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# Stochastic Models for Rock Instability in Tunnels

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SUMMARY

We consider a tunnel of rectangular cross-section driven in hard rock containing three joint sets (series of parallel discontinuity planes that traverse the rock mass). If three planes intersect at a point just above the tunnel, a tetrahedron is formed whose base appears as a triangle in the tunnel roof. If the base triangle is contained entirely between the tunnel walls, the tetrahedral block is said to be kinematically unstable and may require support. Overlapping tetrahedra may also occur. In this paper we begin with the simplifying assumption that the spacings within each joint set are exponentially distributed, a model previously considered by Lang (1979), Warburton (1980), Pahl (1981), and consistent with the empirical findings of Priest and Hudson (1976). The principal results concern the total volume and base area of the overbreak (kinematically unstable rock), the distribution of apex height and the number of apexes, together with the number of simple blocks, composite blocks and key blocks. A similar model based on three independent gamma processes is developed and shows that increased regularity of the spacings leads to an increase in the volume of overbreak. Finally, it is shown how some of the results may be applied to circular and semi-circular tunnels.

*Keywords*: EXPONENTIAL DISTRIBUTION; GAMMA DISTRIBUTION; JOINT SET; KEY BLOCK; KINEMATIC INSTABILITY; OVERBREAK; POINT PROCESS OF PLANES; RENEWAL PROCESS: ROCK BOLT; ROCK MECHANICS; ROCK SUPPORT; TUNNEL SUPPORT

## 1. INTRODUCTION

Consider a tunnel driven in jointed rock. Blocks that fall from the roof or blocks that would fall if not secured by rock bolts are of considerable interest to tunnel engineers. Our concern in this paper is with estimating the total volume of unstable blocks, the distribution of block volumes and heights, and the number, length and strength of rock bolts required to support the rock.

Our basic model involves the notion of a *joint set* which is a series of parallel discontinuity planes that traverse the rock mass. Rock failure occurs by separation of the blocks at the discontinuity planes or by sliding between divergent planes. Other modes of failure involving rock deformation or sliding between parallel planes are not considered. Under this model, no failure occurs unless the rock mass contains at least three joint sets. Initially, for simplicity, we restrict attention to the simplest non-trivial case of a flat-roofed tunnel in a rock mass containing exactly three joint sets. The tunnel walls are assumed to be stable so that failure occurs only in the roof (which need not be horizontal).

Under the exponential model for joint sets introduced in Section 3, we compute the expected total volume of kinematically unstable blocks and the expected number of simple blocks, composite blocks and key blocks per unit length of tunnel. Furthermore, we give the apex height distribution and the expected number of apexes per unit length of tunnel. These quantities are useful in determining the number, length and strength of rock bolts required to support the rock.

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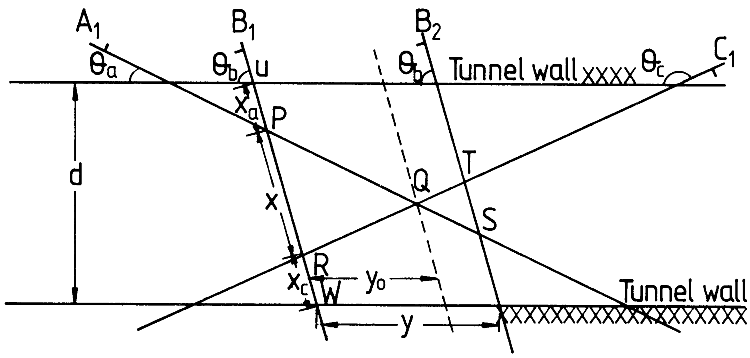
The exponential model for joint spacings has been used by Warburton (1980) and by Pahl (1981) and partially confirmed by the empirical findings of Priest and Hudson (1976). However, to investigate the effect of increased regularity of spacings, we introduce in Section 4, a model based on three gamma processes which generalizes the exponential model. The qualitative conclusion is that increased regularity of spacings increases the volume of overbreak.

In Section 5 it is shown how the results can be applied to circular and semi-circular tunnels but the assumption of three joint sets is crucial.

## 2. DEFINITIONS

Extensive use is made of Fig. 1 which gives a diagram of the tunnel roof in plan. The three-dimensional aspects of the problem are absorbed into trigonometric constants $K_H$ and $K_V$ introduced in Section 3. The three sets of discontinuity planes appear in plan as three sets of parallel lines labelled $A$, $B$, $C$, with subscripts to distinguish lines in the same set. The three angles, $0 < \theta_a < \theta_b < \theta_c < \pi$ are measured from the tunnel direction in the plane of the roof.



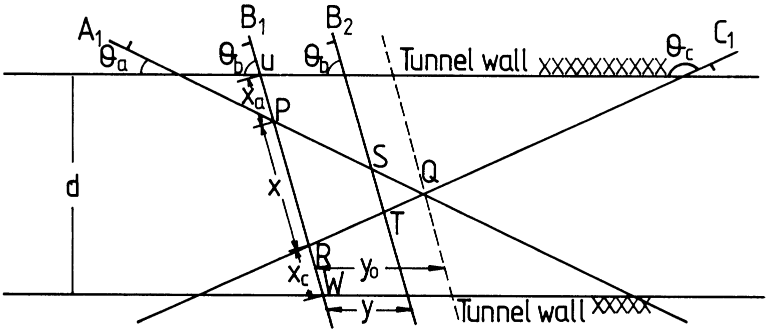




Fig. 1a (top). A plan of the tunnel roof showing a single unstable tetrahedral block, *PQR*.

Fig. 1b (bottom). A plan of the tunnel roof illustrating one composite block, *PQR* containing two simple blocks, *SQT* and *PSTR*.

Discontinuity lines in the $B$ set play a special role so that the choice of labels is important. If the tunnel direction were changed, the labels would also be changed in order to satisfy $0 < \theta_a < \theta_b < \theta_c < \pi$. Dip directions are indicated in Fig. 1 by means of tick marks. The base triangle $SQT$ in Fig. 1b corresponds to an apex-up tetrahedron and is kinematically unstable;

the corresponding triangle in Fig. 1a corresponds to an apex-down tetrahedron that has previously been excavated and is of no further concern. We now define five terms that are used throughout this paper.

A *block* is any mass of rock bounded either by discontinuity planes entirely or by discontinuity planes and by the excavation face.

A tetrahedral block is said to be *kinematically unstable* if its base triangle is contained entirely in the tunnel roof. All the component blocks contained within such a tetrahedron are also kinematically unstable.

A *simple block* is a block containing no internal discontinuities.

A *key block* is a kinematically unstable simple tetrahedral block.

A *composite block* is a kinematically unstable block bounded by stable blocks and by the excavation face.

The assumption that blocks may not slide between parallel faces is reasonable because surface asperities and friction are usually sufficient to ensure stability. Key blocks are important because, in principle, only these blocks need be supported to avoid a collapse. Thus the number of rock bolts required is at least as great as the number of key blocks. However, rock bolts through the key block must penetrate into stable rock and the force must be sufficient to support neighbouring unstable blocks.

In Fig. 2, for example, there are two composite blocks with two key blocks in the first and one in the second. Note that each $B$-line intersects at most one composite block whereas each $A$ and $C$ line may intersect several composite blocks. The number of kinematically unstable simple blocks in Fig. 2 is 14. This is one more than the number of simple block areas visible in the plan because

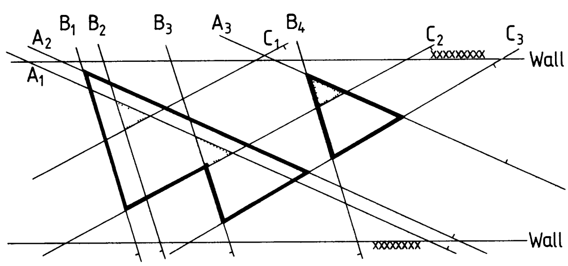



Fig. 2. A segment of tunnel roof in plan showing 2 composite blocks, 3 key blocks and 14 unstable simple blocks.

the unstable simple block bounded by $A_1$, $A_2$, $B_1$, $B_2$, $C_1$, $C_2$, does not appear in the plan. The number of kinematically unstable simple blocks can be found by counting triangles; 4 on $B_1$, 4 each on $B_2$ and $B_3$ and 2 on $B_4$.

## 3. A MODEL BASED ON POISSON POINT PROCESSES

The discontinuity lines in the plan intersect the tunnel wall and so define a point process, one for each joint set. In this section we assume that the points so generated form three independent Poisson processes with rates $\lambda_a$, $\lambda_b$, $\lambda_c$ respectively. Choose $B_1$ as in Fig. 1 intersecting the tunnel walls at $U$ and $W$. Write $UW = x_0 = d/\sin\theta_b$ where $d$ is the tunnel diameter. Let $A_1$ be the first line in the $A$ set encountered as we move from $U$ towards $W$. Similarly $C_1$ is the first line in the $C$ set going from $W$ towards $U$. The frequencies of the $A$ and $C$ processes measured parallel to $B$ are

$$\phi_a = \lambda_a \sin(\theta_b - \theta_a)/\sin\theta_a \text{ and } \phi_c = \lambda_c \sin(\theta_c - \theta_b)/\sin\theta_c.$$

Under the Poisson model, the recurrence random variables $X_a = UP$ and $X_c = WR$ are exponentially distributed with parameters $\phi_a$ and $\phi_c$ respectively. Define the base length $X = PR$ of the triangle $PQR$ by

$$X = \max(x_0 - X_a - X_c, 0). \tag{1}$$

The results that follow are given in terms of the moments of $X$, $\mu_1 = E(X)$, $\mu_2 = E(X^2)$, and its Laplace transform $M_X(t)$, which may be written

$$M_X(t) = 1 + \int\int f_a(z) f_c(u - z)\{\exp(tu - tx_0) - 1\}\, dz\, du \tag{2}$$

with integration over the region $0 \leqslant z \leqslant u \leqslant x_0$. In (2), $f_a$ and $f_c$ are the recurrence densities of $X_a$ and $X_c$ respectively. Direct calculation in the case of exponential recurrence densities gives

$$M_X(t) = \frac{\phi_a\phi_c(\phi_a - \phi_c)e^{-tx_0} + \phi_c t(\phi_c - t)e^{-\phi_a x_0} - \phi_a t(\phi_a - t)e^{-\phi_c x_0}}{(\phi_a - \phi_c)(\phi_a - t)(\phi_c - t)}, \tag{3}$$

$$\mu_1 = x_0 - \frac{\phi_a + \phi_c}{\phi_a\phi_c} + \left(\phi_a^2 e^{-\phi_c x_0} - \phi_c^2 e^{-\phi_a x_0}\right) \Big/ \left\{\phi_a\phi_c(\phi_a - \phi_c)\right\} \tag{4}$$

$$\mu_2 = x_0^2 - 2x_0\left(\frac{\phi_a + \phi_c}{\phi_a\phi_c}\right) + 2\left(\frac{\phi_a\phi_c}{\phi_a - \phi_c}\right)\left\{\phi_c^{-3}(1 - e^{-\phi_c x_0}) - \phi_a^{-3}(1 - e^{-\phi_a x_0})\right\}. \tag{5}$$

In fact the distribution of $X$ involves a condensation

$$p_0 = \frac{\phi_a\phi_c}{\phi_a - \phi_c}\left[\frac{1}{\phi_c}\exp(-\phi_c x_0) - \frac{1}{\phi_a}\exp(-\phi_a x_0)\right] \tag{6}$$

at the origin and a density

$$\frac{\phi_a\ \phi_c}{\phi_a - \phi_c}\left[\exp\{-\phi_c(x_0 - x)\} - \exp\{-\phi_a(x_0 - x)\}\right] \tag{7}$$

over the interval $0 < x \leqslant x_0$. Appropriate limits of (3)-(7) must be used if $\phi_a$, $\phi_c$ and $t$ are not all distinct.

### 3.1. *Mean Base Area and Volume of Kinematically Unstable Blocks*

The main difficulty here is to avoid double counting. To do so we compute, for fixed base length $X = x$, the expected volume and base area of unstable rock lying between $B_1$ and $B_2$. The distance, $y$, between $B_1$ and $B_2$ determines whether the area involved is the triangle $PQR$ as in Fig. 1a of the trapezium $PSTR$ in Fig. 1b: the critical value is $y_0 = K_A x/\sin\theta_b$ where $K_A$ is a constant defined below.

Introduce trigonometric constants $K_H$, $K_A$ and $K_V = K_A K_H$ such that the perpendicular height, base area and volume of the kinematically unstable tetrahedron in Fig. 1a are

$$H(x) = K_H x,$$

$$A(x,y) = \tfrac{1}{2} K_A x^2 \quad y \geqslant y_0 \tag{8}$$

and

$$V(x,y) = \tfrac{1}{6} K_V x^3 \quad y \geqslant y_0.$$

For $y \leqslant y_0$, corresponding to Fig. 1b, the area and volume associated with the trapezium $PSTR$ are obtained by subtraction

$$A(x,y) = \tfrac{1}{2} K_A x^2 \, [2\,y/y_0 - (y/y_0)^2]$$

$$V(x,y) = \tfrac{1}{6} K_V x^3 \, [3\,y/y_0 - 3(y/y_0)^2 + (y/y_0)^3].$$

Now write

$$t = K_A \, \lambda_b/\sin \theta_b \tag{9}$$

and average over $Y$ with $X = x$ fixed, giving

$$A(x) = K_A \, [\exp(-tx) - 1 + tx]/t^2$$

$$V(x) = -K_V \, [\exp(-tx) - 1 + tx - t^2x^2/2]/t^3.$$

Averaging over $X$, we find that the expected base area and volume of kinematically unstable rock per unit length of tunnel are

$$\mu_A = K_A \lambda_b \, [M_X(t) - 1 + \mu_1 t]/t^2 \tag{10}$$

and

$$\mu_V = -K_V \lambda_b \, [M_X(t) - 1 + \mu_1 t - \mu_2 t^2/2]/t^3. \tag{11}$$

The above formulae continue to apply even when successive spacings in each joint set are not independent (Cox, 1962), but the marginal distribution of spacings must be exponential and the three joint sets must be mutually independent.

### 3.2. *The Mean Number of Simple Blocks, Composite Blocks and Key Blocks*

It is not difficult to see that the number of unstable simple blocks is equal to the number of unstable base triangles counting all overlapping triangles. A straightforward calculation shows that the expected number of these triangles per unit length of tunnel is

$$\tfrac{1}{2} x_0^2 \, \phi_a \, \phi_c \, \lambda_b. \tag{12}$$

This formula holds under weaker conditions than (10) and (11). The assumption of exponentiality is not required but the three joint sets must be mutually independent.

We now turn our attention to composite blocks. Let $E_j$ be the event that the unstable block generated by $B_j$ is not degenerate and is not intersected by $B_{j+1}$. The number of composite blocks in a given length of tunnel is the number of such events and the expected number per unit length of tunnel under the Poisson model is

$$\lambda_b \, (M_X(t) - p_0). \tag{13}$$

Key blocks are more important but more difficult to count than simple blocks or composite blocks. Choose a line $B_1$ intersected by $N_a$ $A$'s and $N_c$ $C$'s defining a sequence of length $N_\bullet = N_a + N_c$. The number of switches is the number of occurrences of the ordered pair $AC$ and is related to the runs statistic of non-parametric theory by no. of switches = [(no. of runs − 1)/2]. Provided that there is no interference from further lines in the $B$ set, the number of key blocks on $B_1$ is the number of switches. Under the Poisson model and conditional on $N_a$ and $N_c$, the number of switches has the hypergeometric distribution with mean $N_a N_c/N_\bullet$.

The base length, $X$, associated with any ordered pair is a random variable with probability density, $N_\bullet(1 - x/x_0)^{N_\bullet - 1}/x_0$, $0 \leqslant x \leqslant x_0$. If the base length is sufficiently large no key block is formed on $B_1$ because of interference from $B_2$. In fact, for fixed $x$, the probability that a key block is formed is $\exp(-tx)$. Thus the expected number of key blocks formed at $B_1$ is

$$E\left\{ \frac{N_a N_c}{N_\bullet} \int_0^{x_0} \frac{N_\bullet}{x_0} (1 - x/x_0)^{N_\bullet - 1} e^{-tx} dx \right\}.$$

Evaluating this expression we find that the expected number of key blocks per unit length of tunnel is

$$\mu_{KB} = \lambda_b \, \phi_a \phi_c x_0^2 \, \{\exp(-m_0) - 1 + m_0\}/m_0^2, \tag{14}$$

where $m_0 = x_0(\phi_a + \phi_c + t)$ is the expected number of discontinuity lines of all types intersecting a maximal triangle whose base length $x_0$ extends from $U$ to $W$ (Fig. 1). Expression (14) is the only one in this paper that depends on the assumption that successive spacings in each joint set be independent.

### 3.3. *Distribution of Apex Height*

A tetrahedron with base length $X$ on $B_1$ has apex height $K_H X$ where $X$ is distributed according to (7) normalized by the factor $(1 - p_0)^{-1}$. Thus

$$E(K_H X \mid X > 0) = K_H \mu_1/(1 - p_0). \tag{15}$$

However, this is not the average apex height because, for example, the averaging procedure leading to (15) would count the height of the apex associated with both $PQR$ and $SQT$ in Fig. 1b. The tetrahedron with base $SQT$ should be excluded because there is only one apex.

A more refined analysis leads to the following. Define

$$\xi_a = \left\{ \frac{\lambda_a \lambda_c}{\phi_c} - \frac{\lambda_a \lambda_b}{\phi_a} - \frac{\lambda_a \lambda_c}{\phi_a} \right\}^{-1}; \; \xi_c = \left\{ \frac{\lambda_a \lambda_c}{\phi_c} + \frac{\lambda_b \lambda_c}{\phi_c} - \frac{\lambda_a \lambda_c}{\phi_a} \right\}^{-1};$$

$$\phi_{ac} = (\lambda_a + \lambda_b + \lambda_c)\, \phi_a \phi_c/(\lambda_a \phi_c + \lambda_c \phi_a);$$

$$\lambda_{ac} = \lambda_a \lambda_c (\lambda_a + \lambda_b + \lambda_c)/(\lambda_a + \lambda_c);$$

and

$$p_0^* = \lambda_{ac} \left\{ \frac{\xi_c}{\phi_c} e^{-\phi_c x_0} - \frac{\xi_a}{\phi_a} e^{-\phi_a x_0} + \frac{(\xi_a - \xi_c)}{\phi_{ac}} e^{-\phi_{ac} x_0} \right\}.$$

Then the apex-height distribution $f_H(h)$ is given by

$$f_H(h) = \frac{\lambda_{ac}}{K_H (1 - p_0^*)} \left\{ \xi_c e^{-\phi_c (x_0 - x)} - \xi_a e^{-\phi_a (x_0 - x)} + (\xi_a - \xi_c)\, e^{-\phi_{ac}(x_0 - x)} \right\},$$

where $x = h/K_H$. The range for $h$ is $(0, K_H x_0]$. Furthermore, the average height is given by

$$E(H) = K_H \lambda_{ac} \{ \xi_c g(\phi_c) - \xi_a g(\phi_a) + (\xi_a - \xi_c)\, g(\phi_{ac}) \}/(1 - p_0^*) \tag{16}$$

where $g(\phi) = x_0 \phi^{-1} - \phi^{-2}(1 - e^{-\phi x_0})$. Higher moments are readily computed in terms of the quantities described above. Alternatively, if these calculations are to be used to help select rock bolts of appropriate lengths it may be better to sketch $f_H(h)$ or the corresponding cumulative distribution function. Fig. 3 illustrates the shape of some density functions encountered.

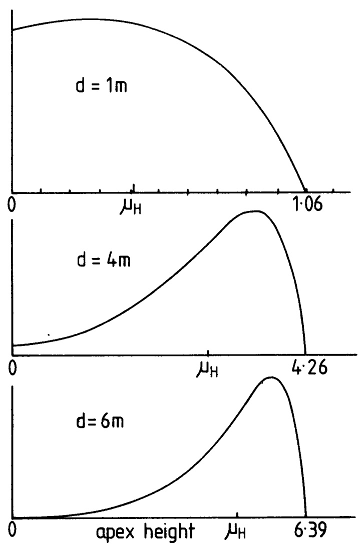



Fig. 3. Apex height densities for tunnels of diameters $d = 1, 4, 6$ metres.

Finally, the expected number of apexes per unit length of tunnel can be shown to be

$$\lambda_b (\lambda_a + \lambda_c) (1 - p_0^*)/(\lambda_a + \lambda_b + \lambda_c). \tag{17}$$

### 3.4. *A Numerical Example*

Suppose that the three angles are $(\theta_a, \theta_b, \theta_c) = (30°, 110°, 130°)$ with corresponding rates $(\lambda_a, \lambda_b, \lambda_c) = (0.7, 1.0, 2.0)$ giving $K_A = 0.3420$, $t = 0.3640$, $\phi_a = 1.3787$, $\phi_c = 0.8930$. Take $K_H = 1$ and consider tunnels of diameters $d = 1$, 4, 6 m. These values correspond in part to those used by Lang (1979). We find the following values.

TABLE 1

| *Expected value per unit length of tunnel of* | *Equation* | *Tunnel diameter* 1 m | 4 m | 6 m |
|---|---|---|---|---|
| Roof area ($m^2$) | (10) | 0.0132 | 0.8973 | 2.2570 |
| Volume ($m^3$) | (11) | 0.0031 | 1.0313 | 4.3076 |
| No. simple blocks | (12) | 0.6971 | 11.1539 | 25.0962 |
| No. composite blocks | (13) | 0.2804 | 0.3871 | 0.2089 |
| No. key blocks | (14) | 0.3306 | 1.8111 | 2.8053 |
| Apex height (m) | (15) | 0.43 | 2.62 | 4.59 |
| Apex height (m) | (16) | 0.44 | 2.80 | 4.84 |
| No. apexes | (17) | 0.2908 | 0.7073 | 0.7270 |

## 4. A MODEL BASED ON THREE GAMMA PROCESSES

The methods described in Section 3 may be used in the case of point process other than the Poisson. We describe briefly the calculations involved for gamma processes where the spacings in each joint set are independent random variables with means $\lambda_a^{-1}$, $\lambda_b^{-1}$ and $\lambda_c^{-1}$ in the tunnel direction and indices $\nu_a, \nu = \nu_b, \nu_c$, assumed to be integers. Defining $\phi_a$ and $\phi_c$ as in Section 3 the recurrence density $f_a(x)$ for the gamma process with parameters $\phi_a, \nu_a$ is

$$f_a(x) = \phi_a \exp(-\nu_a\phi_a x) \sum_{0}^{\nu_a - 1} (\nu_a\phi_a x)^j/j!.$$

Using a similar expression for $f_c(x)$, the Laplace transform, $M_X(s)$, is found by substituting into (2) and integrating. The results that follow are given in terms of $M_X(s)$ and its derivatives evaluated at $s = \nu t = \nu K_A \lambda_b/\sin\theta_b$.

### 4.1. *Mean Base Area and Volume*

After lengthy but elementary calculations, we find that the mean base area and volume of kinematically unstable rock per unit length of tunnel are

$$\mu_A = \tfrac{1}{2} K_A\lambda_b \left[ \sum_{j=0}^{\nu-1} \frac{(-1)^j}{j!} (\nu t)^j (\nu - j)(\nu + 1 - j) M_X^{(j)}(\nu t) + 2\nu^2\mu_1 t - \nu(\nu + 1) \right] /(\nu t)^2 \quad (18)$$

and

$$\mu_V = -\tfrac{1}{6} K_V\lambda_b \left[ \sum_{j=0}^{\nu-1} \frac{(-1)^j}{j!} (\nu t)^j (\nu - j)(\nu + 1 - j)(\nu + 2 - j) M_X^{(j)}(\nu t) - 3\nu^3\mu_2 t^2 \right. \quad (19)$$

$$\left. + 3\nu^2 (\nu + 1)\mu_1 t - \nu(\nu + 1)(\nu + 2) \right] /(\nu t)^3,$$

where $\mu_j = M_X^{(j)}(0)$. If $t$ is small we may expand $M_X(\nu t)$ in a Taylor series about $t = 0$ giving the approximations

$$\mu_A = \tfrac{1}{2} K_A\lambda_b [\mu_2 + O(t^{\nu-1})], \quad \mu_V = \tfrac{1}{6} K_V\lambda_b [\mu_3 + O(t^{\nu-1})], \quad (20)$$

which are appropriate if planes in the $B$ set are widely spaced.

It is instructive to examine the limits of (18) and (19) as $\nu_a$, $\nu_b$, $\nu_c$ all become large corresponding to increasingly regular joint sets. For simplicity we take $t$ to be small and assume that the tunnel is wide so that $t \ll x_0^{-1} \ll \phi_a\phi_c/(\phi_c + \phi_c)$. Under these conditions (20) may be used with $\mu_2 = \mu_1^2 + \sigma^2$, $\mu_3 = \mu_1^3 + 3\mu_1\sigma^2$ where $\mu_1 = x - (\phi_a^{-1} + \phi_c^{-1})/2$ and $\sigma^2 = (\phi_a^{-2} + \phi_c^{-2})/12$.

For a simple numerical comparison we take $(\theta_a, \theta_b, \theta_c) = (30°, 110°, 130°)$, $d = 4$, $(\lambda_a, \lambda_b, \lambda_c) = (0.7, 0.5, 2.0)$ and $K_H = 1$ which give $x_0 = 4.26$, $t = 0.18$ and $tx_0 = 0.77$. Expression (20) for regularly spaced joint sets gives $\mu_A = 0.963$, $\mu_V = 1.099$ compared with 0.529 and 0.588 for the Poisson processes of Section 3. The qualitative conclusion from this and from other numerical comparisons is that regularity tends to increase $\mu_A$ and $\mu_V$ and, as in this example, the effect can be considerable.

## 5. FURTHER DEVELOPMENTS

Further developments are possible in a number of areas the most important of which are as follows:

(i) circular tunnels: a simple geometrical argument can be made to show how (11) may be used for circular and horseshoe shaped tunnels where there is no clear distinction between roof and wall. Details of the argument were given in a previous version of this paper but are now omitted for the sake of brevity;

(ii) statistical problems of estimating the necessary parameters for linear probes and the effect on the calculations of the errors so induced;

(iii) the effects of joint planes that are of finite extent (Piteau, 1973).

(iv) the effects of introducing a fourth or fifth joint set.

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