frac{RV_{t+1} - BV_{t+1}}{\sqrt{((\frac{\pi}{2})^2 + \pi - 5)\Delta QP_{t+1}}}, \quad (36)$$

$$z_{QP,t} = \frac{\log(RV_{t+1}) - \log(BV_{t+1})}{\sqrt{((\frac{\pi}{2})^2 + \pi - 5)\Delta \frac{QP_{t+1}}{BV_{t+1}^2}}}, \quad (37)$$

$$z_{QP,lm,t} = \frac{\log(RV_{t+1}) - \log(BV_{t+1})}{\sqrt{((\frac{\pi}{2})^2 + \pi - 5)\Delta \max(1, \frac{QP_{t+1}}{BV_{t+1}^2})}}, \quad (38)$$

where QP_{t+1} is the realized Quad-Power Quarticity [Barndorff-Nielsen and Shephard (2003a)], with

$$QP_{t+1} = h\mu_1^{-4} \sum_{j=4}^h |r_{(t+j).\Delta,\Delta} || r_{(t+(j-1).\Delta,\Delta} || r_{(t+(j-2).\Delta,\Delta} || r_{(t+(j-3).\Delta,\Delta} |,$$

and $\mu_1 = \sqrt{\frac{2}{\pi}}$.

For each time t , the statistics $z_{QP,l,t}$, $z_{QP,t}$, and $z_{QP,lm,t}$ follow a Normal distribution $N(0, 1)$ as $\Delta \rightarrow 0$, under the assumption of no jumps. The results of testing for jumps in our data are plotted in Figures 12-15. Figure 12 represents the Quantile-Quantile Plot (hereafter QQ Plot) of the relative measure of jumps given by equation (5). The other Figures, see 13, 14, and 15, represent the QQ Plots of the $z_{QP,l,t}$, $z_{QP,t}$, and $z_{QP,lm,t}$ statistics, respectively. When there are no jumps, we expect that the blue and red lines in Figures 12-15 will coincide. However, as these figures show, the two lines are clearly distinct, indicating the presence of jumps in our data. Therefore, we will present our results for both realized volatility and bipower variation.

5.2 Estimation of causality measures

We apply short-run and long-run causality measures to quantify the strength of relationships between return and volatility. We use *OLS* to estimate the $VAR(p)$ models described in sections 3 and 3.2 and the Akaike information criterion to specify their orders. To obtain consistent estimates of the causality measures we simply replace the unknown parameters by their estimates.¹⁰ We calculate causality measures for various horizons $h = 1, \dots, 20$. A higher value for a causality measure indicates a stronger causality. We also compute the corresponding nominal 95% bootstrap confidence intervals according to the procedure described in the Appendix.

5.3 Results

We examine several empirical issues regarding the relationship between volatility and returns. Because volatility is unobservable and high-frequencies data were not available,

¹⁰See proof of consistency of the estimation in Dufour and Taamouti (2005).

these issues have been addressed before mainly in the context of volatility models. Recently, Bollerslev et al. (2006) looked at these relationships using high frequency data and realized volatility measures. As they emphasize, the fundamental difference between the leverage and the volatility feed-back explanations lies in the direction of causality. The leverage effect explains why a low return causes higher subsequent volatility, while the volatility feedback effect captures how an increase in volatility may cause a negative return. However, they studied only correlations between returns and volatility at various leads and lags and not causality relationships between both. The concept of causality introduced by Granger (1969) necessitates an information set and is conducted in the framework of a model between the variables of interest. Moreover, it is also important economically to measure the strength of this causal link and to test if the effect is significantly different from zero. In measuring causal relationship, aggregation is of course a major problem. Low frequency data may mask the true causal relationship between the variables. Looking at high-frequency data offers an ideal setting to isolate, if any, causal effects. Formulating a VAR model to study causality allows also to distinguish between the immediate or current effects between the variables and the effects of the lags of one variable on the other. It should be emphasized also that even for studying the relationship at daily frequencies, using high-frequency data to construct daily returns and volatilities provides better estimates than using daily returns as most previous studies have done. Since realized volatility is an approximation of the true unobservable volatility we study the robustness of the results to another measure, the bipower variation, which is robust to the presence of jumps.

Our empirical results will be presented mainly through graphs. Each figure will report the causality measure on the vertical axis while the horizon will appear on the horizontal axis. We also draw in each figure the 95% bootstrap confidence intervals. With five-minute intervals we could conceivably estimate the VAR model at this frequency. However if we wanted to allow for enough time for the effects to develop we would need a large number of lags in the VAR model and sacrifice efficiency in the estimation. This problem arises in studies of volatility forecasting. Researchers have use several schemes to group five-minute intervals, in particular the HAR-RV or the MIDAS schemes¹¹.

¹¹The HAR-RV scheme, in which the realized volatility is parameterized as a linear function of the lagged realized volatilities over different horizons has been proposed by Müller et al. (1997) and Corsi (2003). The MIDAS scheme, based on the idea of distributed lags, has been analyzed and estimated by Ghysels, Santa-Clara and Valkanov (2002).

We decided to aggregate the returns at hourly frequency and study the corresponding intradaily causality relationship. between returns and volatility. As illustrated in figures 24 (log realized volatility) and 25 (log bipower variation), we find that the leverage effect is statistically significant for the first four hours but that it is small in magnitude. The volatility feedback effect in hourly data is negligible at all horizons [see tables 4 and 5].

Using daily observations, calculated with high frequency data, we measure a strong leverage effect for the first three days. This result is the same with both realized and bipower variations [see figures 22 and 23]. The volatility feedback effect is found to be negligible at all horizons [see tables 6 and 7]. By comparing these two effects, we find that the leverage effect is more important than the volatility feedback effect [see figures 30 and 31]. The comparison between the leverage effects in hourly and daily data reveal that this effect is more important in daily then in hourly returns [see figures 32 and 33].

If the feedback effect from volatility to returns is almost-non-existent, it is apparent in figures 26 and 27 that the instantaneous causality between these variables exists and remains economically and statistically important for several days. This means that volatility has a contemporaneous effect on returns, and similarly returns have a contemporaneous effect on volatility. These results are confirmed with both realized and bipower variations. Furthermore, as illustrated in figures 28 and 29, dependence between volatility and returns is also economically and statistically important for several days.

Since only the causality from returns to volatility is significant, it is important to check if negative and positive returns have a different impact on volatility. To answer this question we have calculated the causality measures from centered and non centered positive and negative returns to volatility. The empirical results are graphed in figures 34-45 and reported in tables 8 – 11. We find a much stronger impact of bad news on volatility for several days. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons. Figures 46 and 47 make it possible to compare for both realized and bipower variations the impact of bad and good news on volatility. As we can see, bad news have more impact on volatility than good news at all horizons.

Finally, to study the temporal aggregation effect on the relationship between returns and volatility, we compare the conditional dependence between returns and volatility at several levels of aggregation: one hour, one day, two days, 3 days, 6 days, 14 days, and 21 days. The empirical results show that the dependence between returns and volatility is an

increasing function of temporal aggregation [see Figure 50]. This is still true for the 21 first days, after which the dependence decreases.

6 Conclusion

In this paper we analyze and quantify the relationship between volatility and returns with high-frequency equity returns. Within the framework of a vector autoregressive linear model of returns and realized volatility or bipower variation, we quantify the dynamic leverage and volatility feedback effects by applying short-run and long-run causality measures proposed by Dufour and Taamouti (2005). These causality measures go beyond simple correlation measures used recently by Bollerslev, Litvinova, and Tauchen (2006).

Using 5-minute observations on S&P 500 Index futures contracts, we measure a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility feedback effect is found to be negligible at all horizons

We also use causality measures to quantify and test statistically the dynamic impact of good and bad news on volatility. First, we assess by simulation the ability of causality measures to detect the differential effect of good and bad news in various parametric volatility models. Then, empirically, we measure a much stronger impact for bad news at several horizons. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons.

References

- [1] Andersen, T.G., T. Bollerslev, and F.X. Diebold. (2003). "Some Like it Smooth, and Some Like it Rough: Untangling Continuous and Jump Components in Measuring, Modeling, and Forecasting Asset Return Volatility," *Working Paper*.
- [2] Andersen, T.G., T. Bollerslev, F.X. Diebold, and H. Ebens. (2001). "The Distribution of Stock Return Volatility," *Journal of Financial Economics*, 61, 1, 43-76.
- [3] Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys. (2001). "The Distribution of Realized Exchange Rate Volatility," *Journal of the American Statistical Association*, 96, 42-55.
- [4] Andersen, T.G., T. Bollerslev, and F.X. Diebold. (2003). "Parametric and Non-Parametric Volatility Measurement," *Handbook of Financial Econometrics* (L.P Hansen and Y. Ait-Sahalia, eds.). Elsevier Science, New York, forthcoming.
- [5] Andersen, T.G. and T. Bollerslev. (1998). "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts," *International Economic Review*, 39, 885-905.
- [6] Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys. (2001). "The Distribution of Realized Exchange Rate Volatility," *Journal of American Statistical Association*, 96, 42-55.
- [7] Andersen, T. G., T. Bollerslev, F. X. Diebold, and C. Vega. (2003). "Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange," *American Economic Review*, 93, 38-62.
- [8] Ang, A. and J. Liu. (2006). "Risk, Return, and Dividends," *forthcoming Journal of Financial Economics*.
- [9] Balduzzi, P., E. J. Elton, and T. C. Green. (2001). "Economic News and Bond Prices: Evidence from the U.S. Treasury Market," *Journal of Financial and Quantitative Analysis*, 36, 523-544.

- [10] Barndorff-Nielsen, O.E. and N. Shephard. (2002a). “Econometric Analysis of Realized Volatility and its Use in Estimating Stochastic Volatility Models,” *Journal of the Royal Statistical Society*, 64, 253-280.
- [11] Barndorff-Nielsen, O.E. and N. Shephard. (2002b). “Estimating Quadratic Variation Using Realized Variance,” *Journal of Applied Econometrics*, 17, 457-478.
- [12] Barndorff-Nielsen, O.E. and N. Shephard. (2003c). “Power and Bipower Variation with Stochastic and Jumps,” Manuscript, Oxford University.
- [13] Barndorff-Nielsen, O.E., S.E. Graversen, J. Jacod, M. Podolskij, and N. Shephard. (2005). “A Central Limit Theorem for Realized Power and Bipower Variations of Continuous Semimartingales.” Working Paper, Nuffield College, Oxford University; forthcoming in Yu Kabanov and Robert Liptser (eds.), *From Stochastic Analysis to Mathematical Finance, Festschrift for Albert Shiryaev*. New York: Springer-Verlag.
- [14] Bekaert, G. and G. Wu. (2000). “Asymmetric Volatility and Risk in Equity Markets,” *The Review of Financial Studies*, 13, 1-42.
- [15] Black, F. (1976). “Studies of Stock Price Volatility Changes,” *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistics*, 177-181.
- [16] Bollerslev, T. and H. Zhou. (2005). “Volatility Puzzles: A Unified Framework for Gauging Return-Volatility Regressions,” *Journal of Econometrics*, forthcoming.
- [17] Bollerslev, T., U. Kretschmer, C. Pigorsch, and G. Tauchen. (2005). “A Discrete-Time Model for Daily S&P500 Returns and Realized Variations: Jumps and Leverage Effects,” *Working Paper*.
- [18] Bollerslev, T., J. Litvinova, and G. Tauchen. (2006). “Leverage and Volatility Feedback Effects in High-Frequency Data,” *Journal of Financial Econometrics* 4 (3), 353-384.
- [19] Bouchaud, J-P., A. Matacz, and M. Potters. (2001). “Leverage Effect in Financial Markets: The Retarded Volatility Model,” *Physical Review Letters*, 87, 228 701.

- [20] Brandt, M. W., and Q. Kang (2004). On the Relationship Between the Conditional Mean and Volatility of Stock Returns: A Latent VAR Approach, *Journal of Financial Economics*, 72, 2004, 217-257.
- [21] Campbell, J. and L. Hentschel. (1992). “No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns,” *Journal of Financial Economics*, 31, 281-331.
- [22] Christie, A.A. (1982). “The Stochastic Behavior of Common Stock Variances- Value, Leverage and Interest Rate Effects,” *Journal of Financial Economics*, 3, 145-166.
- [23] Comte, F. and E. Renault. (1998). “Long Memory in Continuous Time Stochastic Volatility Models,” *Mathematical Finance*, 8, 291-323.
- [24] Corsi, F. (2003). “A Simple Long Memory Model of Realized Volatility,” *Manuscript, University of Southern Switzerland*.
- [25] Cutler, D. M., J. M. Poterba, and L. H. Summers. (1989). “What Moves Stock Prices?,” *The Journal of Portfolio Management*, 15, 4-12.
- [26] Dacorogna, M.M., R. Gençay, U. Müller, R.B. Olsen, and O.V. Pictet. (2001). “An Introduction to High-Frequency Finance,” *San Diego: Academic Press*.
- [27] Dufour J-M. and E. Renault. (1998). “Short-run and long-run causality in time series: Theory,” *Econometrica* 66(5), 1099-1125.
- [28] Dufour J-M. and A. Taamouti. (2006). “Nonparametric Short and Long Run Causality Measures,” in *Proceedings of the 2006 Meetings of the American Statistical Association, Business and Economic Statistics*, forthcoming.
- [29] Dufour J-M. and A. Taamouti. (2005). “Short and Long Run Causality Measures: Theory and Inference,” *Working Paper*.
- [30] Engle, R.F and V.K. Ng. (1993). “Measuring and Testing the Impact of News on Volatility,” *Journal of Finance*, 48, 1749-1778.
- [31] French, M., W. Schwert, and R. Stambaugh. (1987). “Expected Stock Returns and Volatility,” *Journal of Financial Economics*, 19, 3-30.

- [32] Ghysels, E., P. Santa-Clara and R. Valkanov. (2002). “The MIDAS Touch: Mixed Data Sampling Regression,” *Discussion Paper UCLA and UNC*.
- [33] Ghysels, E., P. Santa-Clara, and R. Valkanov. (2004). “There is a risk-return trade-off after all”, *Journal of Financial Economics*, 76, 509-548.
- [34] Glosten, L. R., R. Jagannathan, and D. E. Runkle. (1993). “On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks,” *Journal of Finance*, 48, 1779-1801.
- [35] Gouriéroux, C., and A. Monfort. (1992). “Qualitative threshold ARCH models,” *Journal of Econometrics* 52, 159-200.
- [36] Granger, C. W. J. (1969). “Investigating causal relations by econometric models and cross-spectral methods,” *Econometrica* 37, 424–459.”
- [37] Guo, H., and R. Savickas. (2006). “Idiosyncratic Volatility, Stock Market Volatility, and Expected Stock Returns,” *Journal of Business and Economic Statistics*, 24(1), 43-56.
- [38] Hardouvelis, G. A. (1987). “Macroeconomic information and stock prices.” *Journal of Economics and Business*, 39, 131-140.
- [39] Haugen, A. H., E. Talmor, and W. N. Torous. (1991). “The Effect of Volatility Changes on the Level of Stock Prices and Subsequent Expected Returns,” *Journal of Finance*, 46, 985-1007.
- [40] Hull, J. and A. White. (1987). “The pricing of options with stochastic volatilities,” *Journal of Finance*, 42, 281-300.
- [41] Huang, X., and G. Tauchen. (2005). “The Relative Contribution of Jumps to Total Price Variance,” *Working Paper*.
- [42] Huang, X. (2007). “Macroeconomic News Announcements, Financial Market Volatility and Jumps,” *Working Paper*.
- [43] Jacquier, E., N. Polson, and P. Rossi. (2004). “Bayesian Analysis of Stochastic Volatility Models with Leverage Effect and Fat tails,” *Journal of Econometrics*, 122.

- [44] Jain, P. C. (1988). “Response of Hourly Stock Prices and Trading Volume to Economic News,” *The Journal of Business*, 61, 219-231.
- [45] Lamoureux, C., G. Zhou. (1996). “Temporary components of stock returns: what do the data tell us?,” *Review of Financial Studies* 9, 1033–1059.
- [46] Ludvigson, S. C., and S. Ng. (2005). “The Empirical Risk-Return Relation: A Factor Analysis Approach,” Forthcoming, *Journal of Financial Economics*.
- [47] McQueen, G. and V. V. Roley. (1993). “Stock Prices, News, and Business Conditions,” *The Review of Financial Studies*, 6, 683-707.
- [48] Meddahi, N. (2002). “A Theoretical Comparison Between Integrated and Realized Volatility,” *Journal of Applied Econometrics*, 17, 475-508.
- [49] Müller, U., M. Dacorogna, R. Dav, R. Olsen, O. Pictet, and J. von Weizsacker. (1997). “Volatilities of different time resolutions - analyzing the dynamics of market components,” *Journal of Empirical Finance* 4, 213{39.
- [50] Nelson, D. B. (1991). “Conditional Heteroskedasticity in Asset Returns: A New Approach,” *Econometrica*, 59, 347-370.
- [51] Pagan, A.R., and G.W. Schwert. (1990). “Alternative models for conditional stock volatility,” *Journal of Econometrics* 45, 267-290.
- [52] Pearce, D. K. and V. V. Roley (1985). “Stock Prices and Economic News,” *Journal of Business*, 58, 49-67.
- [53] Pindyck, R.S. (1984). “Risk, Inflation, and the Stock Market,” *American Economic Review*, 74, 334-351.
- [54] Schwert, G.W. (1989). “Why Does Stock Market Volatility Change Over Time?” *Journal of Finance*, 44, 1115-1153.
- [55] Schwert, G. W. (1981). “The Adjustment of Stock Prices to Information About Inflation,” *Journal of Finance*, 36, 15-29.

- [56] Turner, C.M., R. Startz, and C.R. Nelson. (1989). "A Markov Model of Heteroskedasticity, Risk and Learning in the Stock Market," *Journal of Financial Economics*, 25, 3-22.
- [57] Whitelaw, R. F. (1994): "Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns," *The Journal of Finance*, 49(2), 515-41.
- [58] Wu, G. (2001). "The determinants of Asymmetric Volatility," *Review of Financial Studies*, 14, 837-859.

Appendix

Following Dufour and Taamouti (2005), we compute the nominal 95% bootstrap confidence intervals of the causality measures as follows:

(1) Estimate by OLS the $VAR(p)$ process given by equation (15) and save the residuals

$$\hat{u}(t) = \begin{pmatrix} r_t \\ RV_t \end{pmatrix} - \hat{\mu} - \sum_{j=1}^p \hat{\Phi}_j \begin{pmatrix} r_{t-j} \\ \ln(RV_{t-j}) \end{pmatrix}, \text{ for } t = p+1, \dots, T,$$

where $\hat{\mu}$ and $\hat{\Phi}_j$ are the OLS regression estimates of μ and Φ_j , for $j = 1, \dots, p$.

(2) Generate $(T-p)$ bootstrap residuals $\hat{u}^*(t)$ by random sampling with replacement from the residuals $\hat{u}(t)$, $t = p+1, \dots, T$.

(3) Generate a random draw for the vector of p initial observations $w(0) = ((r_1, \ln(RV_1))', \dots, (r_p, \ln(RV_p))')'$.

(4) Given $\hat{\mu}$ and $\hat{\Phi}_j$, for $j = 1, \dots, p$, $\hat{u}^*(t)$, and $w(0)$, generate bootstrap data for the dependent variable $(r_t^*, \ln(RV_t^*))'$ from equation:

$$\begin{pmatrix} r_t^* \\ \ln(RV_t^*) \end{pmatrix} = \hat{\mu} + \sum_{j=1}^p \hat{\Phi}_j \begin{pmatrix} r_{t-j}^* \\ \ln(RV_{t-j}^*) \end{pmatrix} + \hat{u}^*(t), \text{ for } t = p+1, \dots, T.$$

(5) Calculate the bootstrap OLS regression estimates

$$\hat{\Phi}^* = (\hat{\mu}^*, \hat{\Phi}_1^*, \hat{\Phi}_2^*, \dots, \hat{\Phi}_p^*) = \hat{\Gamma}^{*-1} \hat{\Gamma}_1^*,$$

$$\hat{\Sigma}_u^* = \sum_{t=p+1}^T \hat{u}^*(t) \hat{u}^{*'}(t) / (T-p),$$

where $\hat{\Gamma}^* = (T-p)^{-1} \sum_{t=p+1}^T w^*(t) w^{*'}(t)$, for $w^*(t) = ((r_t^*, \ln(RV_t^*))', \dots, (r_{t-p+1}^*, \ln(RV_{t-p+1}^*))')'$, $\hat{\Gamma}_1^* = (T-p)^{-1} \sum_{t=p+1}^T w^*(t) (r_{t+1}^*, \ln(RV_{t+1}^*))'$, and

$$\hat{u}^*(t) = \begin{pmatrix} r_t^* \\ \ln(RV_t^*) \end{pmatrix} - \hat{\mu} - \sum_{j=1}^p \hat{\Phi}_j \begin{pmatrix} r_{t-j}^* \\ \ln(RV_{t-j}^*) \end{pmatrix}.$$

(6) Estimate the constrained model of $\ln(RV_t)$ or r_t using the bootstrap sample $\{(r_t^*, \ln(RV_t^*))'\}_{t=1}^T$.

(7) Calculate the causality measures at horizon h , denoted $\hat{C}^{(j)*}(r \xrightarrow{h} \ln(RV))$ and $\hat{C}^{(j)*}(\ln(RV) \xrightarrow{h} r)$, using equations (18) and (19), respectively.

(8) Choose B such $\frac{1}{2}\alpha(B+1)$ is an integer and repeat steps (2) – (7) B times.¹²

¹²Where $1-\alpha$ is the considered level of confidence interval.

(9) Finally, calculate the α and $1-\alpha$ percentile interval endpoints of the distributions of $\hat{C}^{(j)*}(r \xrightarrow[h]{\ln(RV)})$ and $\hat{C}^{(j)*}(\ln(RV) \xrightarrow[h]{r})$.

A proof of the asymptotic validity of the bootstrap confidence intervals of the causality measures is provided in Dufour and Taamouti (2005).

Table 1: Parameter values of different *GARCH* models

	ω	β	α	γ
<i>GARCH</i>	2.7910^{-5}	0.86695	0.093928	–
<i>EGARCH</i>	–0.290306	0.97	0.093928	–0.09
<i>NL-GARCH</i>	2.7910^{-5}	0.86695	0.093928	0.5, 1.5, 2.5
<i>GJR-GARCH</i>	2.7910^{-5}	0.8805	0.032262	0.10542
<i>AGARCH</i>	2.7910^{-5}	0.86695	0.093928	–0.1108
<i>VGARCH</i>	2.7910^{-5}	0.86695	0.093928	–0.1108
<i>NGARCH</i>	2.7910^{-5}	0.86695	0.093928	–0.1108

Note: The table summarizes the parameter values for parametric volatility models considered in our simulations study.

Table 2: Summary statistics for S&P 500 futures returns, 1988-2005.

<i>Variables</i>	<i>Mean</i>	<i>St.Dev.</i>	<i>Median</i>	<i>Skewness</i>	<i>Kurtosis</i>
<i>Five – minute</i>	$6.9505e - 006$	0.000978	$0.00e - 007$	–0.0818	73.9998
<i>Hourly</i>	$1.3176e - 005$	0.0031	$0.00e - 007$	–0.4559	16.6031
<i>Daily</i>	$1.4668e - 004$	0.0089	$1.1126e - 004$	–0.1628	12.3714

Note: The table summarizes the Five-minute, Hourly, and Daily returns distributions for the S&P 500 index contracts. The sample covers the period from 1988 to December 2005 for a total of 4494 trading days.

Table 3: Summary statistics for daily volatilities, 1988-2005.

<i>Variables</i>	<i>Mean</i>	<i>St.Dev.</i>	<i>Median</i>	<i>Skewness</i>	<i>Kurtosis</i>
RV_t	$8.1354e - 005$	$1.2032e - 004$	$4.9797e - 005$	8.1881	120.7530
BV_t	$7.6250e - 005$	$1.0957e - 004$	$4.6956e - 005$	6.8789	78.9491
$\ln(RV_t)$	–9.8582	0.8762	–9.9076	0.4250	3.3382
$\ln(BV_t)$	–9.9275	0.8839	–9.9663	0.4151	3.2841
J_{t+1}	0.0870	0.1005	0.0575	1.6630	7.3867

Note: The table summarizes the Daily volatilities distributions for the S&P 500 index contracts. The sample covers the period from 1988 to December 2005 for a total of 4494 trading days.

Table 4: Causality Measure of Hourly Feedback Effect: $\ln(RV)$

$C(\ln(RV) \xrightarrow{h} r)$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.00016	0.00014	0.00012	0.00012
95% Bootstrap interval	[0.0000, 0.0007]	[0.0000, 0.0006]	[0.0000, 0.0005]	[0.0000, 0.0005]

Table 5: Causality Measure of Hourly Feedback Effect: $\ln(BV)$

$C(\ln(BV) \xrightarrow{h} r)$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.00022	0.00020	0.00019	0.00015
95% Bootstrap interval	[0.0000, 0.0008]	[0.0000, 0.0007]	[0.0000, 0.0007]	[0.0000, 0.0005]

Table 6: Causality Measure of Daily Feedback Effect: $\ln(RV)$

$C(\ln(RV) \xrightarrow{h} r)$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0019	0.0019	0.0019	0.0011
95% Bootstrap interval	[0.0007, 0.0068]	[0.0005, 0.0065]	[0.0004, 0.0061]	[0.0002, 0.0042]

Table 7: Causality Measure of Daily Feedback Effect: $\ln(BV)$

$C(\ln(BV) \xrightarrow{h} r)$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0017	0.0017	0.0016	0.0011
95% Bootstrap interval	[0.0007, 0.0061]	[0.0005, 0.0056]	[0.0004, 0.0055]	[0.0002, 0.0042]

Note: Tables 4-7 summarize the estimation results of causality measures from hourly realized volatility to hourly returns, hourly bipower variation to hourly returns, daily realized volatility to daily returns, and daily bipower variation to daily returns, respectively. The second row in each table gives the point estimate of the causality measures at $h = 1, \dots, 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Table 8: Measuring the impact of good news on volatility: Centered positive returns, $\ln(RV)$

$$C([r_{t+1-j} - E_{t-j}(r_{t+1-j})]^+ \xrightarrow{h} \ln(RV))$$

$E_t(\widehat{r_{t+1}}) = \frac{1}{15} \sum_{j=1}^{15} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.00076	0.00075	0.00070	0.00041
95% Bootstrap interval	[0.0003, 0.0043]	[0.0002, 0.0039]	[0.0001, 0.0034]	[0, 0.0030]
$E_t(\widehat{r_{t+1}}) = \frac{1}{30} \sum_{j=1}^{30} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.00102	0.00071	0.00079	0.00057
95% Bootstrap interval	[0.00047, 0.00513]	[0.00032, 0.00391]	[0.00031, 0.00362]	[0, 0.00321]
$E_t(\widehat{r_{t+1}}) = \frac{1}{90} \sum_{j=1}^{90} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0013	0.00087	0.00085	0.00085
95% Bootstrap interval	[0.0004, 0.0059]	[0.00032, 0.0044]	[0.0002, 0.0041]	[0.0001, 0.0039]
$E_t(\widehat{r_{t+1}}) = \frac{1}{120} \sum_{j=1}^{120} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0011	0.00076	0.00072	0.00074
95% Bootstrap interval	[0.0004, 0.0054]	[0.00029, 0.0041]	[0.00024, 0.00386]	[0, 0.00388]
$E_t(\widehat{r_{t+1}}) = \frac{1}{240} \sum_{j=1}^{240} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0011	0.00069	0.00067	0.0007
95% Bootstrap interval	[0.0004, 0.0053]	[0.0003, 0.0041]	[0.0002, 0.0035]	[0, 0.0034]

Note: The table summarizes the estimation results of causality measures from centered positive returns to realized volatility under five estimators of the average returns. In each of the five small tables, the second row gives the point estimate of the causality measures at $h = 1, \dots, 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Table 9: Measuring the impact of good news on volatility: Centred positive returns, $\ln(BV)$

$$C([r_{t+1-j} - E_{t-j}(r_{t+1-j})]^+ \xrightarrow{h} \ln(RV))$$

$\widehat{E}_t(r_{t+1}) = \frac{1}{15} \sum_{j=1}^{15} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0008	0.0008	0.00068	0.00062
95% Bootstrap interval	[0.00038, 0.0045]	[0.00029, 0.0041]	[0.00021, 0.0035]	[0, 0.0034]
$\widehat{E}_t(r_{t+1}) = \frac{1}{30} \sum_{j=1}^{30} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0012	0.00076	0.00070	0.00072
95% Bootstrap interval	[0.0005, 0.0053]	[0.0003, 0.0041]	[0.0002, 0.0039]	[0.0001, 0.0038]
$\widehat{E}_t(r_{t+1}) = \frac{1}{90} \sum_{j=1}^{90} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0018	0.0009	0.0008	0.0010
95% Bootstrap interval	[0.0006, 0.0065]	[0.0003, 0.0044]	[0.0002, 0.0041]	[0.0001, 0.0042]
$\widehat{E}_t(r_{t+1}) = \frac{1}{120} \sum_{j=1}^{120} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0016	0.0008	0.0007	0.0009
95% Bootstrap interval	[0.0006, 0.0063]	[0.00026, 0.0047]	[0.0002, 0.0042]	[0.0001, 0.0044]
$\widehat{E}_t(r_{t+1}) = \frac{1}{240} \sum_{j=1}^{240} r_{t+1-j}$				
	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0015	0.0007	0.0006	0.0008
95% Bootstrap interval	[0.0005, 0.0057]	[0.00029, 0.0044]	[0.00020, 0.0038]	[0.0001, 0.0037]

Note: The table summarizes the estimation results of causality measures from centered positive returns to bipower variation under five estimators of the average returns. In each of the five small tables, the second row gives the point estimate of the causality measures at $h = 1, \dots, 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Table 10: Measuring the impact of good news on volatility: Noncentered positive returns, $\ln(RV)$

$C(r^+ \xrightarrow{h} \ln(RV))$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0027	0.0012	0.0008	0.0009
95% Bootstrap interval	[0.0011, 0.0077]	[0.0004, 0.0048]	[0.0002, 0.0041]	[0.0001, 0.0038]

Table 11: Measuring the impact of good news on volatility: Noncentered positive returns, $\ln(BV)$

$C(r^+ \xrightarrow{h} \ln(BV))$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Point estimate	0.0035	0.0013	0.0008	0.0010
95% Bootstrap interval	[0.0016, 0.0087]	[0.0004, 0.0051]	[0.0002, 0.0039]	[0.0001, 0.0043]

Note: Tables 10-11 summarize the estimation results of causality measures from noncentered positive returns to realized volatility and noncentered positive returns to bipower variation, respectively. The second row in each table gives the point estimate of the causality measures at $h = 1, \dots, 4$. The third row gives the 95% corresponding percentile bootstrap interval.

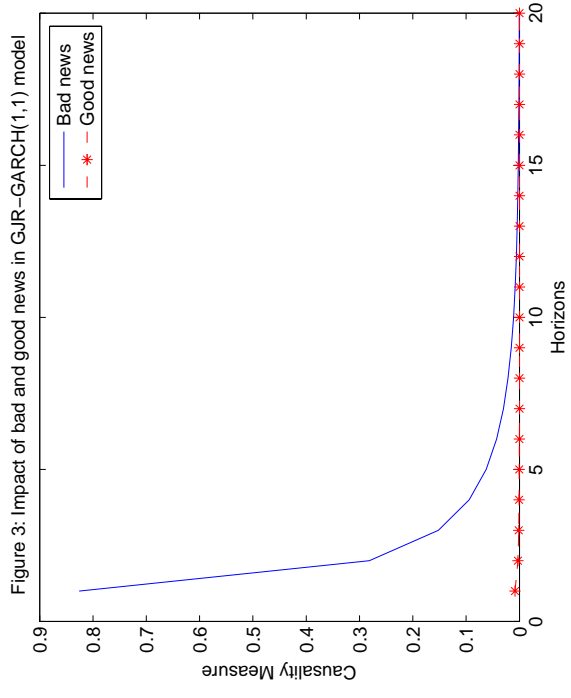
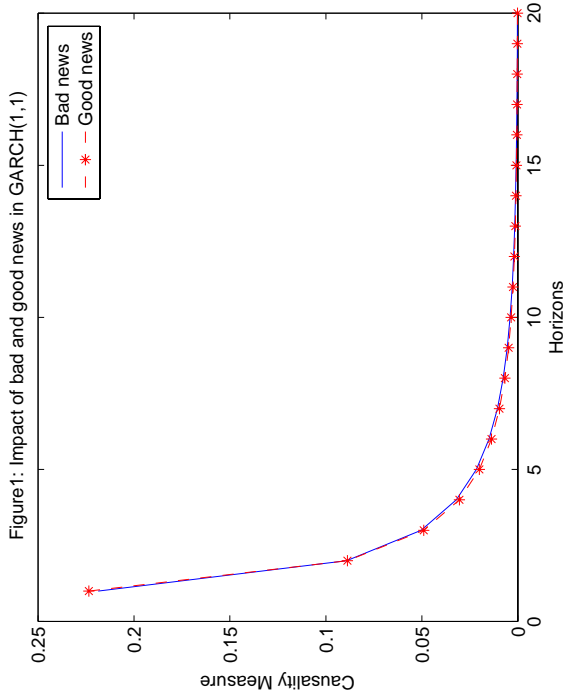
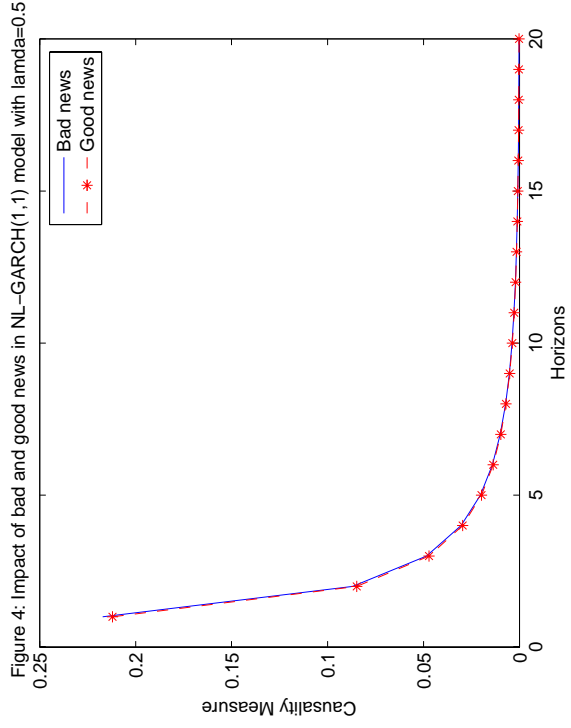
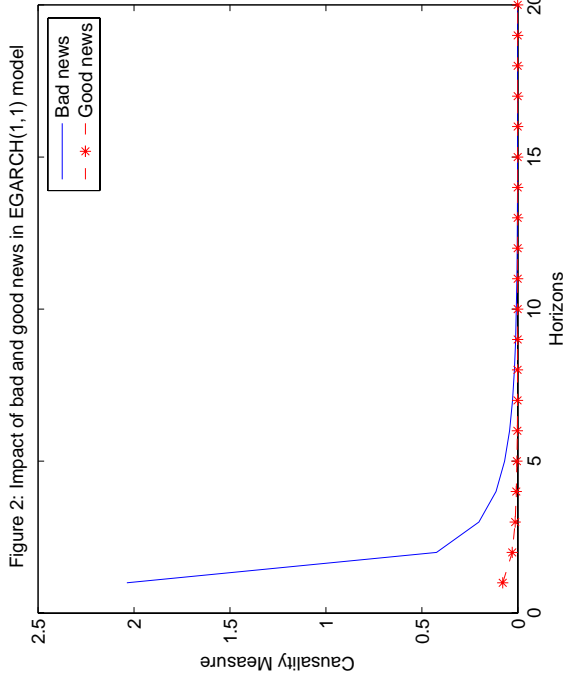


Figure 6: Impact of bad and good news in NL-GARCH(1,1) model with lamda=1.5

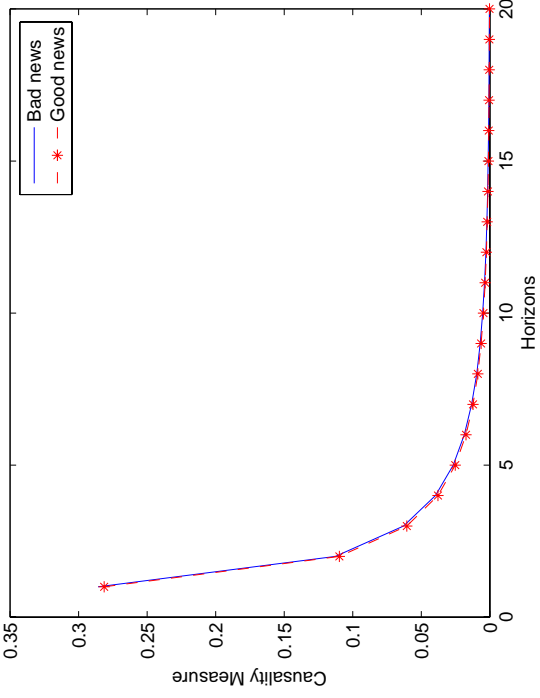


Figure 8: Impact of bad and good news in VGARCH(1,1) model

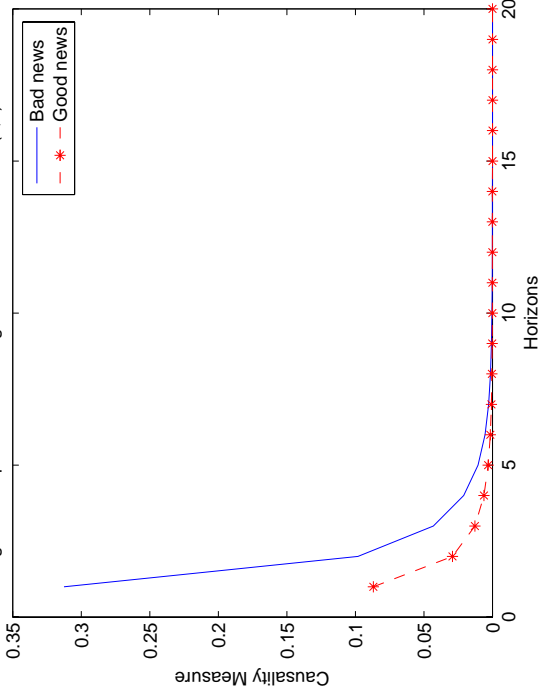


Figure 5: Impact of bad and good news in NL-GARCH(1,1) model with lamda=1

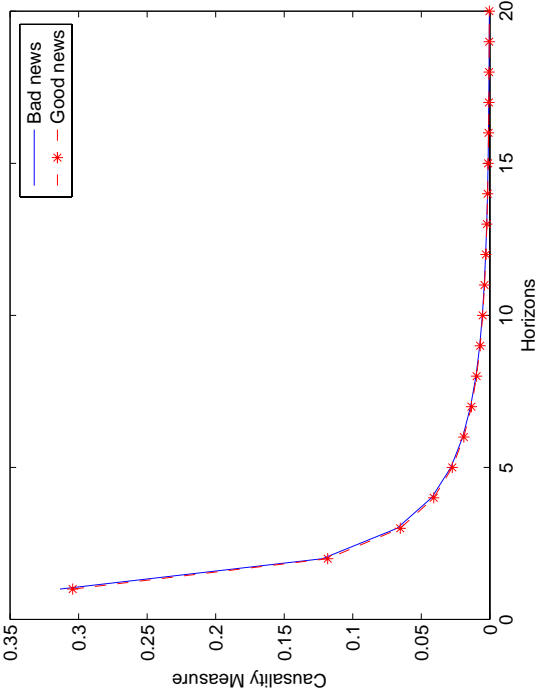


Figure 7: Measuring the impact of bad and good news in AGARCH(1,1)

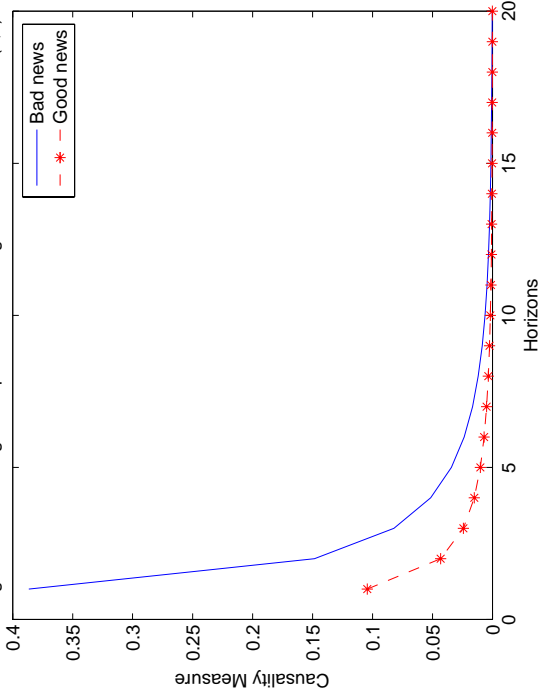


Figure 10: Response of volatility to bad news in different asymmetric GARCH models

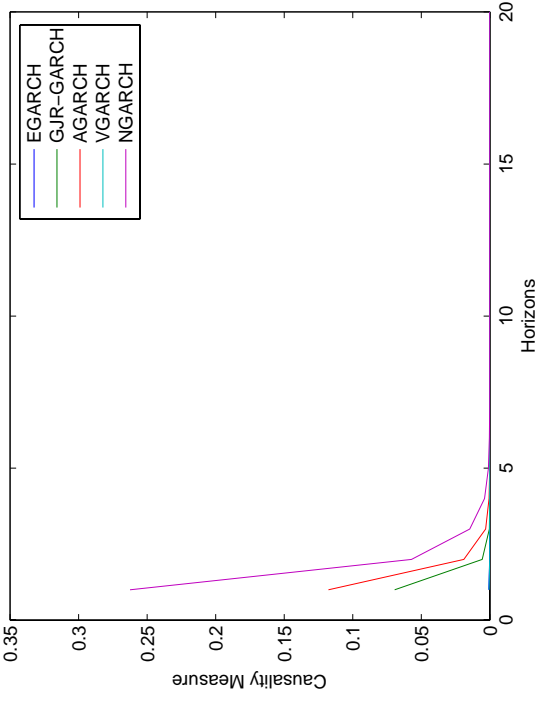
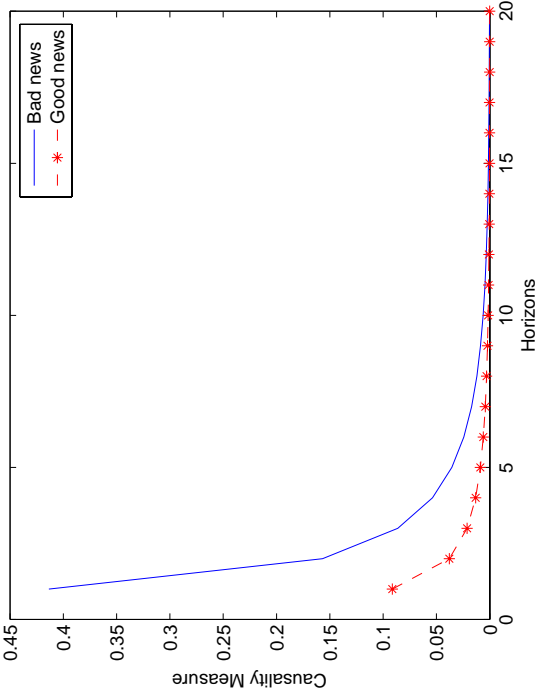
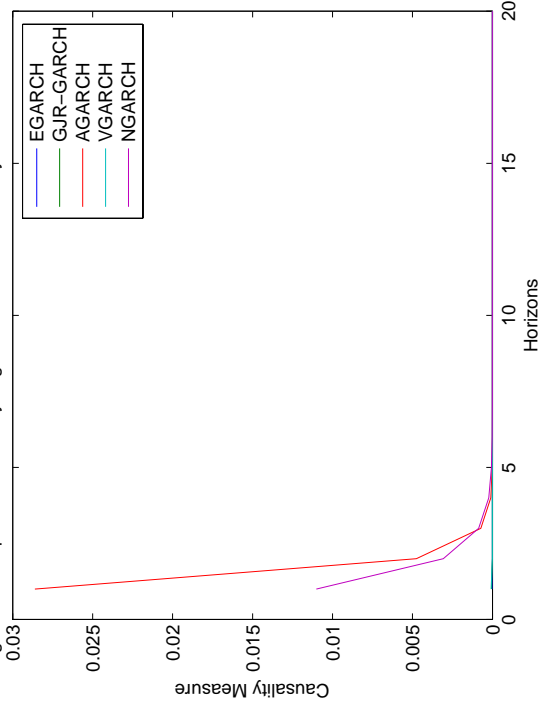


Figure 9: Impact of bad and good news in NGARCH(1,1) model



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Figure 11: Response of volatility to good news in different asymmetric GARCH models



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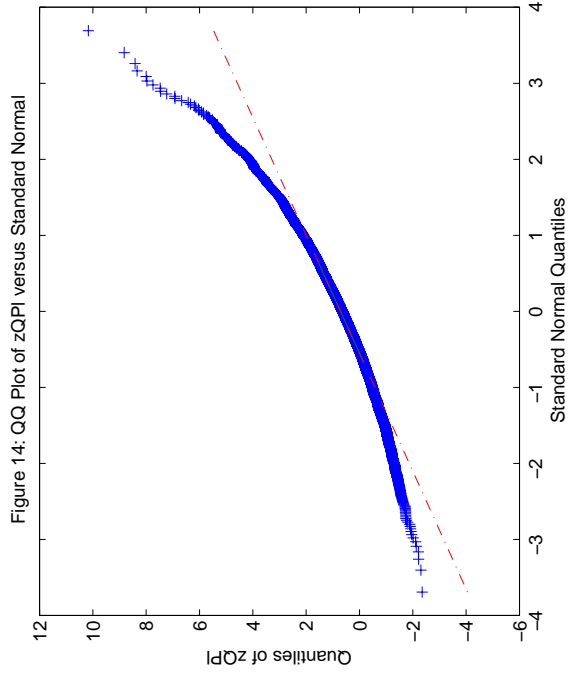
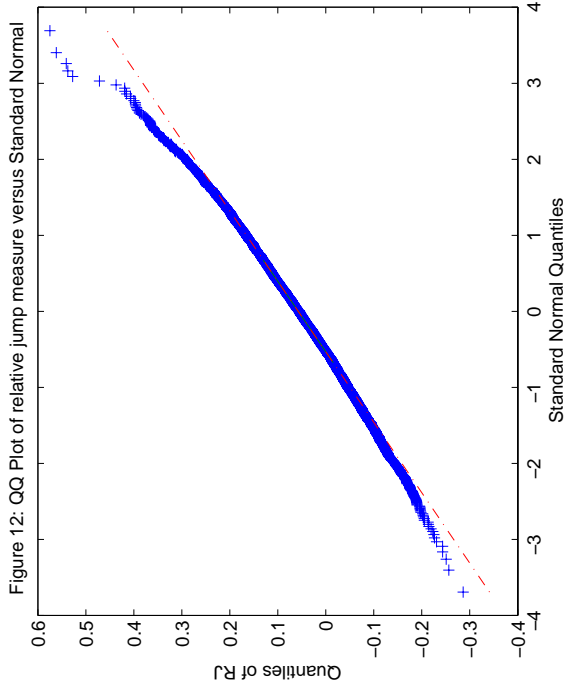
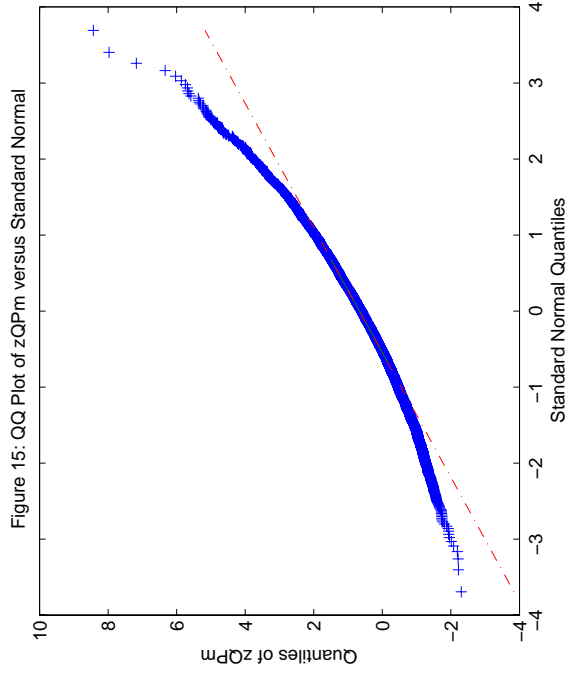
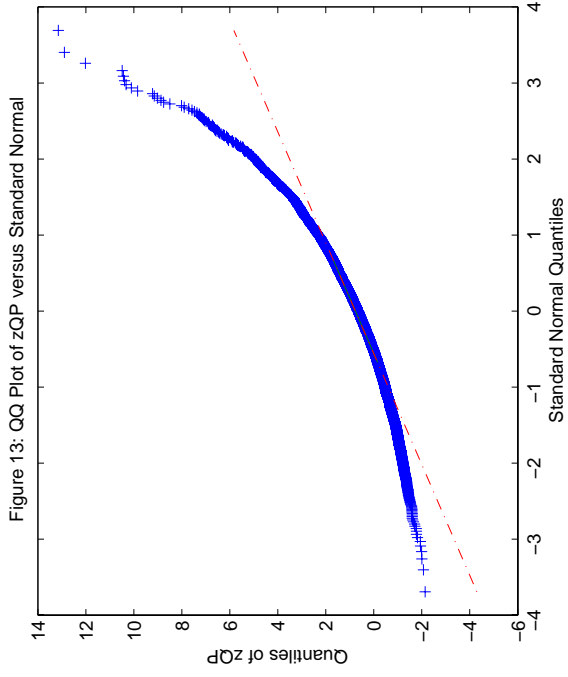


Figure 16: S&P 500 futures, daily returns, 1988-2005

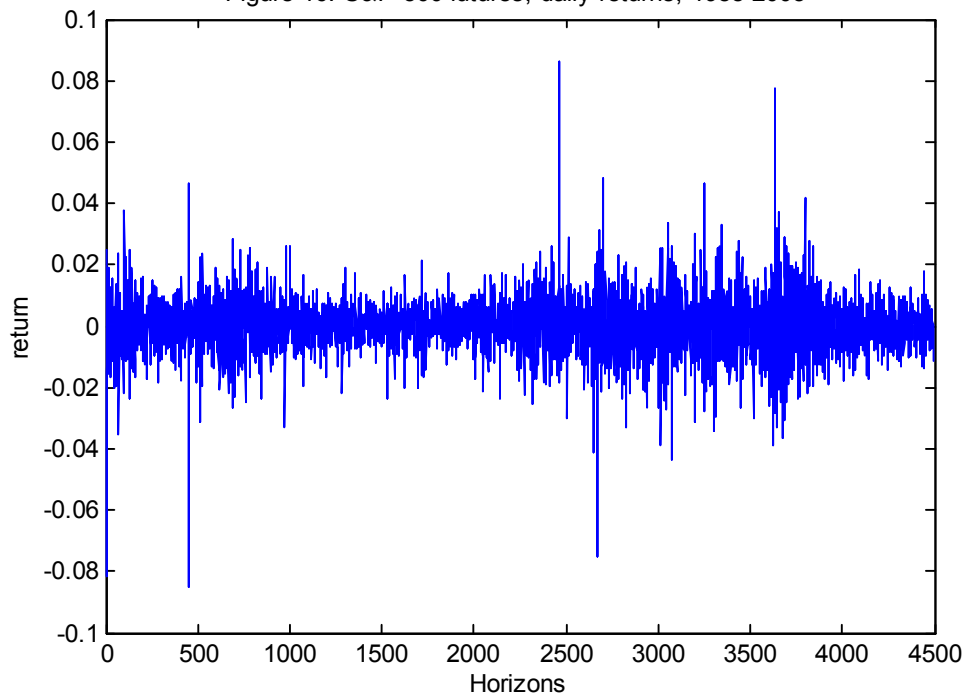


Figure 17: S&P 500 Realized volatility, 1988–2005

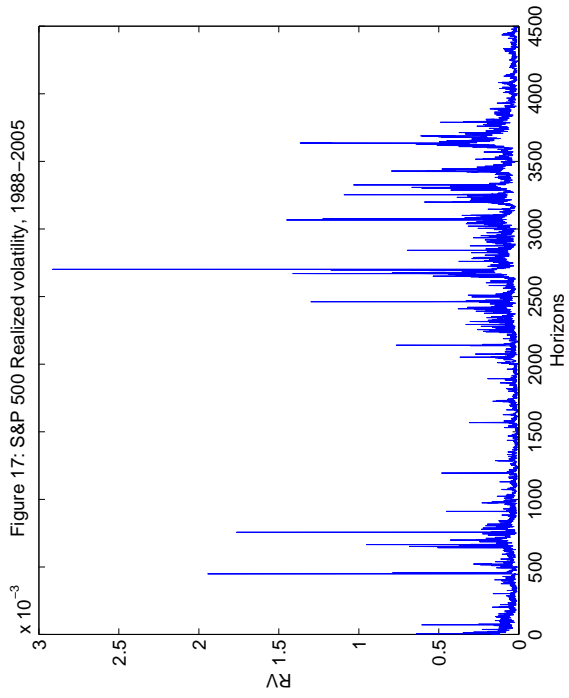


Figure 18: S&P 500 Bipower variation, 1988–2005

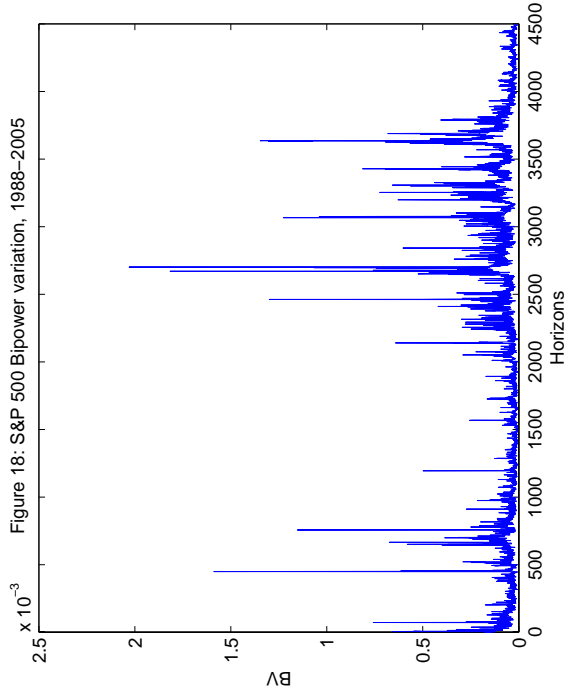


Figure 19: S&P 500 logarithm of Realized volatility, 1988–2005

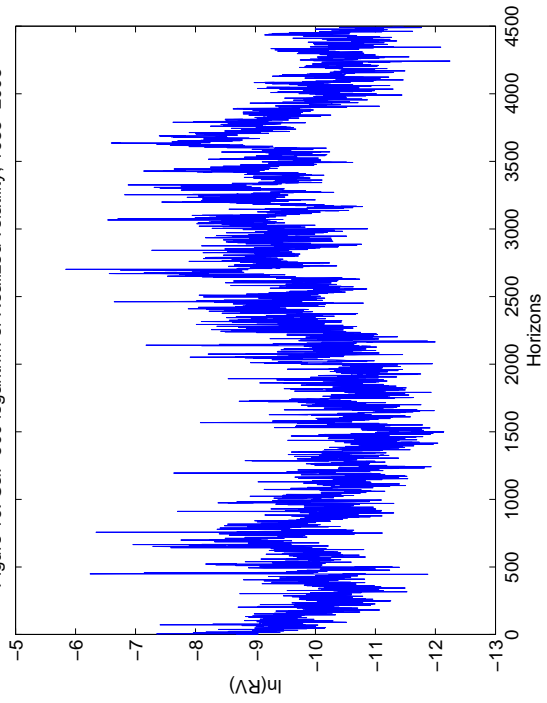


Figure 20: S&P 500 logarithm of Bipower variation, 1988–2005

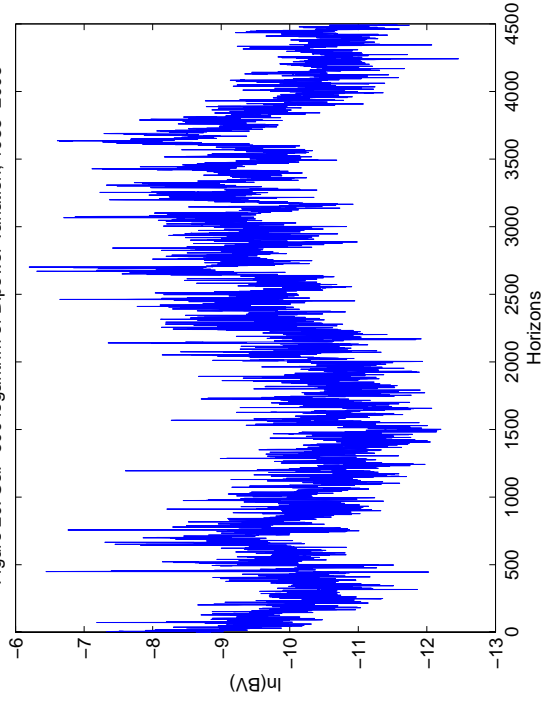
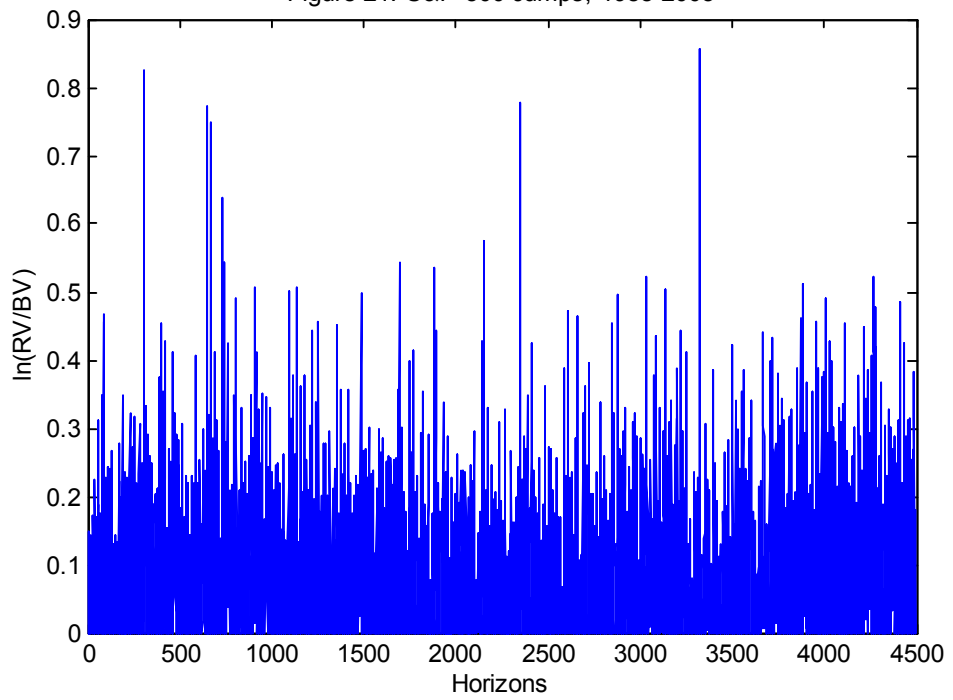


Figure 21: S&P 500 Jumps, 1988-2005



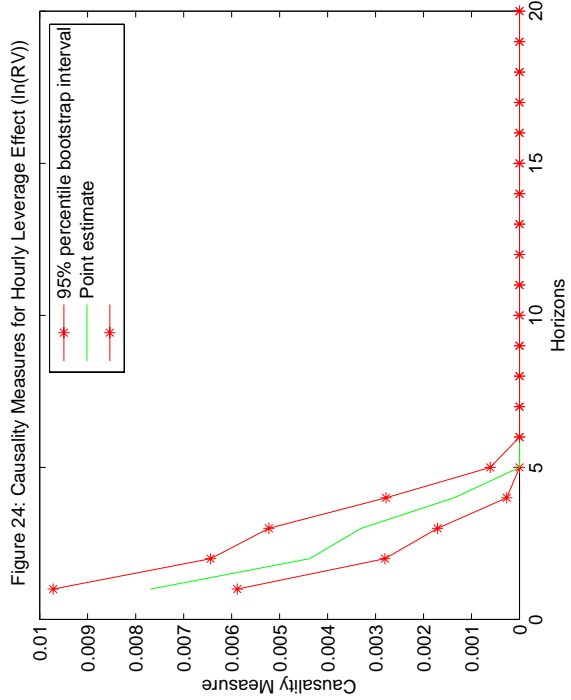
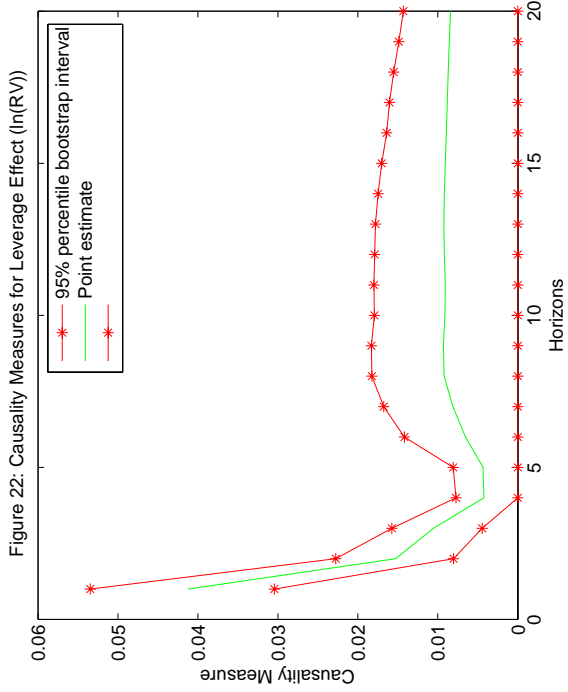
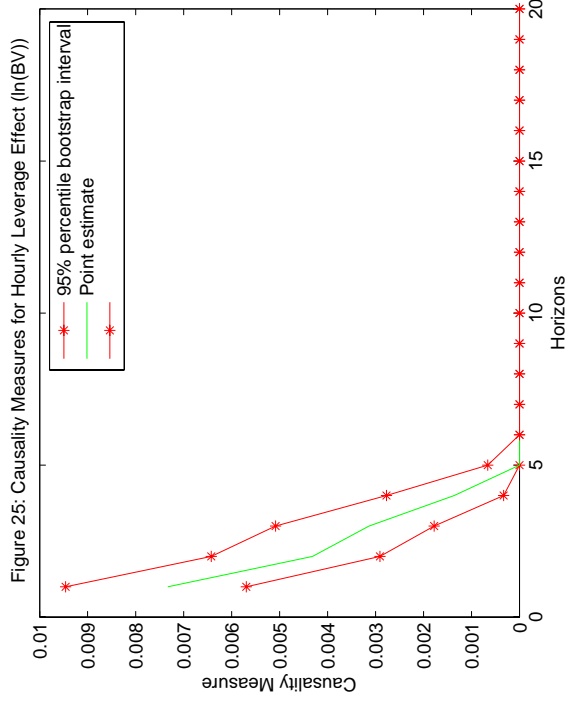
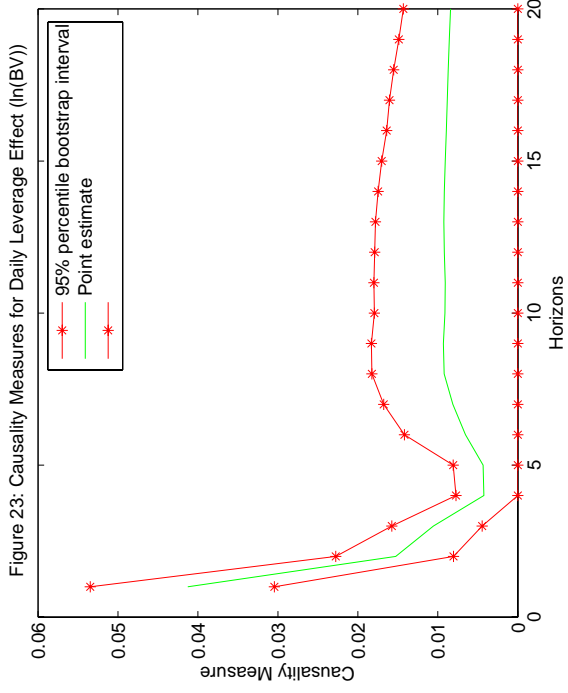


Figure 26: Measures of instantaneous causality between daily return and related volatility

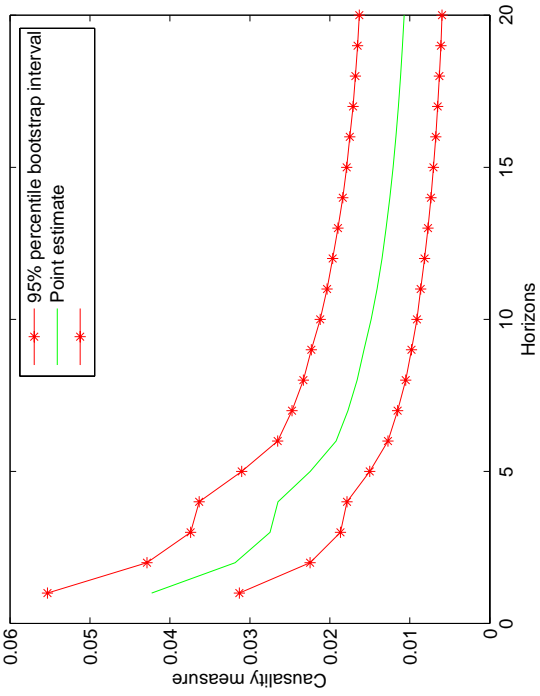


Figure 28: Measures of dependence between daily return and related volatility

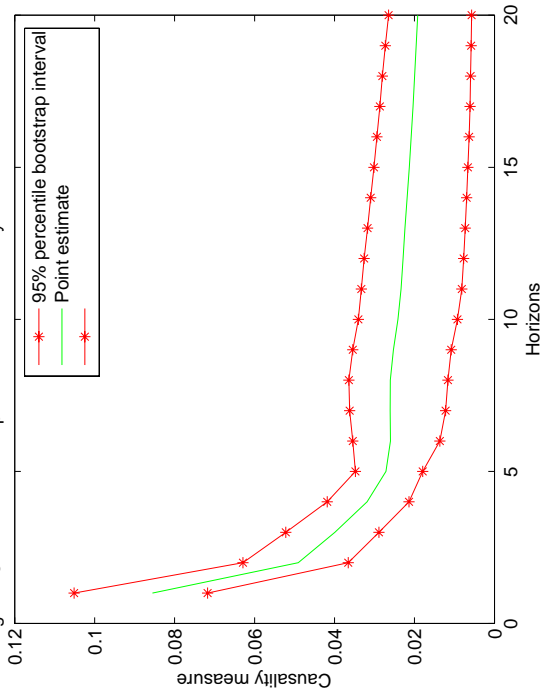


Figure 27: Measures of instantaneous causality between daily return and Bipower variation

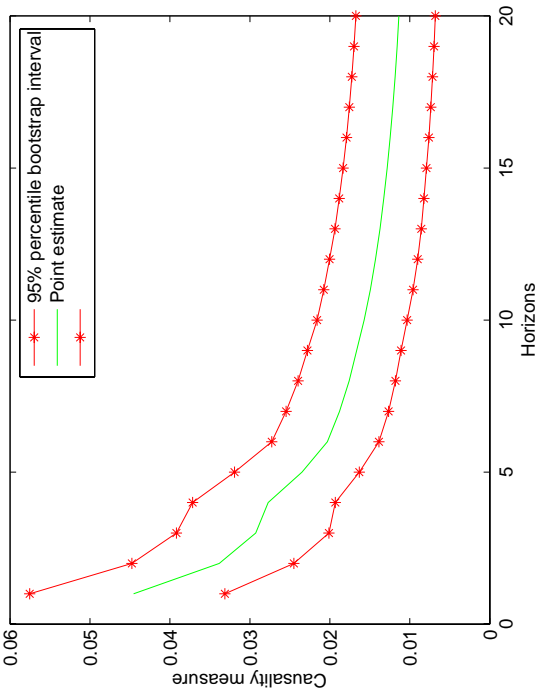


Figure 29: Measures of dependence between daily return and Bipower variation

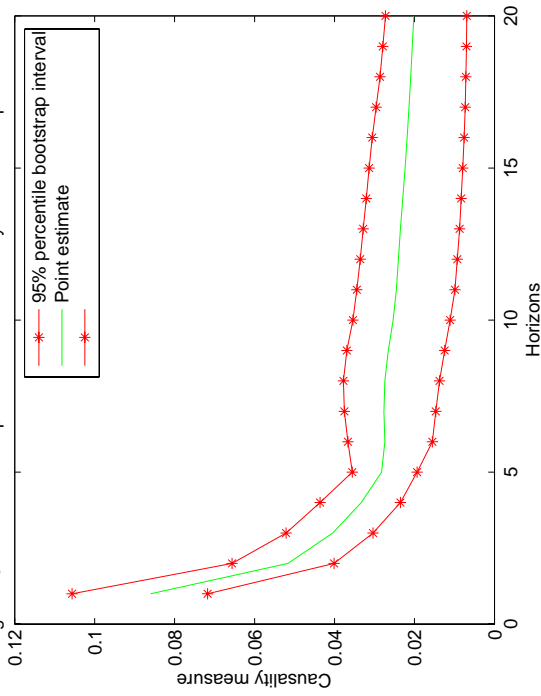


Figure 31: Comparison Between Daily Leverage and Feedback Effects (ln(BV))

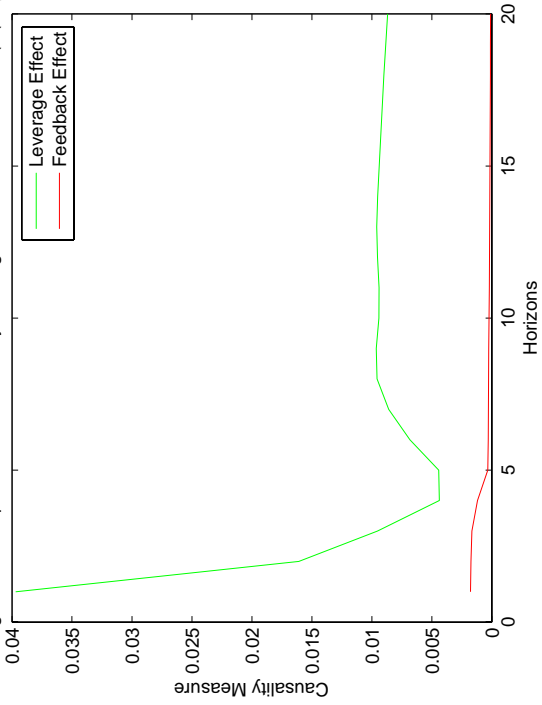


Figure 33: Hourly and Daily Leverage Effect ln(BV)

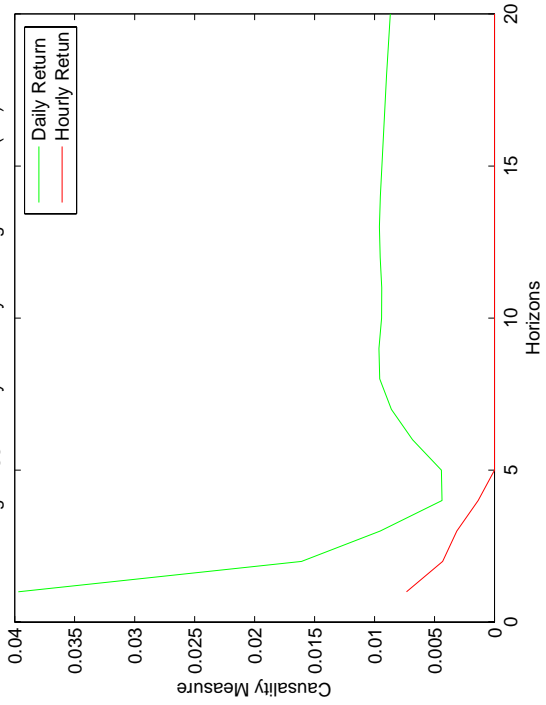


Figure 30: Comparison Between Daily Leverage and Feedback Effects (ln(RV))

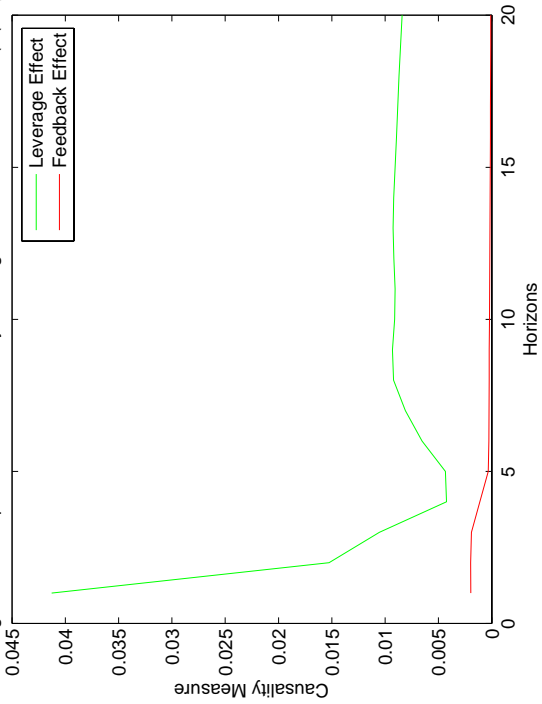
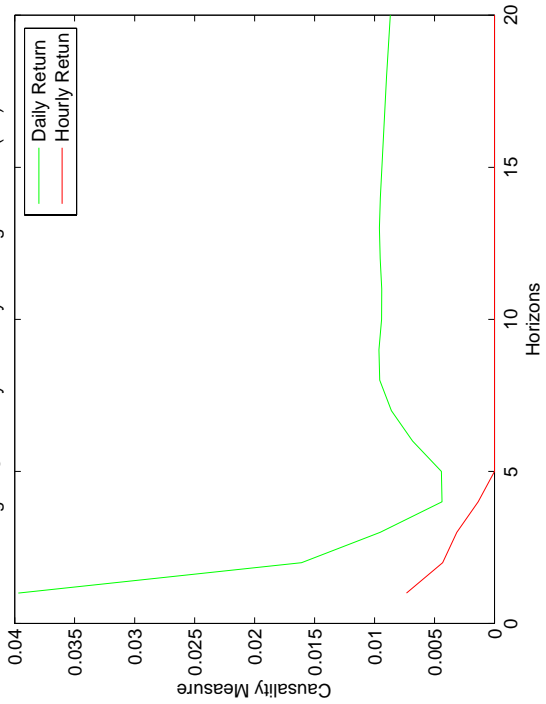
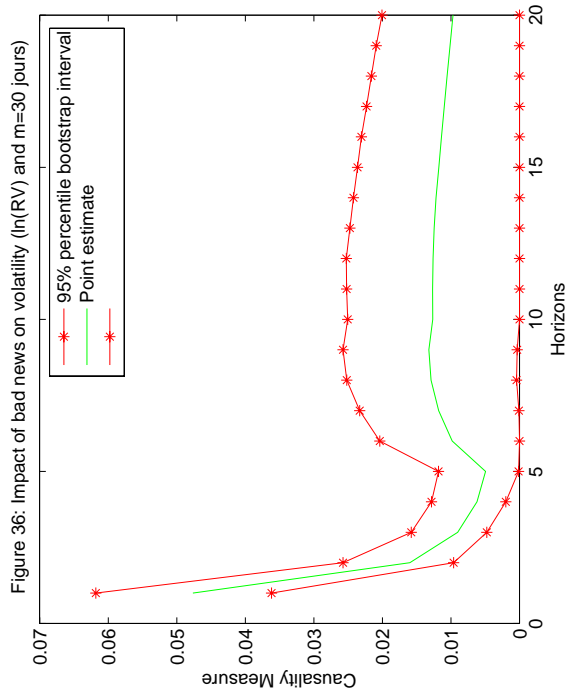
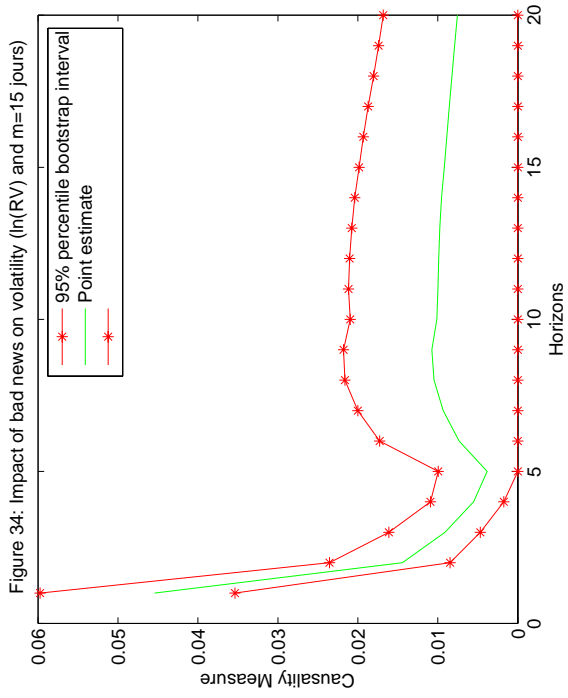
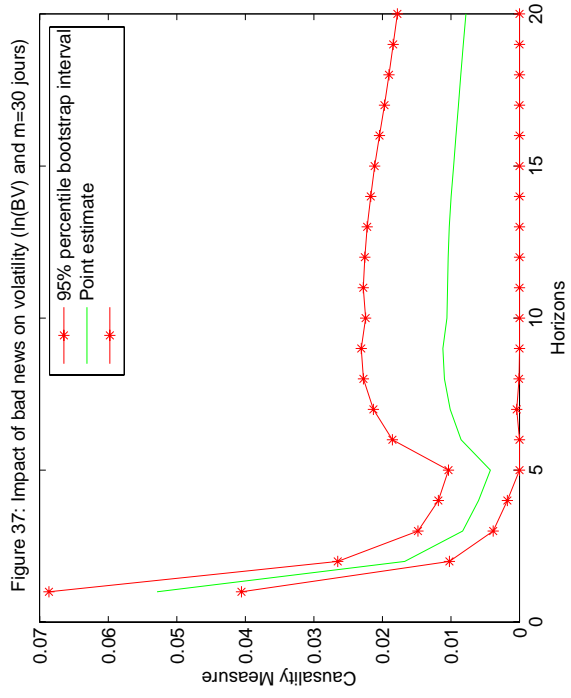
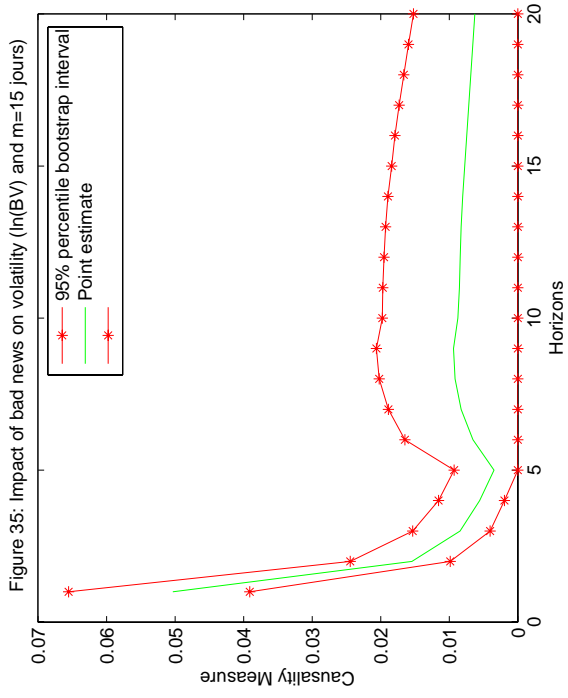
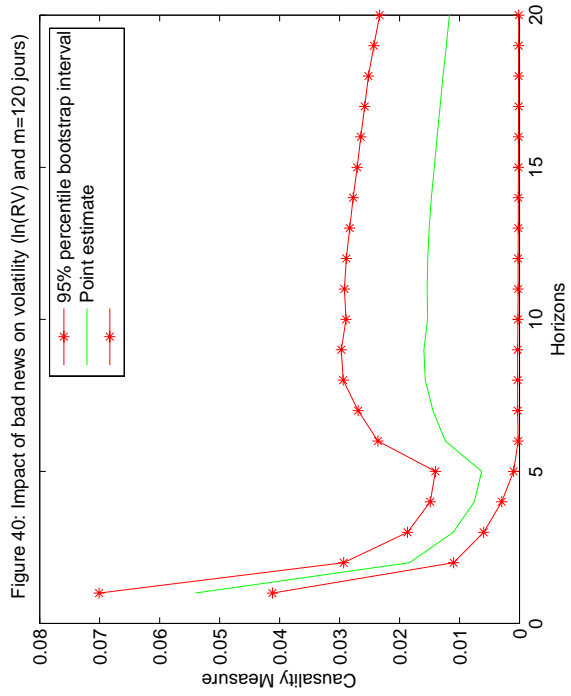
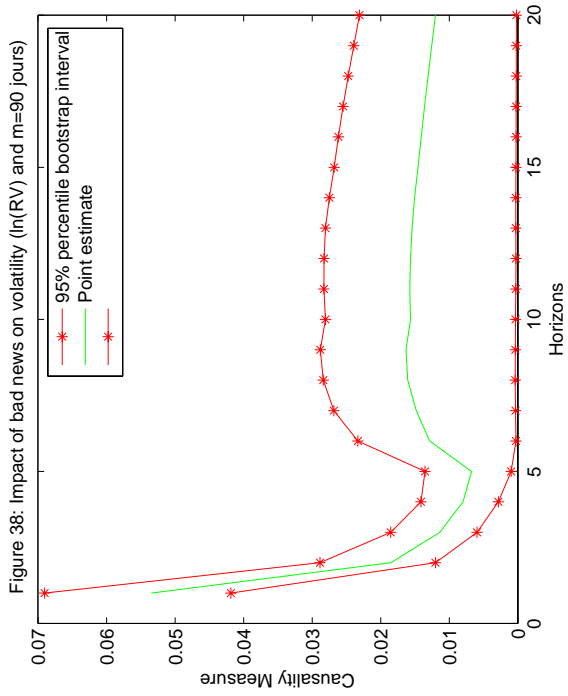
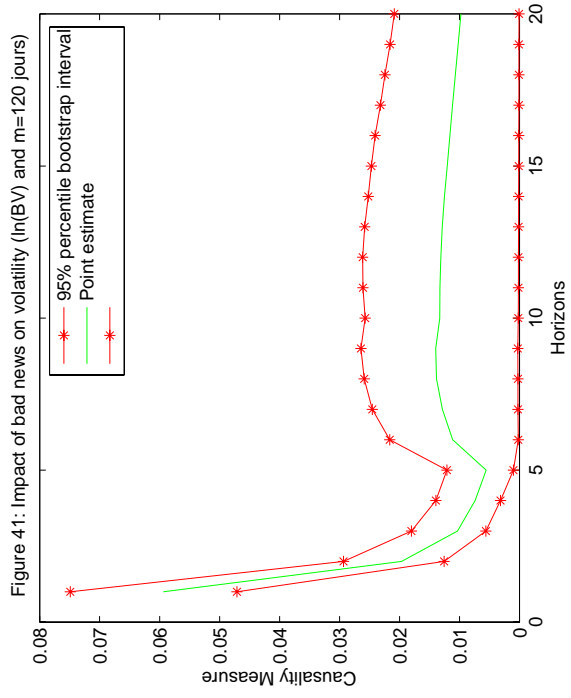
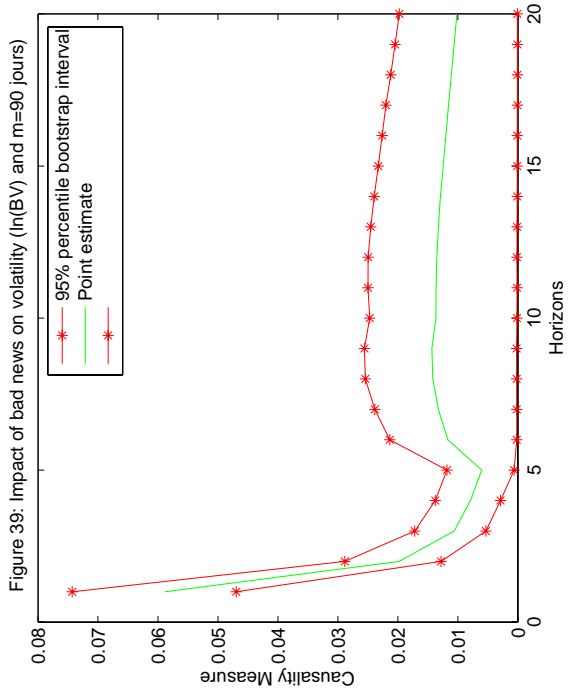
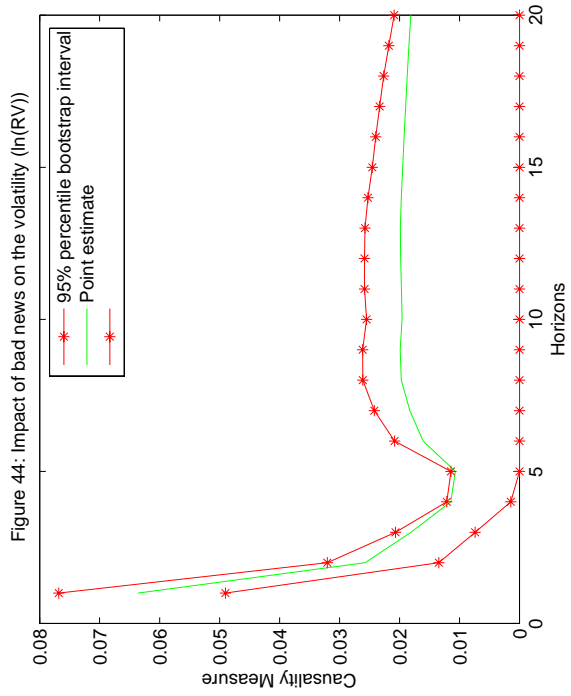
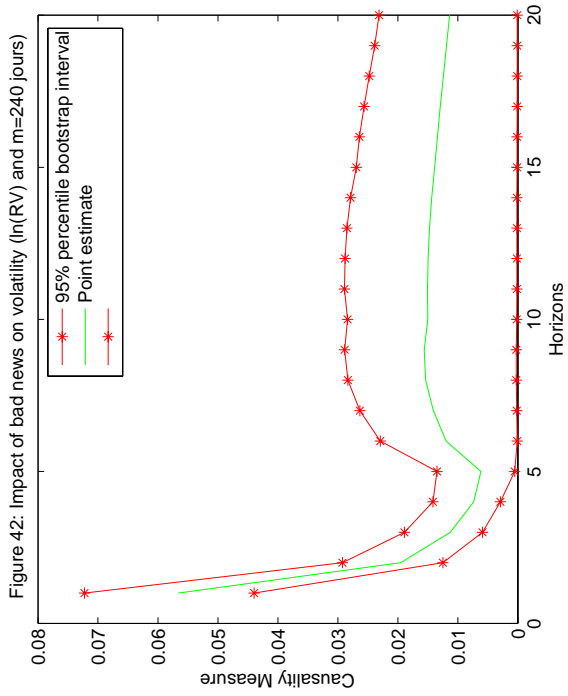
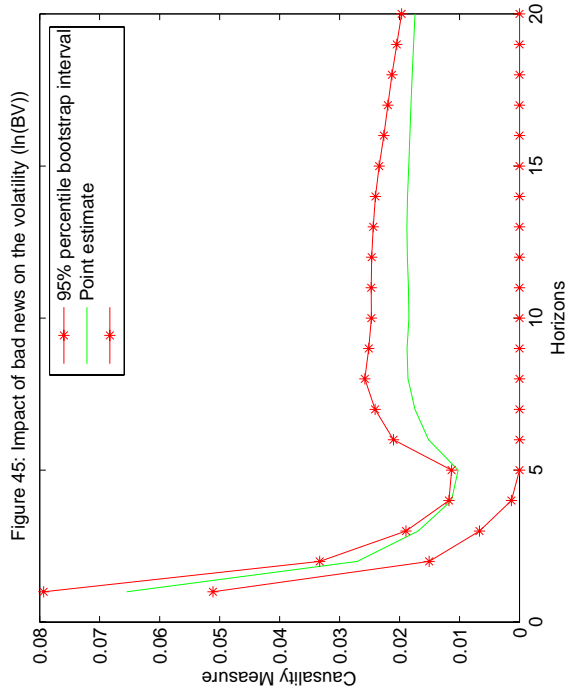
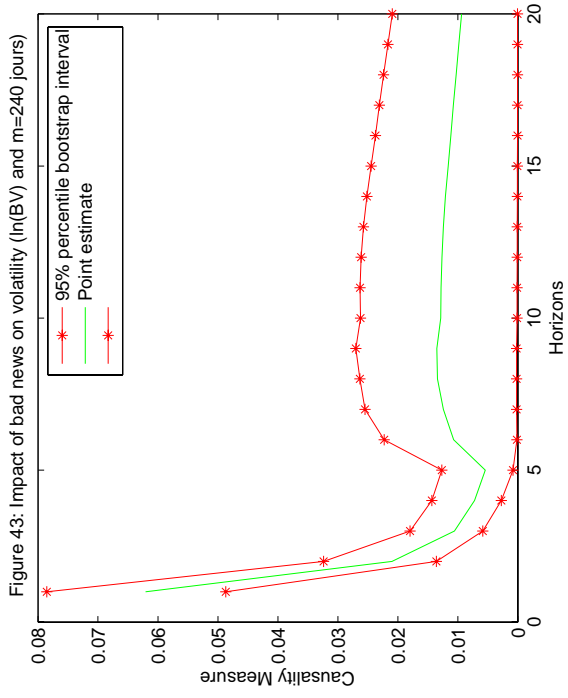


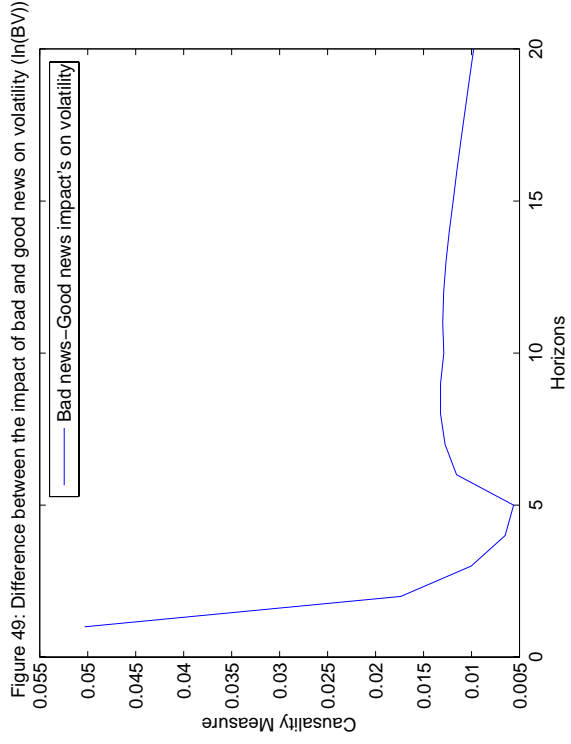
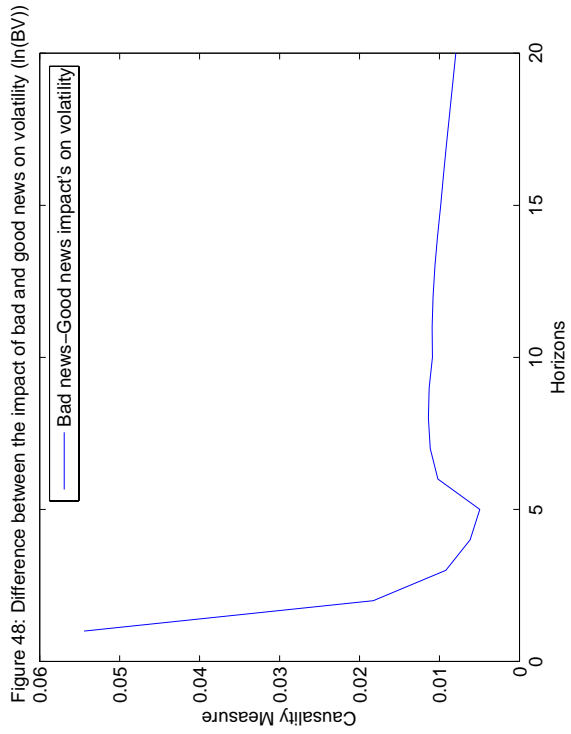
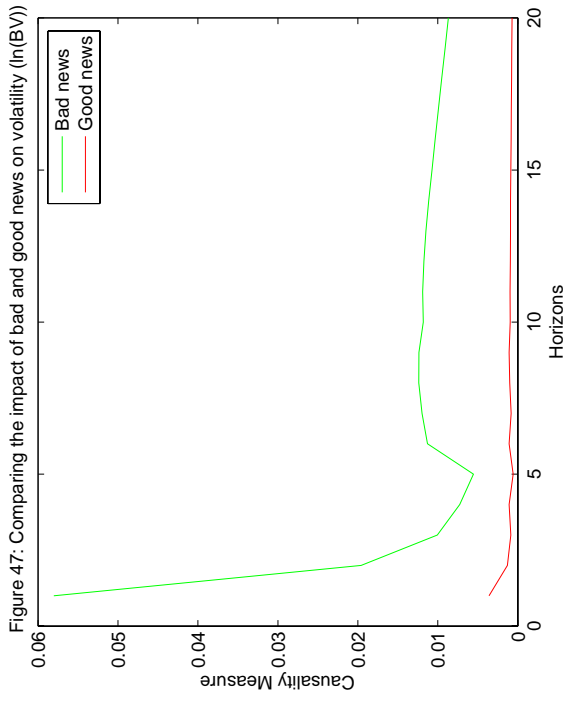
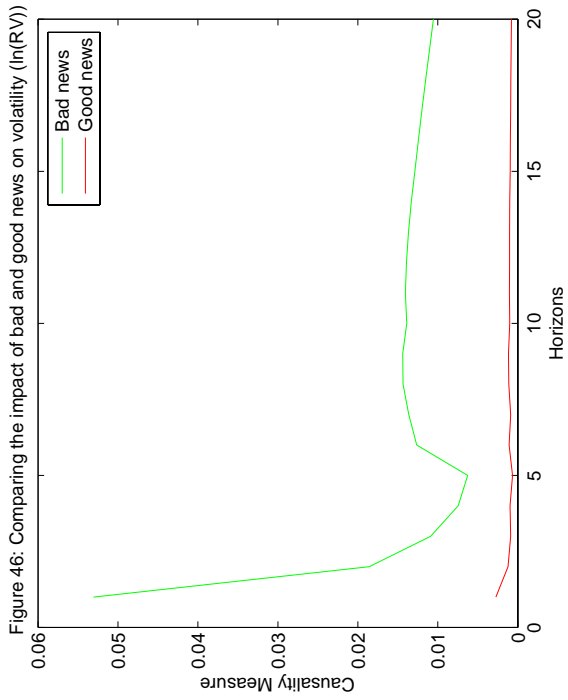
Figure 32: Hourly and Daily Leverage Effect ln(RV)











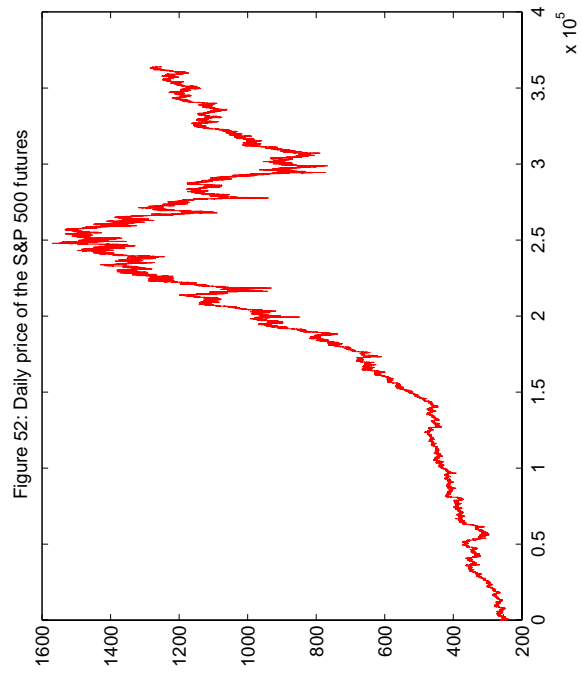


Figure 52: Daily price of the S&P 500 futures

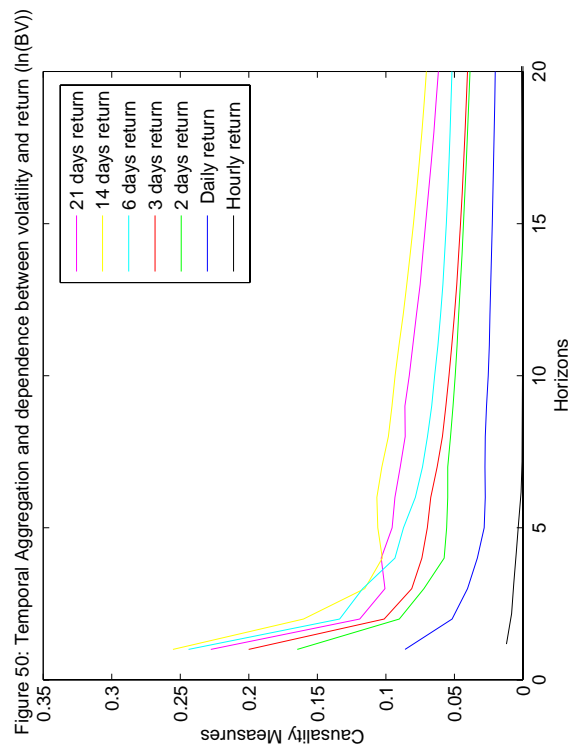


Figure 50: Temporal Aggregation and dependence between volatility and return (ln(BV))