Disentangling jumps from diffusion in equity and electricity markets

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Coauthors

Joint work with:

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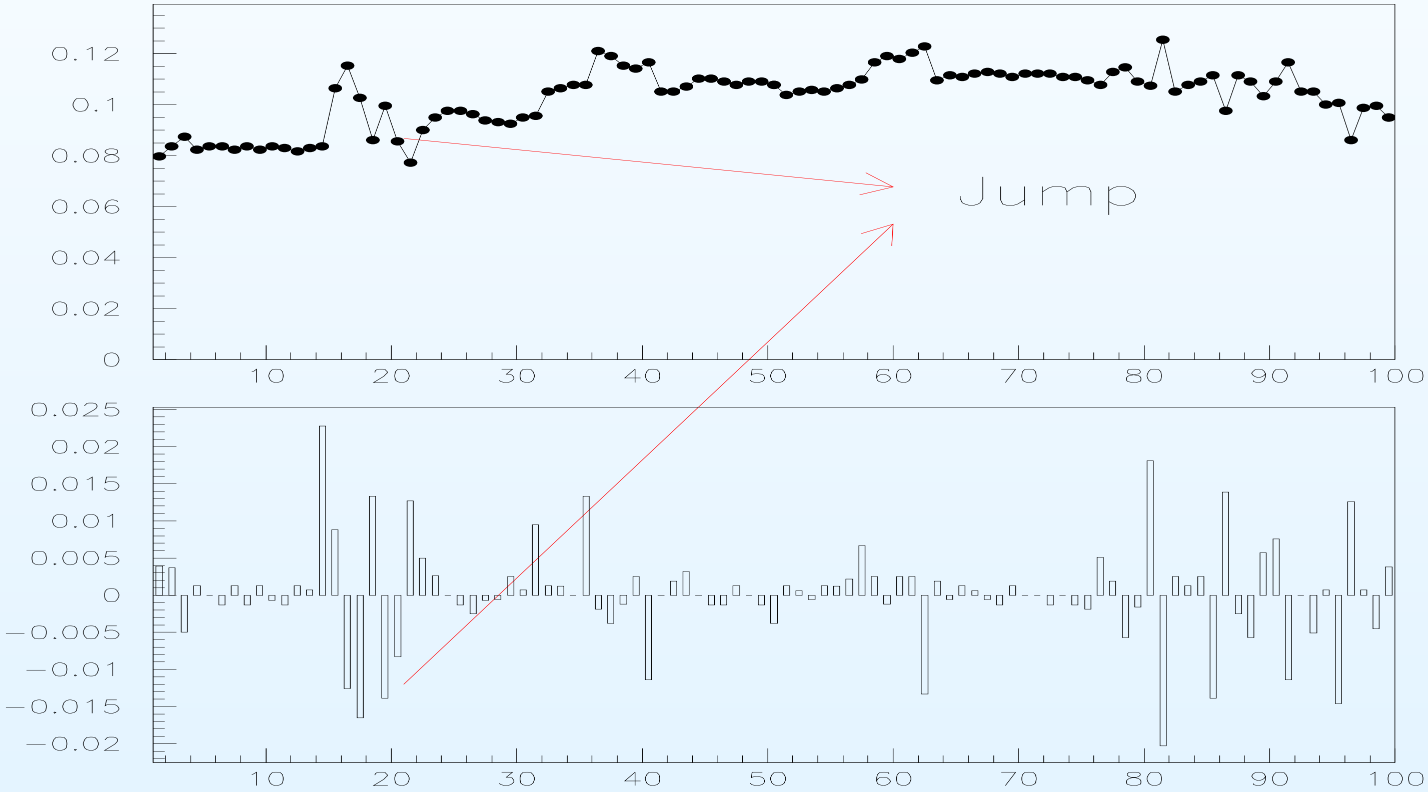
The econometric problem

When we observe variations in the level of any state variable, are these variations continouous or discontinuous?



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Disentangling continuity from abrut changes needs using *infill* asymptotics.

The modulus of continuity

Our idea to disentangle diffusion from jumps is based on the *modulus of continuity* of the Brownian motion:

$$r(\delta) = \sqrt{2\delta \log \frac{1}{\delta}}$$

which has the following property, as established by Lévy:

$$\mathcal{P}\left[\limsup_{\delta \to 0} \frac{\max_{|t-s| \le \delta} |W(t) - W(s)|}{r(\delta)} = 1\right] = 1$$

It measures the *speed* at which the Brownian motion shrinks to zero.

The intuition

When $\delta \longrightarrow 0$, diffusive variations go to zero, while jumps do not.

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Thus, we can identify the jumps as those variations which are larger than a suitable *threshold* $\vartheta(\delta)$ which goes to zero, as $\delta \longrightarrow 0$, slower than $r(\delta)$.

Threshold estimators (Mancini, 2004)

Suppose

$$X = Y + J$$

Where Y is a Brownian martingale plus drift and J is a jump process with non explosive counting process N with $E[N_T] < \infty$ and time horizon $T < \infty$.

If $\vartheta(\delta)$ is a real deterministic function such that

$$\lim_{\delta \to 0} \vartheta(\delta) = 0 \quad \text{and} \quad \lim_{\delta \to 0} \frac{\delta \log \frac{1}{\delta}}{\vartheta(\delta)} = 0$$

then for P-almost all ω, $\exists \bar{\delta}(\omega)$ such that $\forall \delta < \bar{\delta}(\omega)$ we have

$$\forall i = 1, \ldots, n, \quad I_{\{\Delta N = 0\}}(\omega) = I_{\{(\Delta X)^2 \leq \vartheta(\delta)\}}(\omega).$$

The model

We use the above intuition to estimate univariate models of the kind:

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + dJ_t, \qquad t \in [0, T],$$

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Drift and diffusion are level-dependent. J_t can be a finite activity doubly stochastic Poisson process with level-dependent intensity $\lambda(X_t)$, or a Lévy process with finite or infinite activity.

Estimation of the jump process

We first get (in the case of finite activity) an estimate of the whole jump process using:

$$\hat{J}_{1,t} = \sum_{\{i:t_i \le t\}} \hat{\gamma}_{\tau^{(i)}},$$

where

$$\hat{\gamma}_{\tau^{(i)}} \doteq (X_{t_{i+1}} - X_{t_i}) \cdot I_{\{(X_{t_{i+1}} - X_{t_i})^2 > \vartheta(\delta)\}}$$

It is remarkable that we estimate contemporaneously jump times and sizes.

The proposed estimator

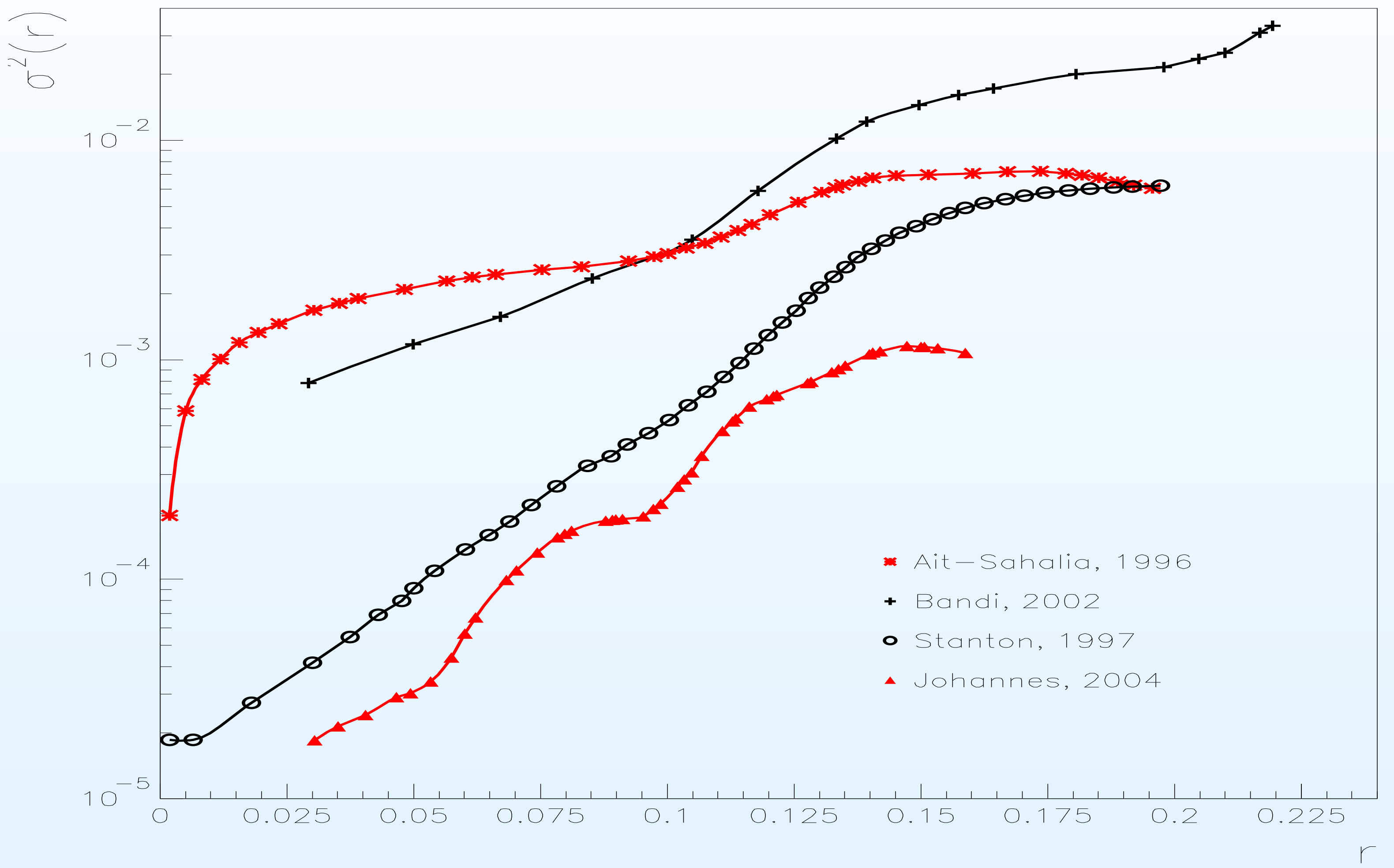
Let X be a jump-diffusion process with finite activity, and assume that:

- $P\{\gamma_\ell = 0\} = 0$;
- $t_i = i\delta$ (equally spaced observations)
- as $\delta \to 0$ both the threshold function $\vartheta(\delta)$ and $\frac{\delta \ln \frac{1}{\delta}}{\vartheta(\delta)}$ tend to zero;
- $nh^4 \to 0$ as $n \to \infty$ and $\exists \beta > 1 : nh^\beta \to \infty$.

Then for all x visited by X

$$\hat{\sigma}^2_{n,h}(x) = \frac{n \sum_{i=1}^{n} K\left(\frac{X_{t_i} - \hat{J}_{1,t_i} - x}{h}\right) (X_{t_{i+1}} - X_{t_i})^2 I_{\{(X_{t_{i+1}} - X_{t_i})^2 \le \vartheta(\delta)\}}}{T \sum_{i=1}^{n} K\left(\frac{X_{t_i} - \hat{J}_{1,t_i} - x}{h}\right)} \to_P \sigma^2(x)$$

Application: The Variance of the short rate



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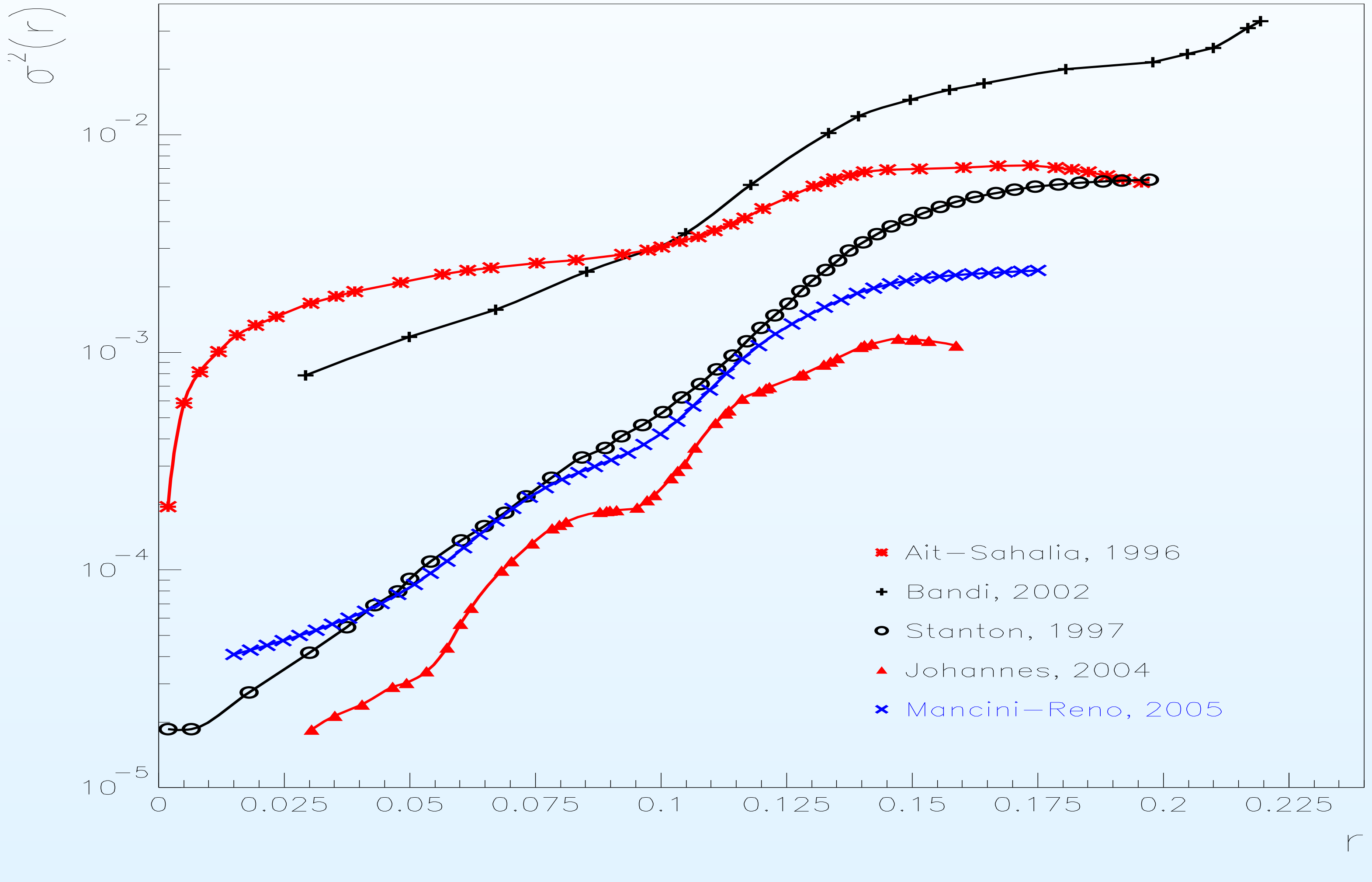

Estimating the drift function

When J is a doubly stochastic Poisson process it is possible to estimate the drift and the jump intensity functions by letting $T \longrightarrow \infty$ and $T/n \longrightarrow 0$.

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The estimator for the drift is:

$$\hat{\mu}_n(x) = \frac{n \sum_{i=1}^{n} K\left(\frac{X_{t_{i-1}} - \hat{J}_{1,t_{i-1}} - x}{h}\right) (X_{t_{i+1}} - X_{t_i}) \cdot I_{\{(X_{t_{i+1}} - X_{t_i})^2 \leq \vartheta(\frac{T}{n})\}}}{T \sum_{i=1}^{n} K\left(\frac{X_{t_{i-1}} - \hat{J}_{1,t_{i-1}} - x}{h}\right)}$$

Then for each x visited by X we have

$$\hat{\mu}_n(x) \longrightarrow_P \mu(x).$$

Estimation of the intensity function

The estimator for $\lambda(x)$ is:

$$\hat{\lambda}_n(x) = \frac{n \sum_{i=1}^{n} K\left(\frac{X_{t_{i-1}} - x}{h}\right) c_{i,n} I_{\{(X_{t_{i+1}} - X_{t_i})^2 > \vartheta(\delta)\}}}{T \sum_{i=1}^{n} K\left(\frac{X_{t_{i-1}} - x}{h}\right)}$$

where $\sup_i |1 - c_{i,n}| \longrightarrow 0$ when $n \longrightarrow \infty$.

Then, for each x visited by X, letting $T \longrightarrow \infty$ and $T/n \longrightarrow 0$,

$$\hat{\lambda}_n(x) \longrightarrow_P \lambda(x).$$

A small sample correction

The coefficients c_i can help in recovering an unbiased estimated intensity by making assumptions on the distribution of jumps.

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Under normality, we choose:

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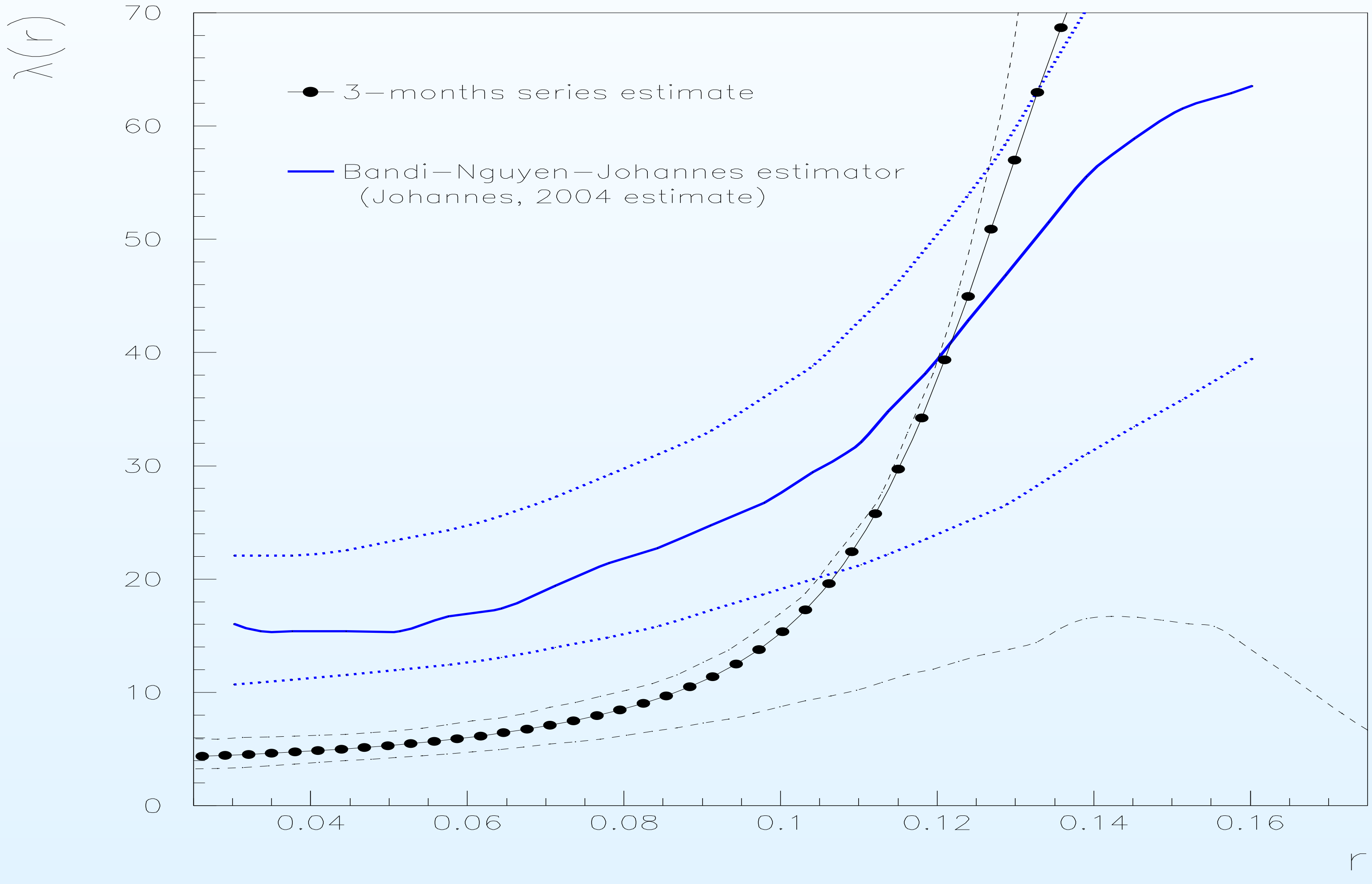
This corrects for the jumps whose size is in the central part of the distribution, which cannot be identified using a threshold in finite samples.

Application: The Intensity of the short rate



Threshold setting

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If ϑ is too small $\longrightarrow$ many variations will be detected as jumps.

If ϑ is too large $\longrightarrow$ many jumps will not be detected.

Threshold setting for equity markets

For equity markets, we use the following iterative nonparametric filter:

$$\vartheta^Z(t) = 9 \cdot \frac{\sum_{i=-L, i\neq 0}^{L} K\left(\frac{i}{L}\right) \left(X_{t+i} - X_{t+i-1}\right)^2 I_{\{(X_{t+i}-X_{t+i-1})^2 < \vartheta^{Z-1}(\delta)\}}}{\sum_{i=-L, i\neq 0}^{L} K\left(\frac{i}{L}\right) I_{\{(X_{t+i}-X_{t+i-1})^2 < \vartheta^{Z-1}(\delta)\}}}$$

where $\vartheta^0(t) = 0$ and $K(x)$ is the Gaussian kernel.

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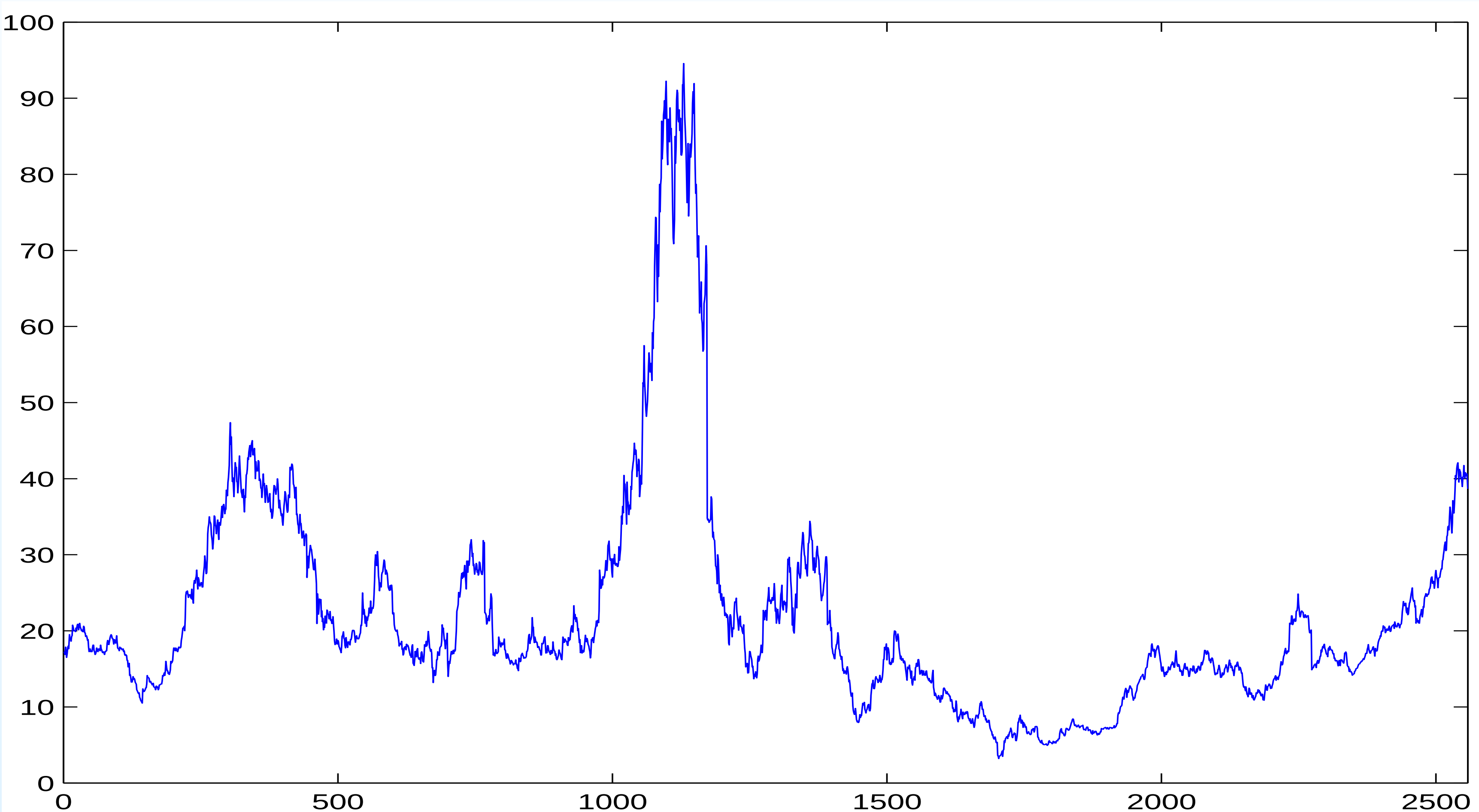
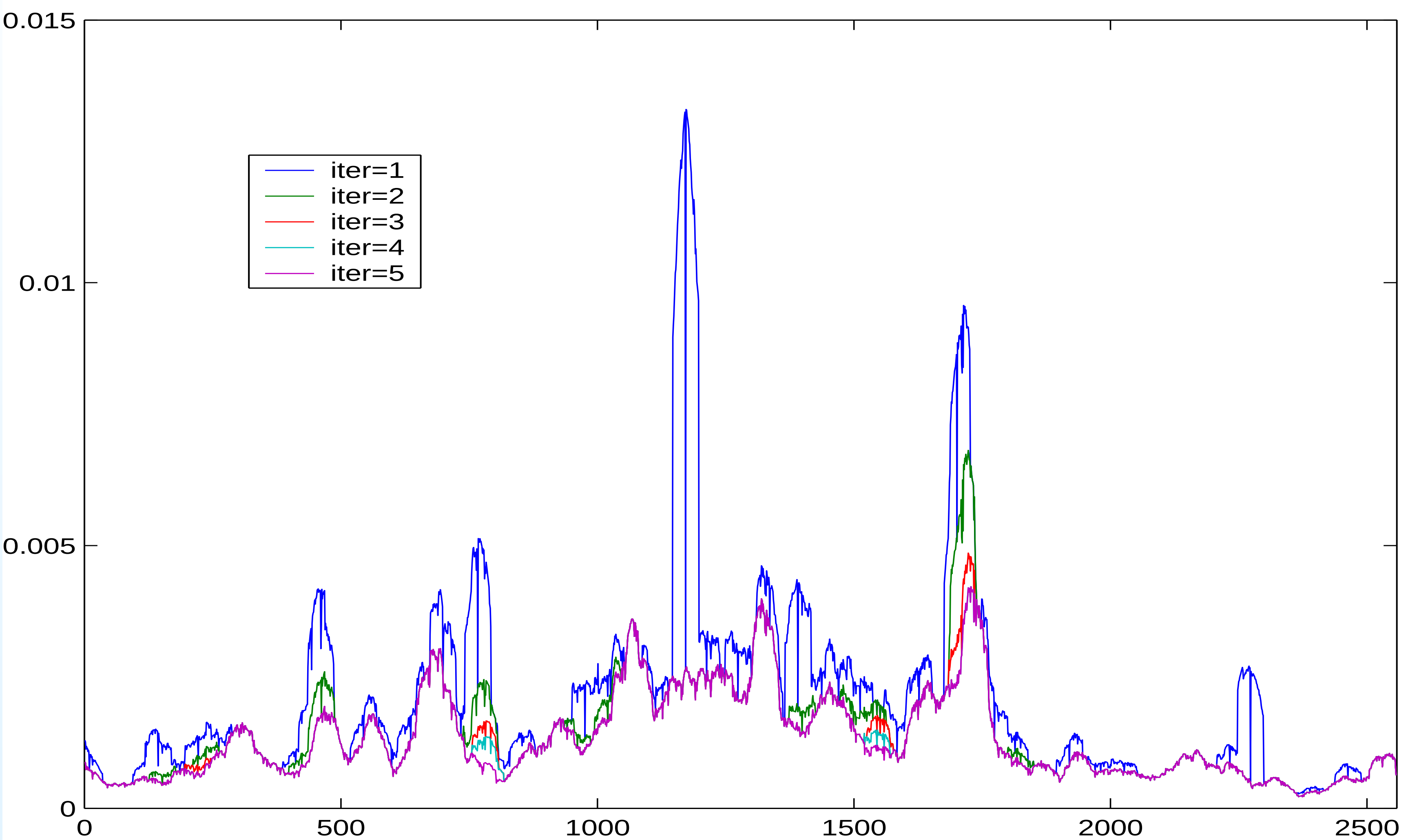
The threshold depends on the smoothing bandwidth L.

Stock price time series example

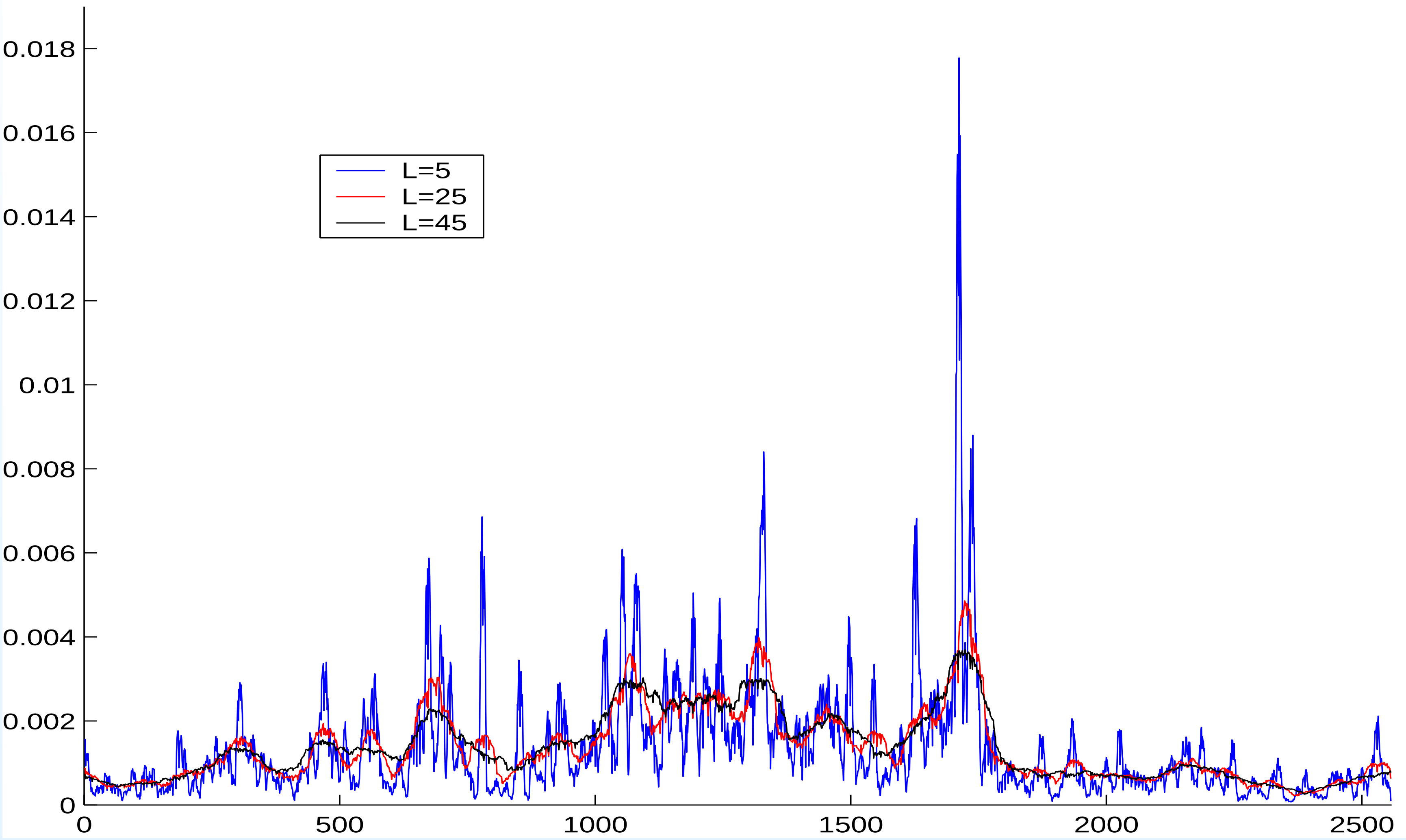
AMD, daily data from 1 March 1996 to 1 March 2006
(2558 observations)



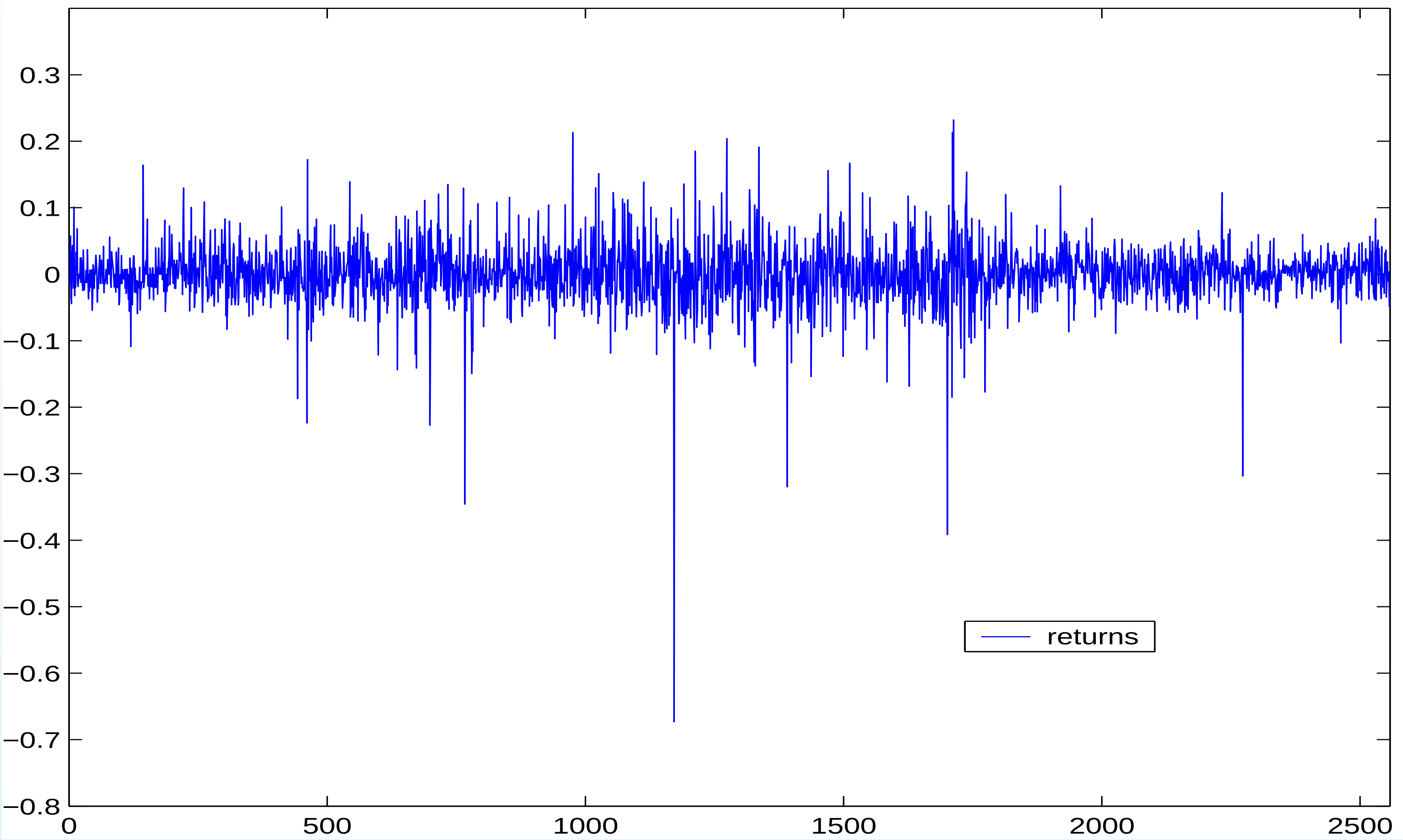
Threshold convergence



Threshold dependence on L



Threshold robustness



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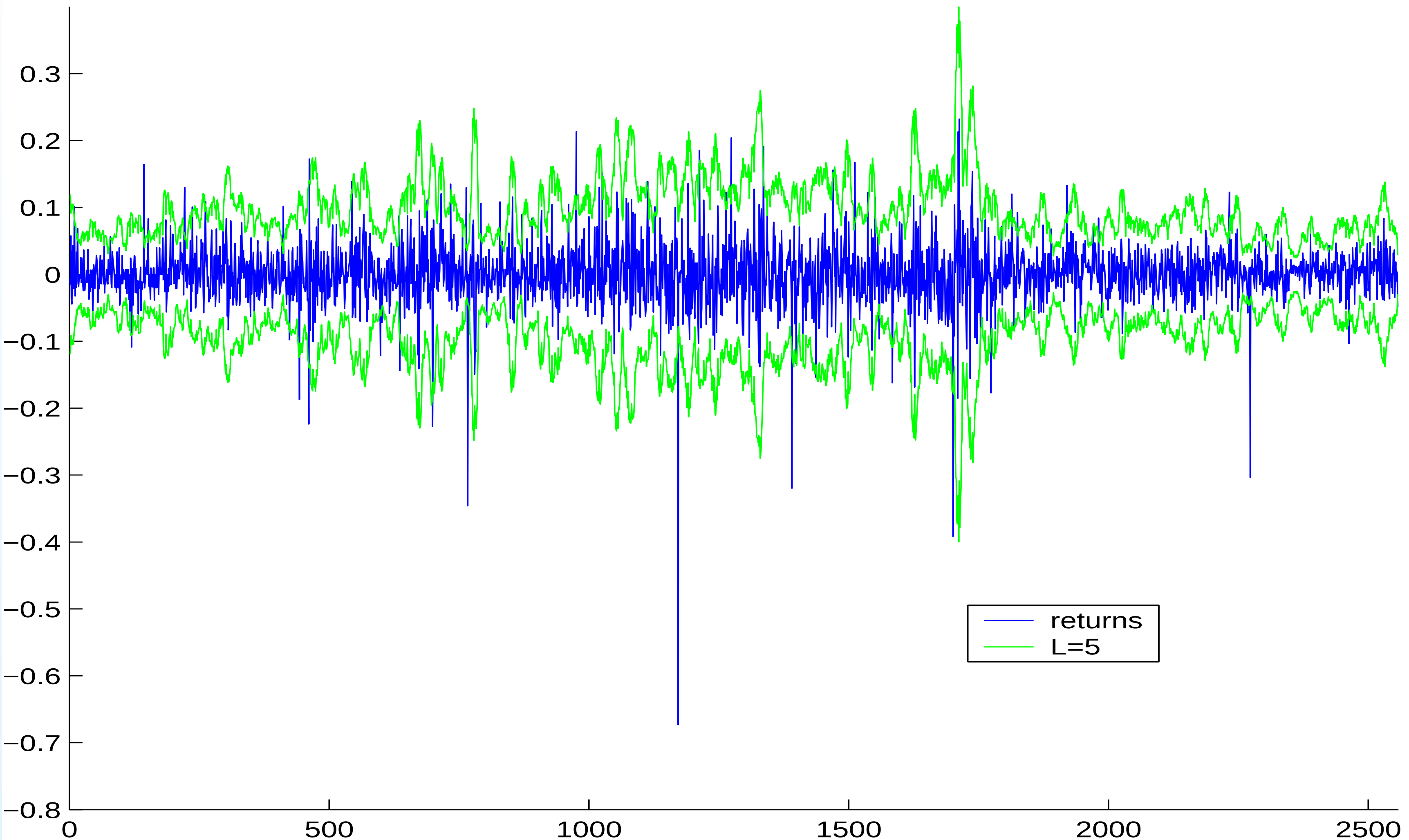

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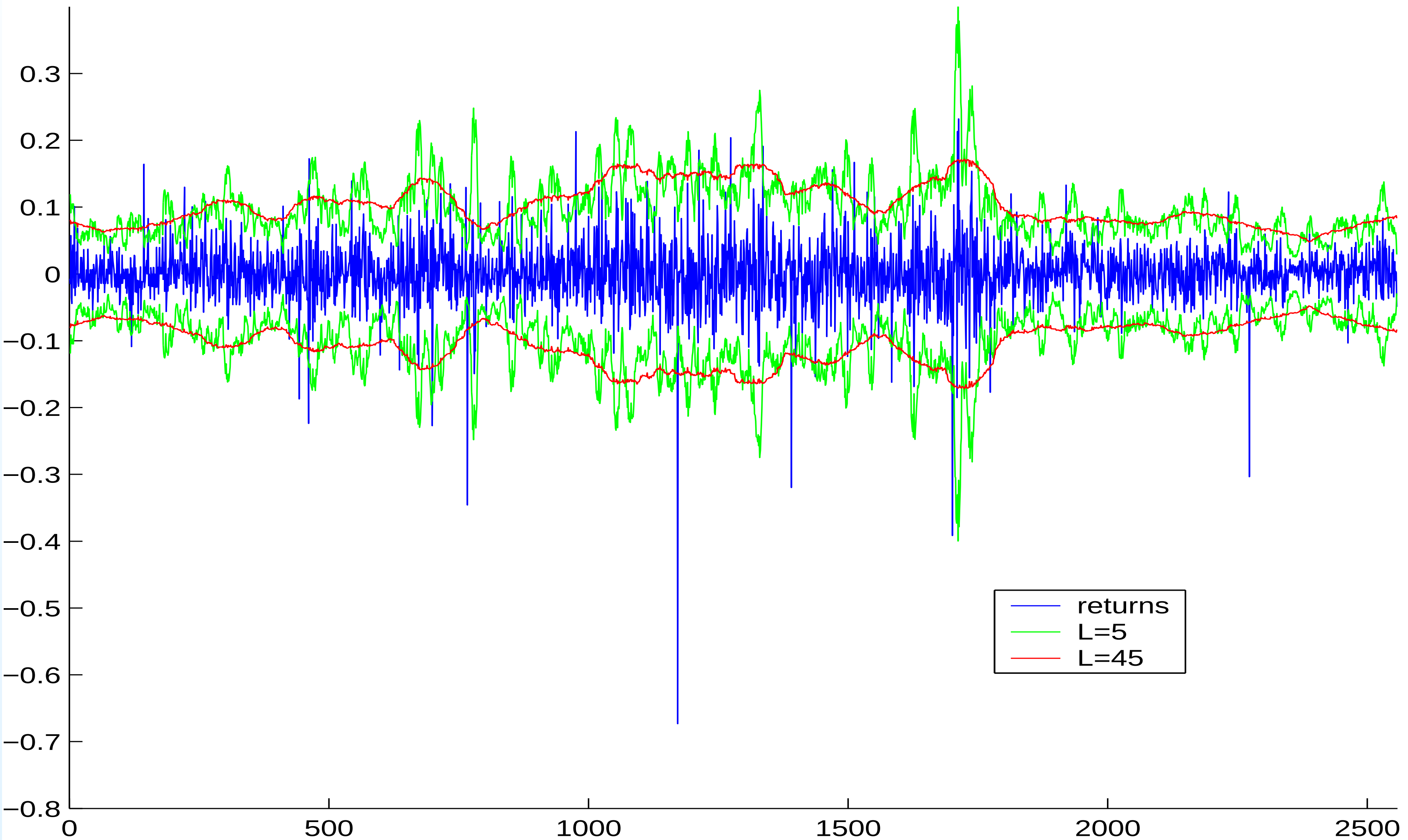

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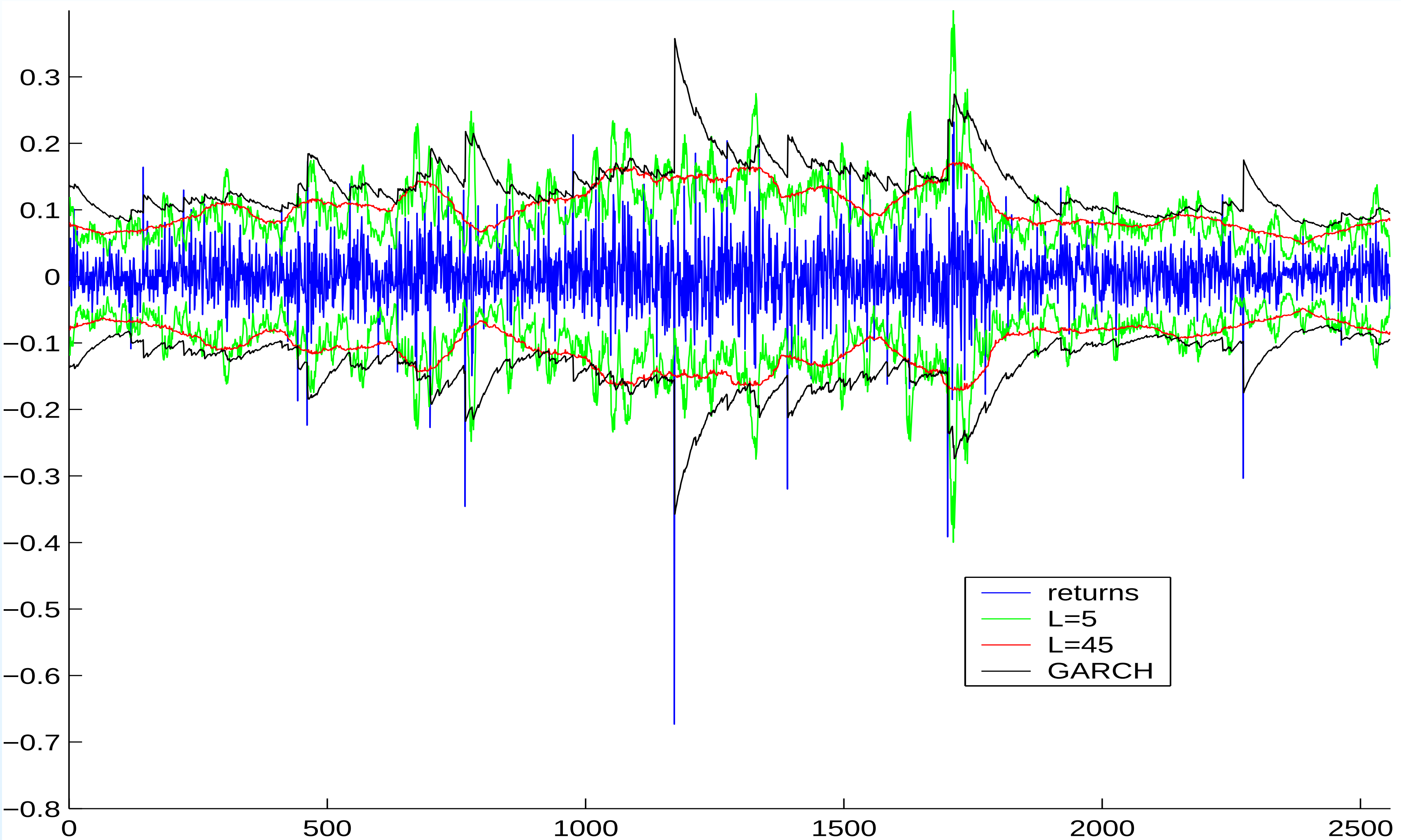
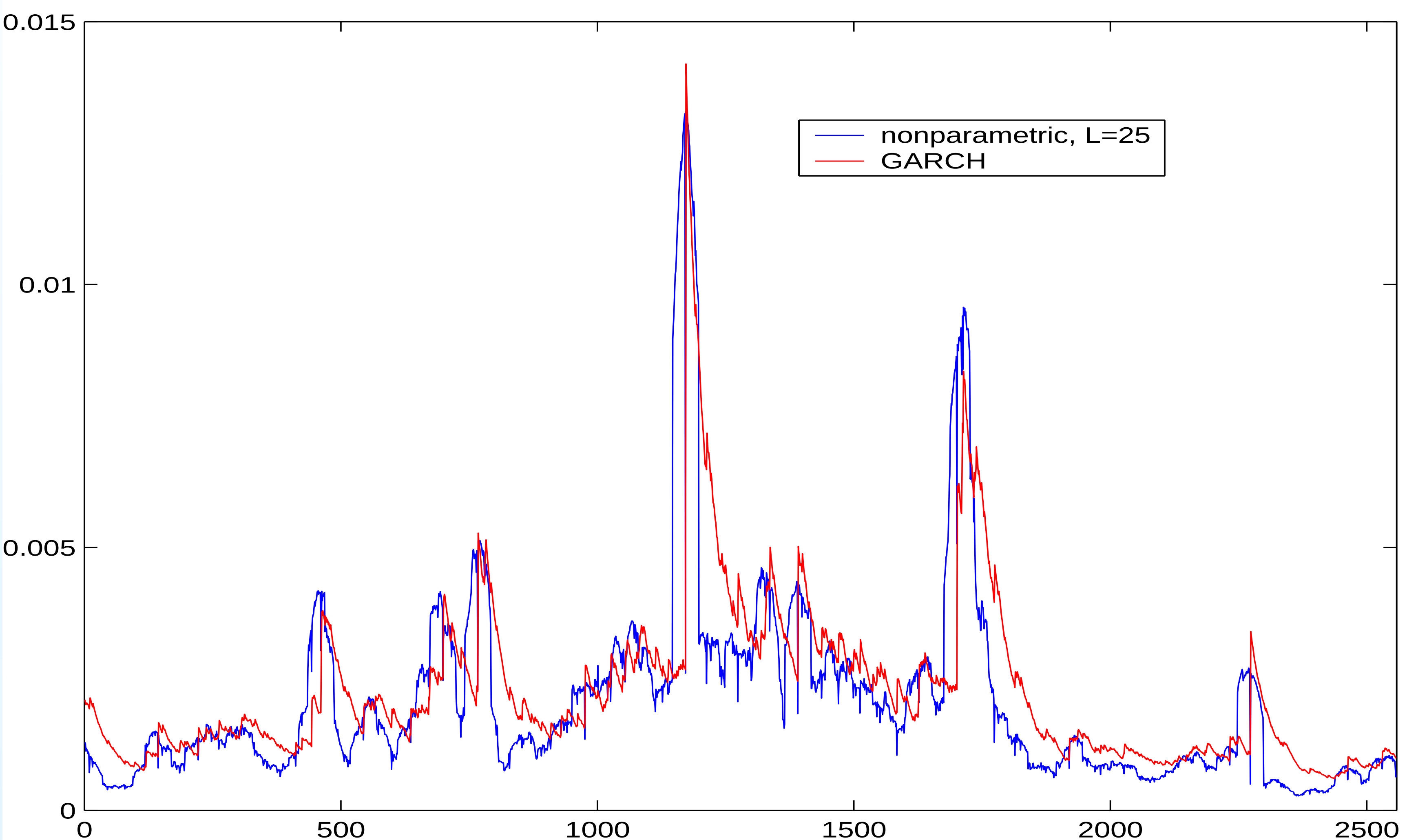


Comparison with GARCH



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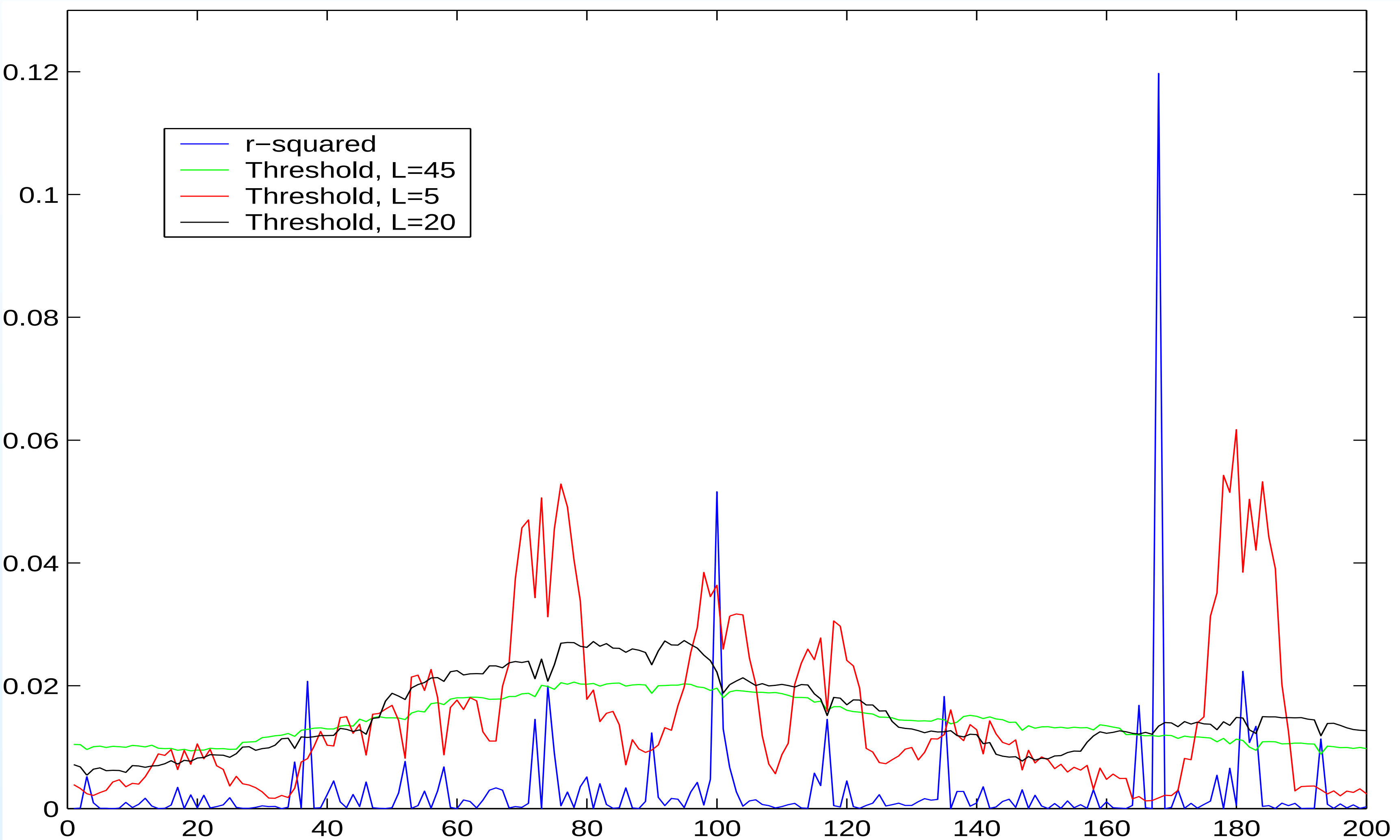
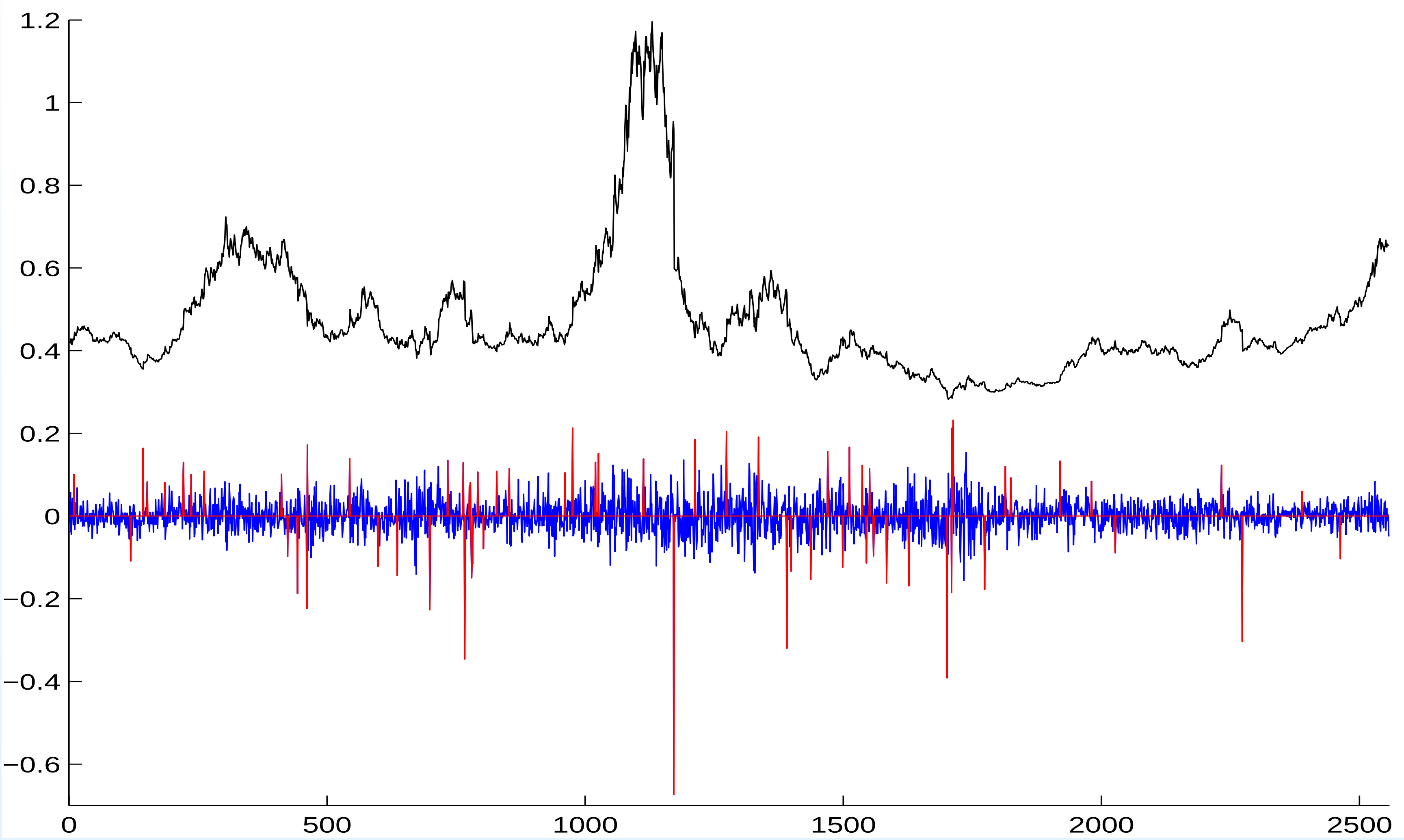


Jump detection



Jumps on stock returns

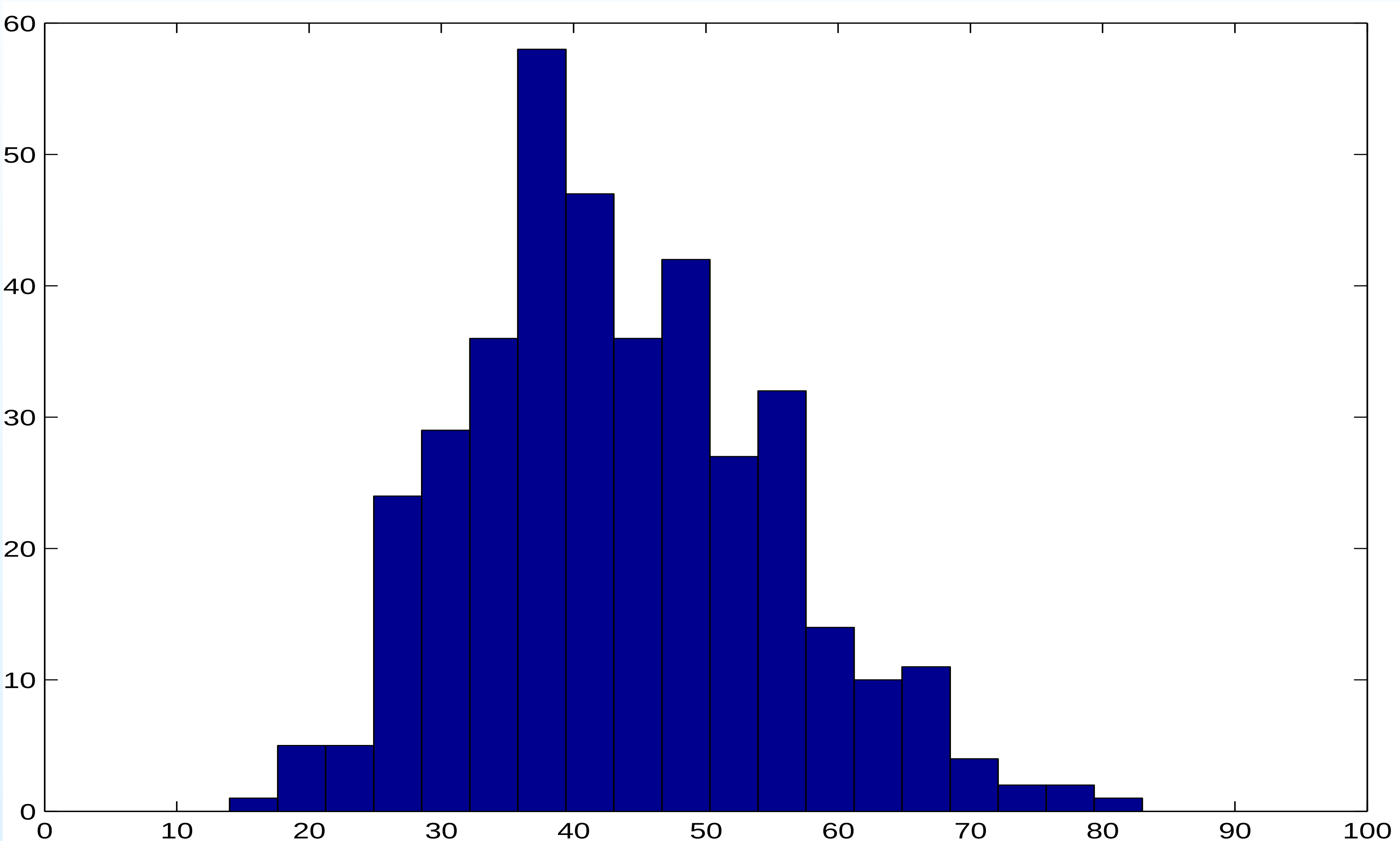
Sample:
386 stocks in the S&P 500 index
10 years of daily observations

AVERAGE NUMBER OF JUMPS

iterations	L=10	L=20	L=30	L=40	L=50
1	38.72	32.27	30.25	29.31	28.68
2	46.62	41.16	39.99	39.64	39.27
3	47.85	43.10	42.18	42.10	42.04
4	48.04	43.50	42.71	42.67	42.80
5	48.05	43.56	42.78	42.81	42.97
6	48.06	43.57	42.80	42.86	43.02

Distribution of jumps among stock

386 stocks, L = 20, six iterations



Electricity prices: a nonparametric approach

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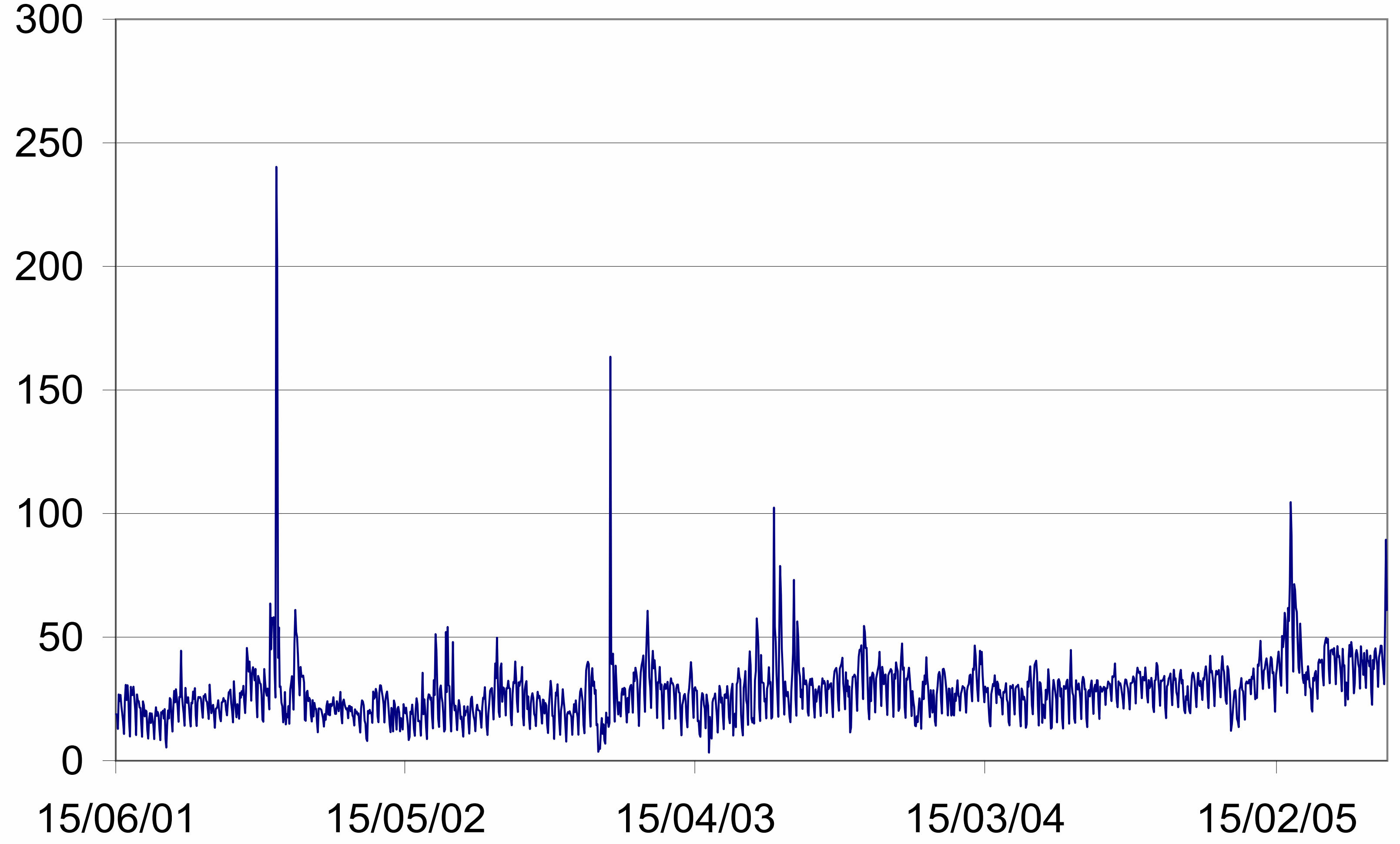
Typically traders distinguish a *normal* status and an *abnormal* status.

We can first separate the two operational regimes; then study the dynamics of the normal status in a nonparametric way.

Electricity prices: the German market



Setting the threshold for jump detection

Given the seasonality of these markets, we first fit a GARCH model:

$$r_t = \mu + \delta r_{t-1} + \sum_{i \in I} c_i D_i + \varepsilon_t \sqrt{h_t}$$

$$h_t = \omega + \alpha r_{t-1}^2 + \beta h_{t-1} + \sum_{i \in I} d_i D_i$$

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We then set $\vartheta(t) = 9 \cdot \hat{h}_t$, where $\hat{h}_t$ is the filtered value of the variance.

A semi-nonparametric approach

Our model can be written as:

$$r_{t+1} = \mu(r_t)\left(1 + \sum_{i \in I} c_i D_i\right) + \sigma(r_t)\left(1 + \sum_{i \in I} d_i D_i\right)\varepsilon_t + dJ_t \qquad (1)$$

where D_i are dummy variables for the day of the week.

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where D_i are dummy variables for the day of the week.

First we separate dJ from continuous variations

A semi-nonparametric approach

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where D_i are dummy variables for the day of the week.

First we separate dJ from continuous variations

Then we estimate the functions μ and σ with an iterative technique.

Estimates of dummy coefficients

If we know $\mu(x), \sigma(x)$ we can estimate the coefficients via maximum likelihood:

$$1 + c_j = \frac{\displaystyle\sum_{t/D_j=1} r_t \frac{\mu\left(r_{t-1}\right)}{\sigma\left(r_{t-1}\right)^2}}{\displaystyle\sum_{t/D_i=1} \frac{\mu\left(r_{t-1}\right)^2}{\sigma\left(r_{t-1}\right)^2}}$$

$$(1 + d_j)^2 = \frac{1}{N_{D_j=1}} \sum_{t/D_{j=1}} \frac{\left(r_t - \mu\left(r_{t-1}\right)\left(1 + c_j\right)\right)^2}{\sigma\left(r_{t-1}\right)^2}$$

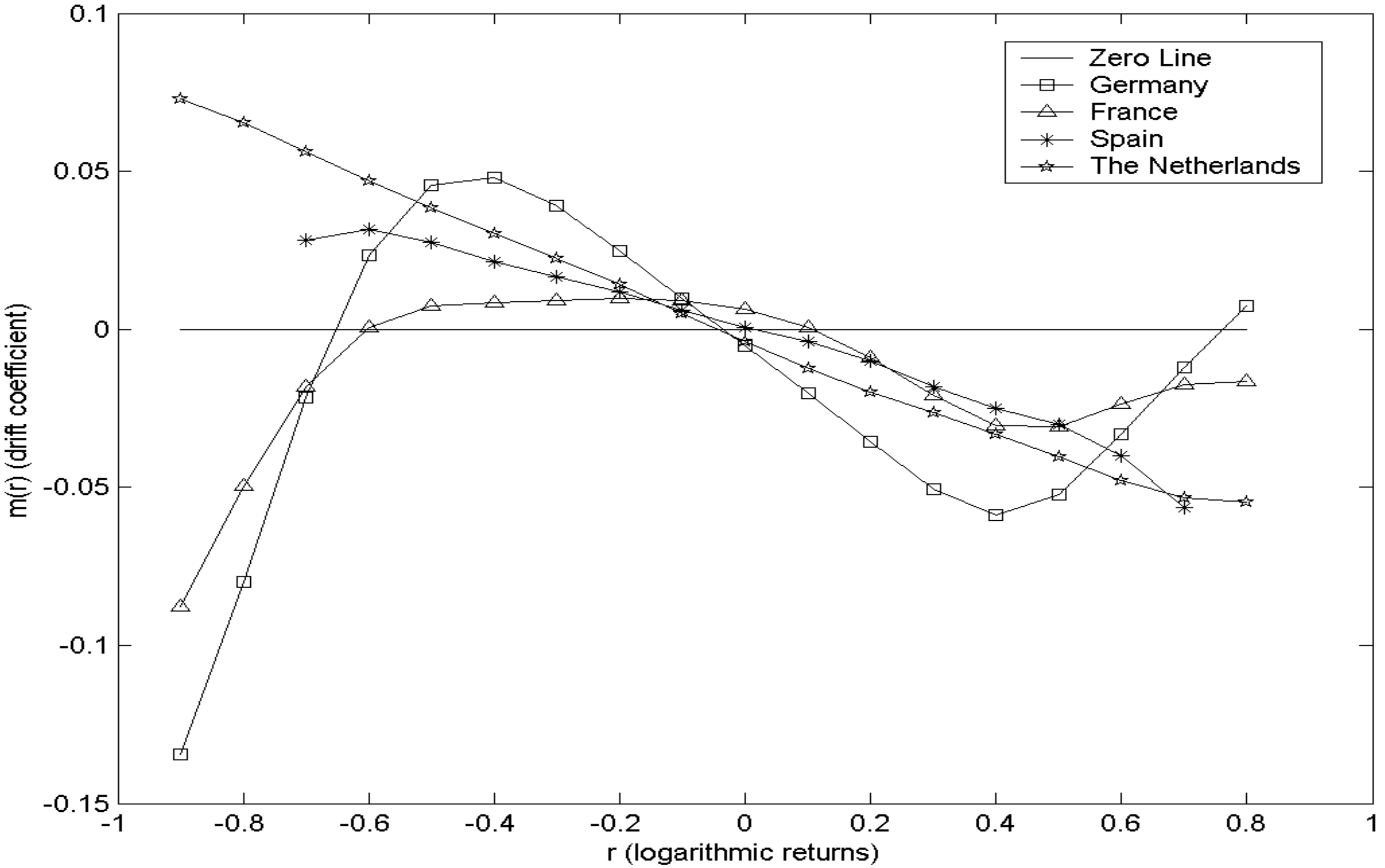
Estimates of drift and volatility

If we know the dummies on weekdays, we can estimate $\mu(x), \sigma(x)$ with the Nadaraya-Watson estimators:

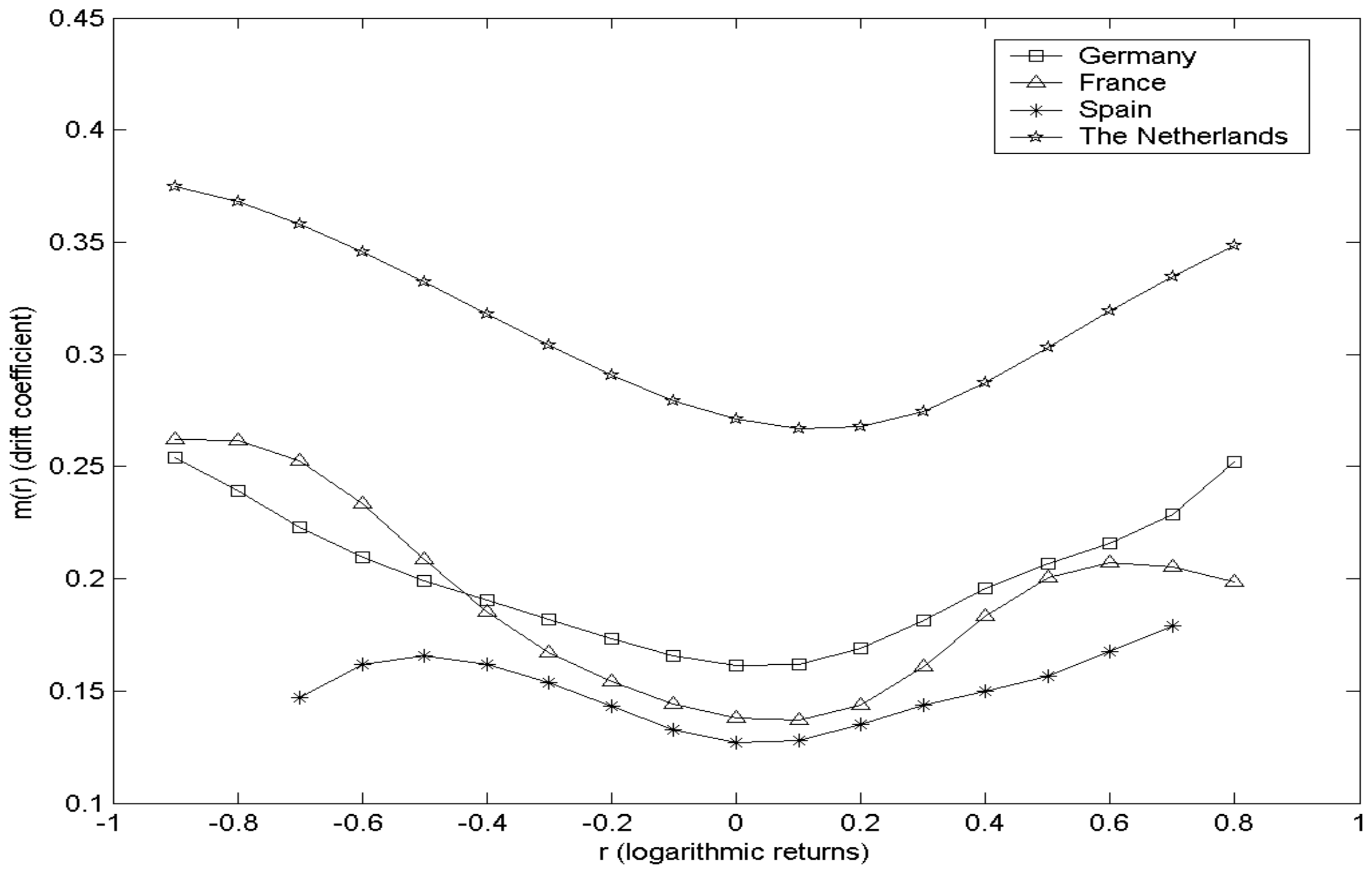
$$\hat{\mu}(x) = \frac{N \sum_{i=1}^{N-1} K\left(\frac{r_i - x}{h}\right) \frac{r_{i+1}}{1+\sum_{q \in I} c_q D_q}}{T \sum_{i=1}^{N} K\left(\frac{r_i - x}{h}\right)} \qquad \text{for all x visited by } r_t$$

$$\hat{\sigma}^2(x) = \frac{N \sum_{i=1}^{N-1} K\left(\frac{r_i - x}{h}\right) \left(\frac{r_{i+1} - \hat{\mu}(r_i)\left(1+\sum_{q \in I} c_q D_q\right)}{1+\sum_{q \in I} d_q D_q}\right)^2}{T \sum_{i=1}^{N} K\left(\frac{r_i - x}{h}\right)} \qquad \text{for all x visited by } r_t.$$

Drift estimates



Volatility estimates



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- On equity, we implement a (nonparametric!) threshold, which displays good robustness on stock return data
- We use the above techniques for estimating a model for electricity price dynamics
- We highlight distinctive common features in the normal status of European electricity markets
- Re-phrasing point one: Work is in progress,

suggestions are welcome!