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It is shown that two visibly distinct distributions can have almost identical moment-generating functions.

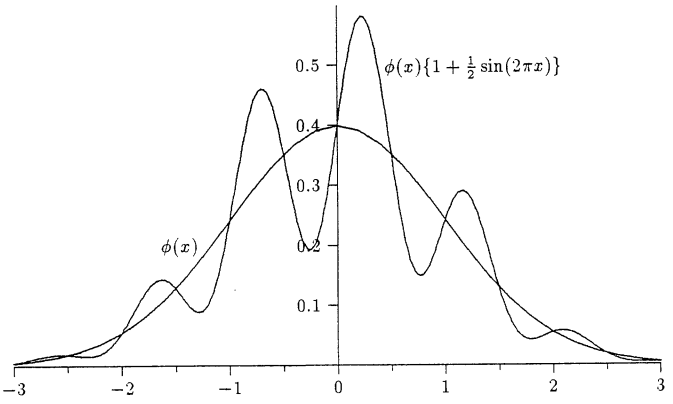

Figure 1. Two Density Functions: Total Variance Distance Is 0.16.

All probabilists agree that two different distributions cannot have the same moment-generating function. Although the distributions in this example are visibly quite different (Fig. 1), the two cumulant-generating functions appear coincident. Even if Figure 2 were plotted by a high-resolution laser printer on top-quality mile-square photographic paper, the difference would be less than one pixel over the entire range. The maximum difference of 1.34×10^{-9} is less than the error often tolerated for purposes of numerical approximation. So the answer to the question posed in the title must be *yes* for mathematical purposes but a resounding *no* for numerical purposes.

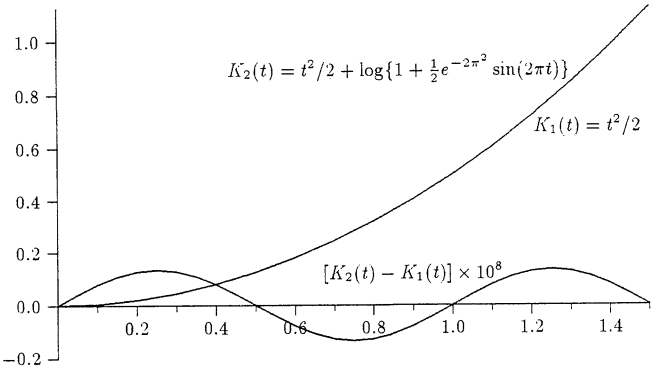

Figure 2. Two Cumulant-Generating Functions and Their Differences (Enlarged by a Factor of 10^8).

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