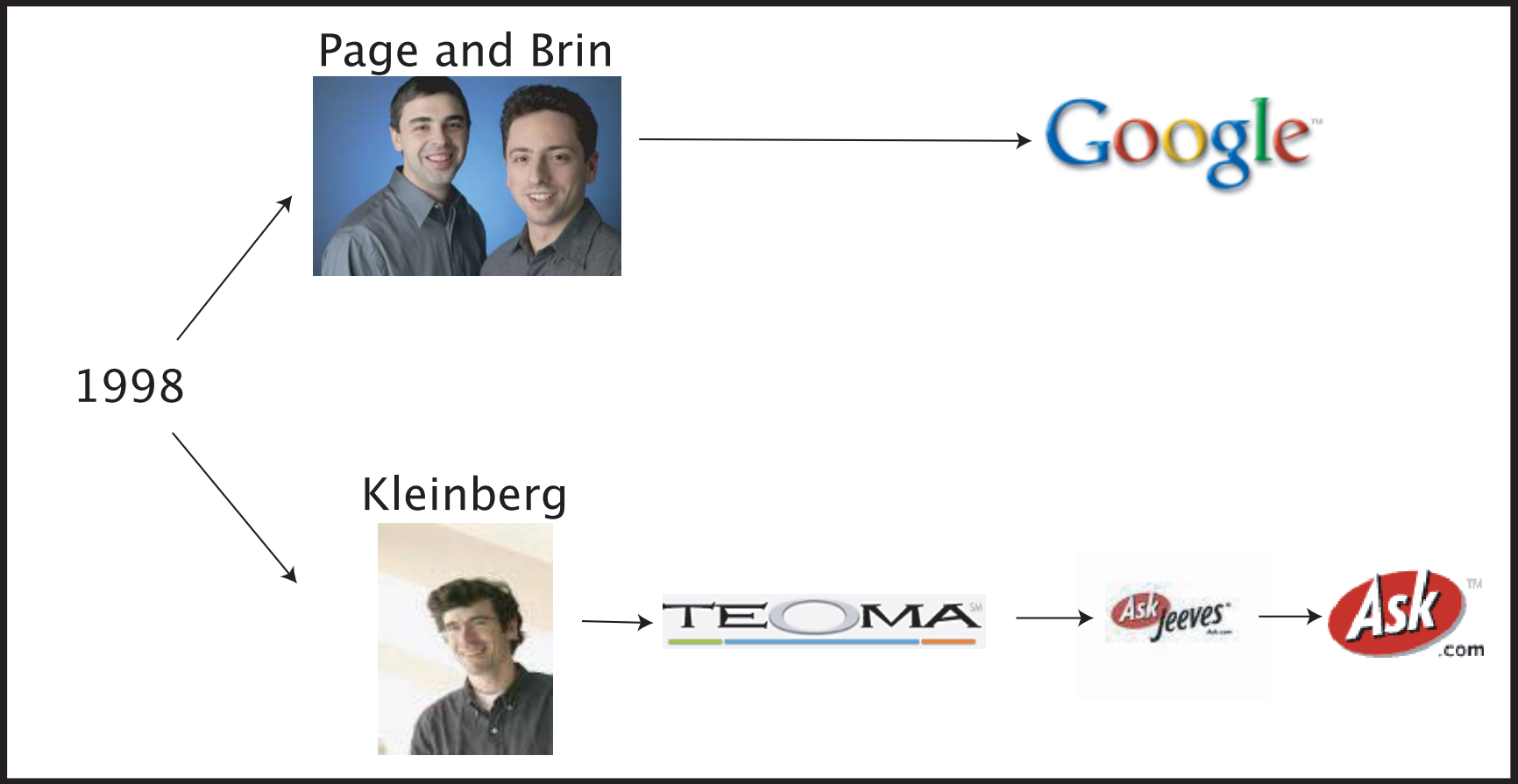
Google's PageRank and Beyond

THE SCIENCE OF SEARCH ENGINE RANKINGS

AMY N. LANGVILLE and CARL D. MEYER

1998: enter Link Analysis

- uses *hyperlink* structure to focus the relevant set
- combine traditional IR score with popularity score



Web Information Retrieval

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A Herculean Task!

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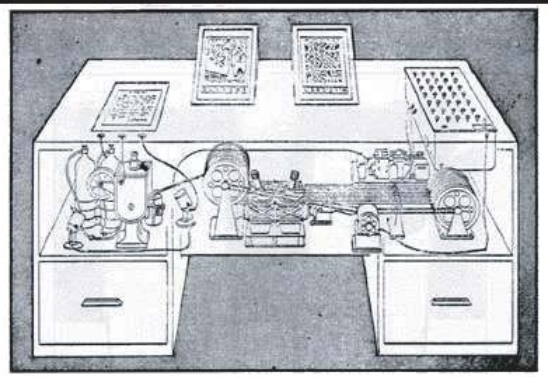
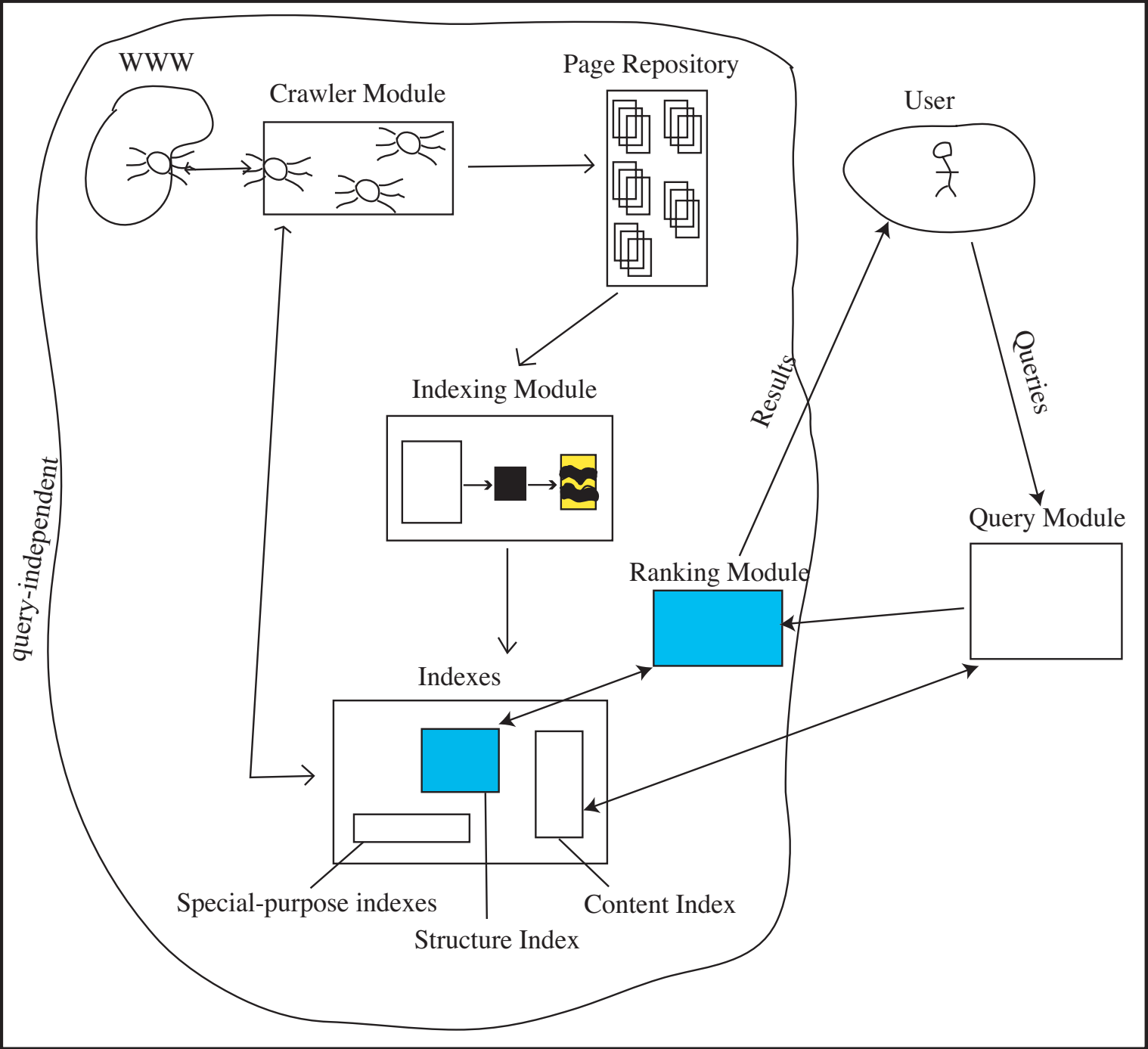
- Ah, but it's *hyperlinked* !
  - Vannevar Bush's 1945 memex



Memex in the form of a desk would instantly bring files and material on any subject to the operator's fingertips. Slanting translucent viewing screens magnify supermicrofilm filed by code numbers. At left is a mechanism which automatically photographs longhand notes, pictures and letters, then files them in the desk for future reference (*LIFE* 19(11), p. 123).

Elements of a Web Search Engine



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Google's PageRank = Google's $$$$$

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business intelligence

Search

Advanced Search
Preferences

Web Results **1** - **10** of about **122,000,000** for **business intelligence**. (**0.10** seconds)

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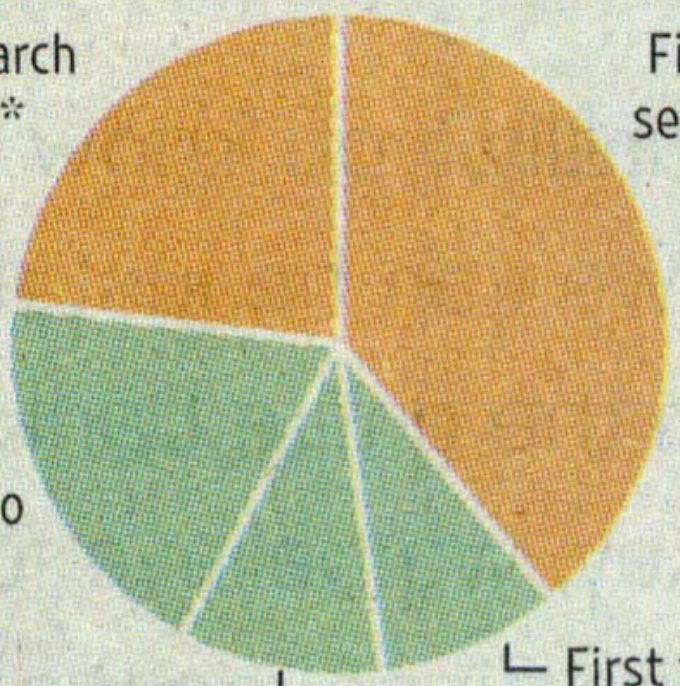
The Wall Street Journal, April 13, 2007

Take Your Pick

Amount of Internet search results that Web surfers typically scan before selecting one



*Top results without reading through the whole page

Note: Sample size is 2,369 people

Sources: JupiterResearch; iProspect

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business intelligence

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PLUS: The Future of Digital Voting
Google founders Larry Page and Sergey Brin

Google's PageRank

(Lawrence Page & Sergey Brin 1998)

The Google Goals

- Create a PageRank $r(P)$ that is not query dependent
  - ▷ Off-line calculations — No query time computation
- Let the Web vote with in-links
  - ▷ But not by simple link counts
    - — One link to P from Yahoo! is important
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- Share The Vote
  - ▷ Yahoo! casts many "votes"
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Start with $r_0(P_i) = 1/n$ for all pages $P_1, P_2, \ldots, P_n$

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$$\ddots$$

$$r_{j+1}(P_i) = \sum_{P \in \mathcal{B}_{P_i}} \frac{r_j(P)}{|P|}$$

In Matrix Notation

After Step k

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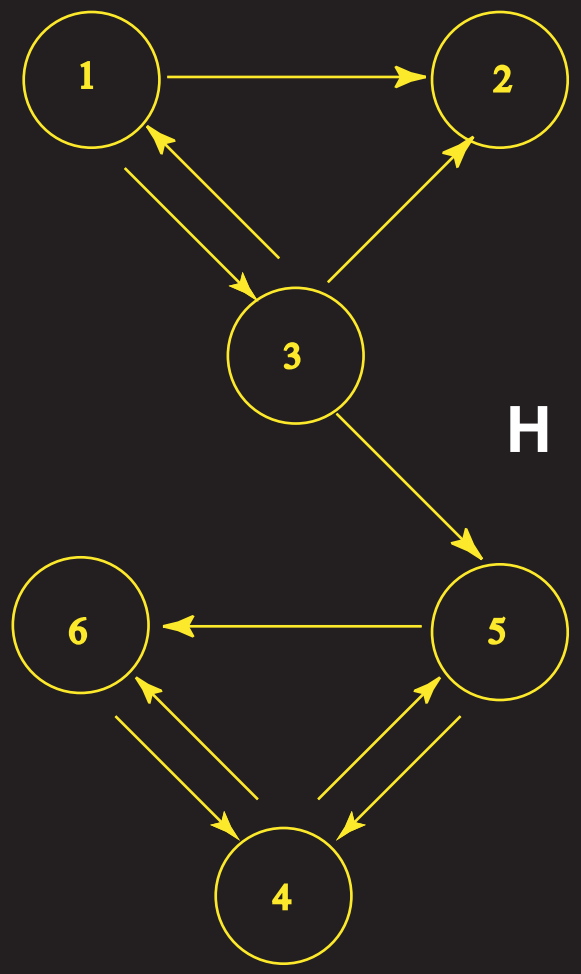
- PageRank vector = $\boldsymbol{\pi}^T = \lim_{k \to \infty} \boldsymbol{\pi}_k^T$ = eigenvector for $\mathbf{H}$

Provided that the limit exists

Tiny Web



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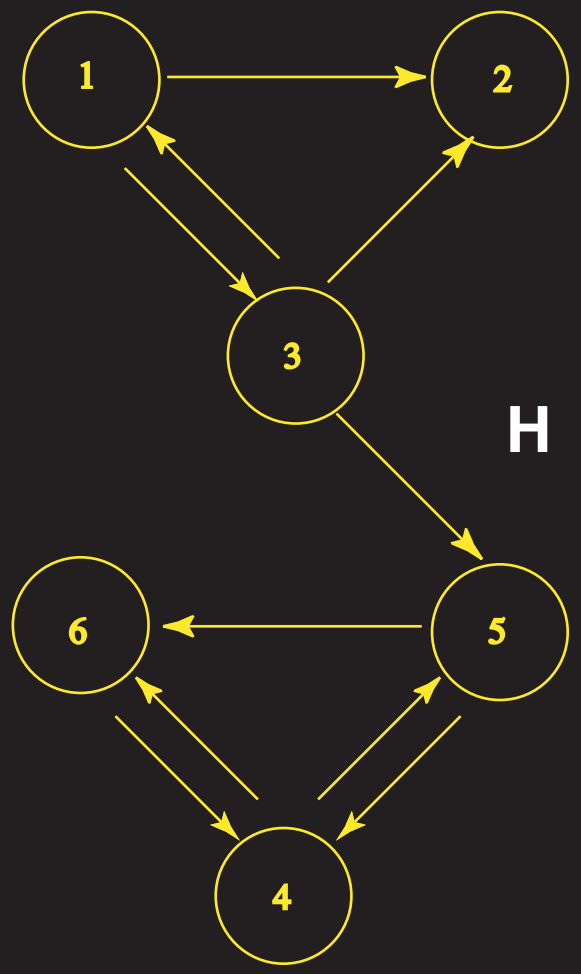

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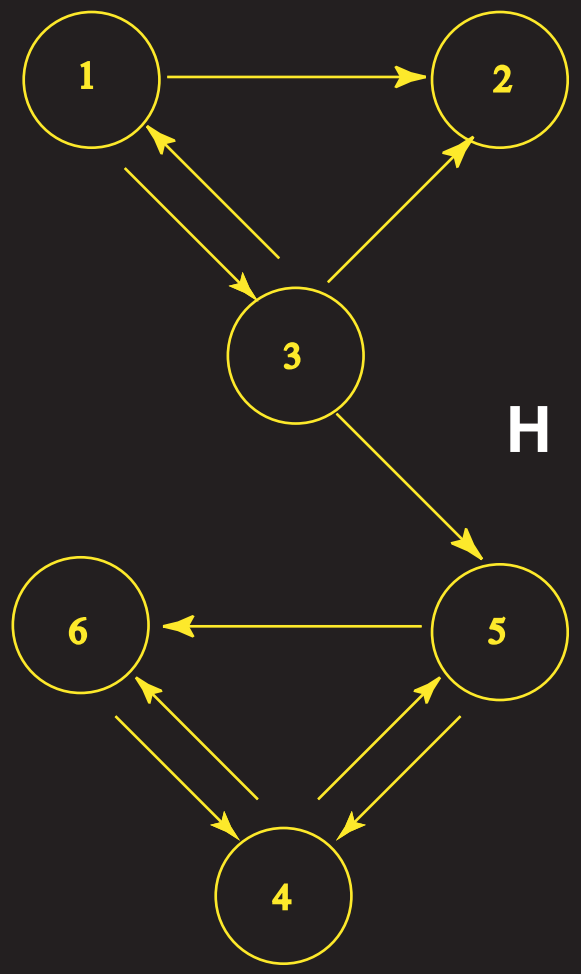

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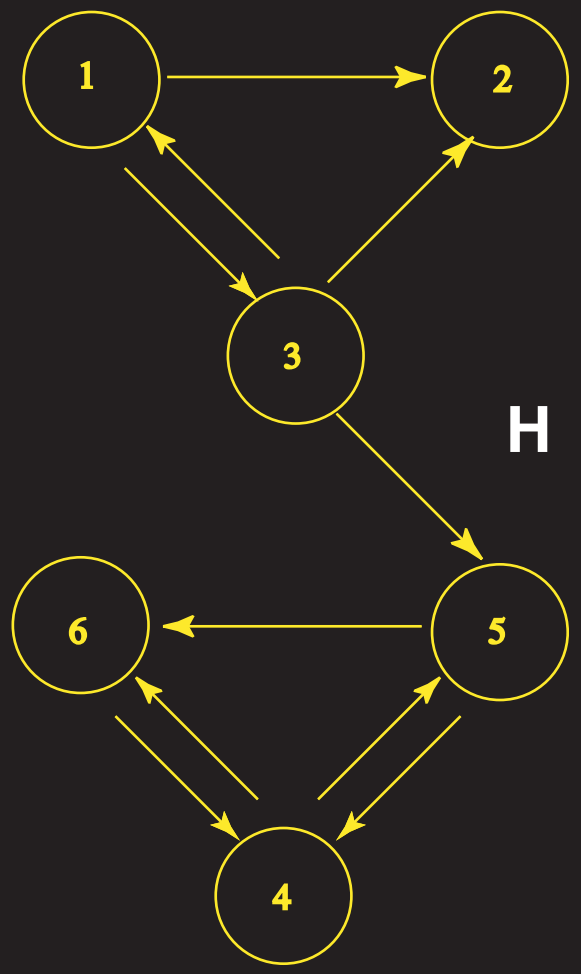

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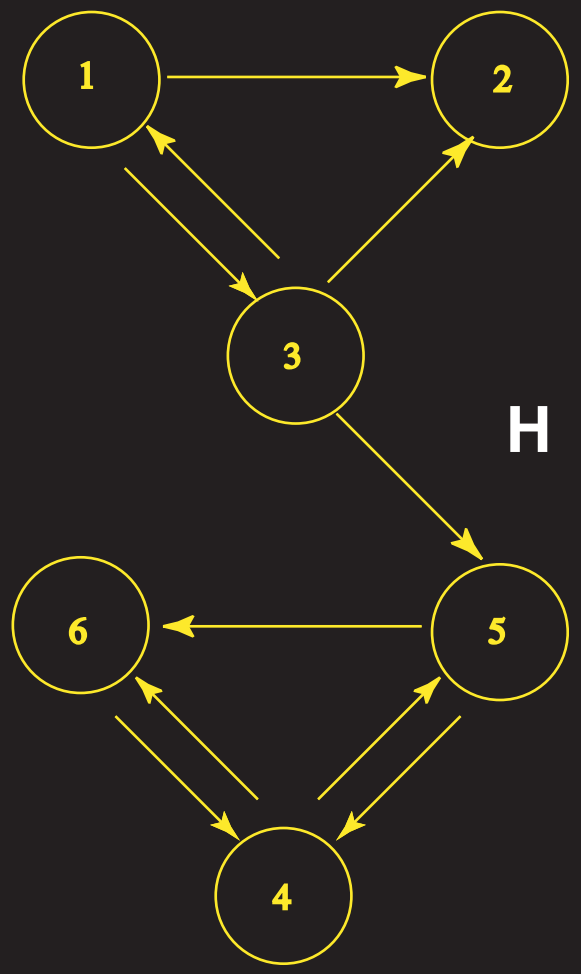

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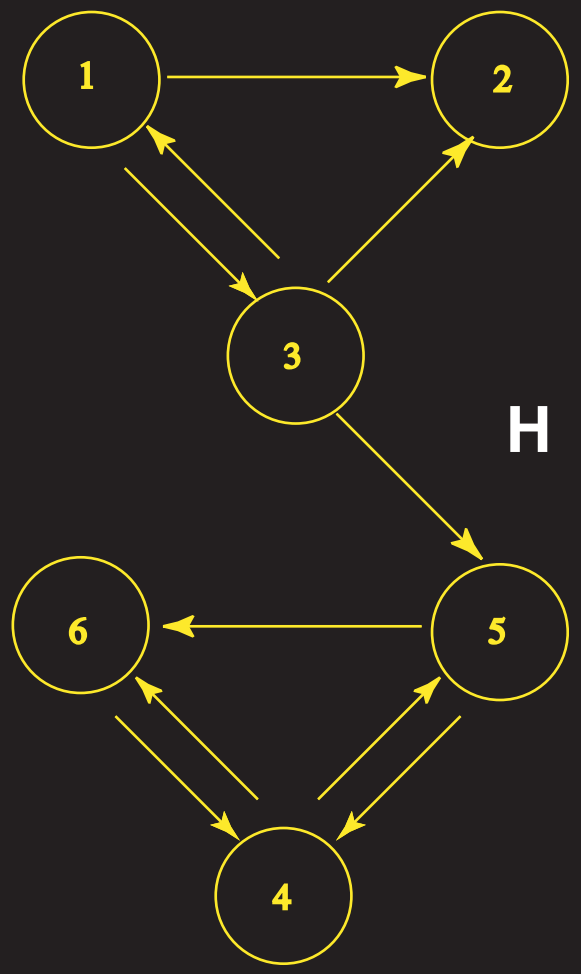

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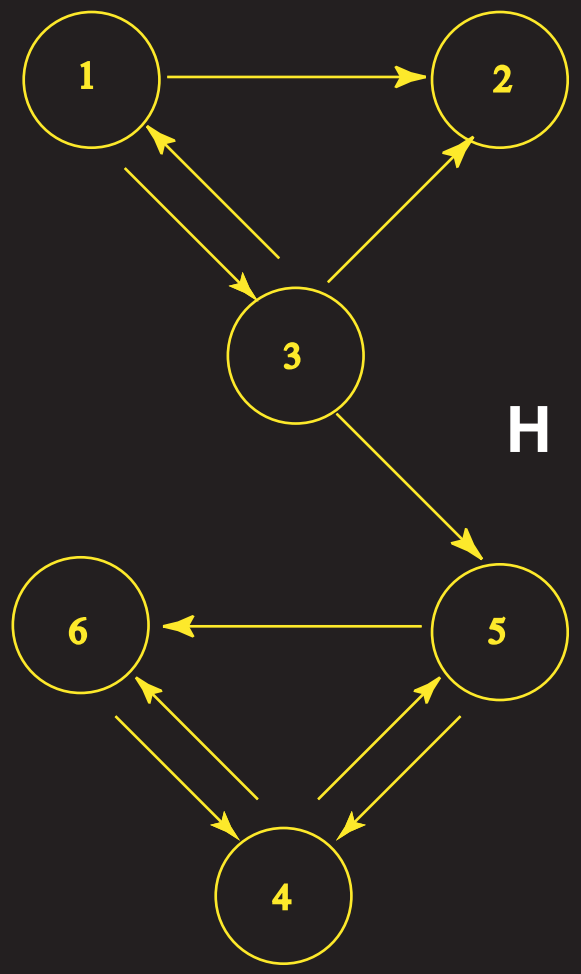

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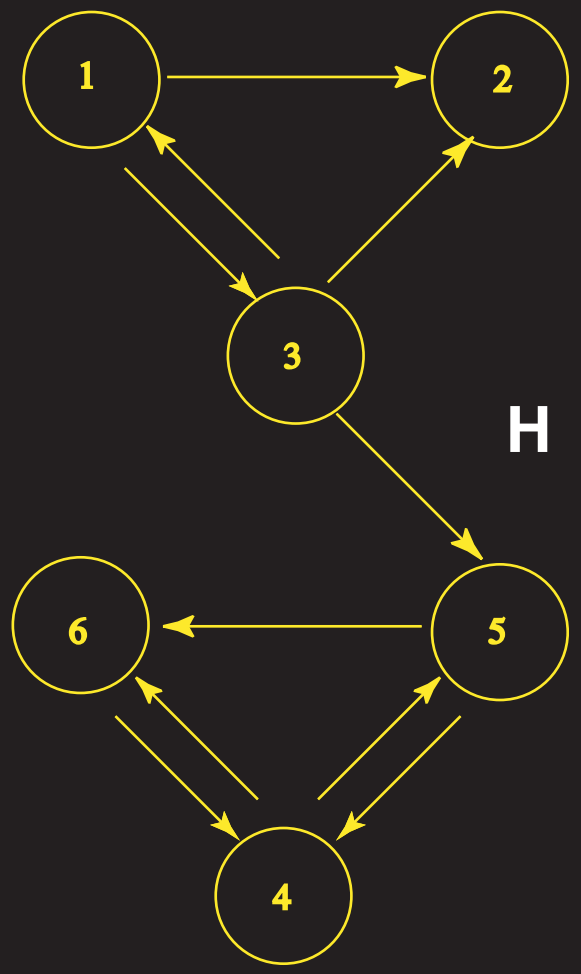

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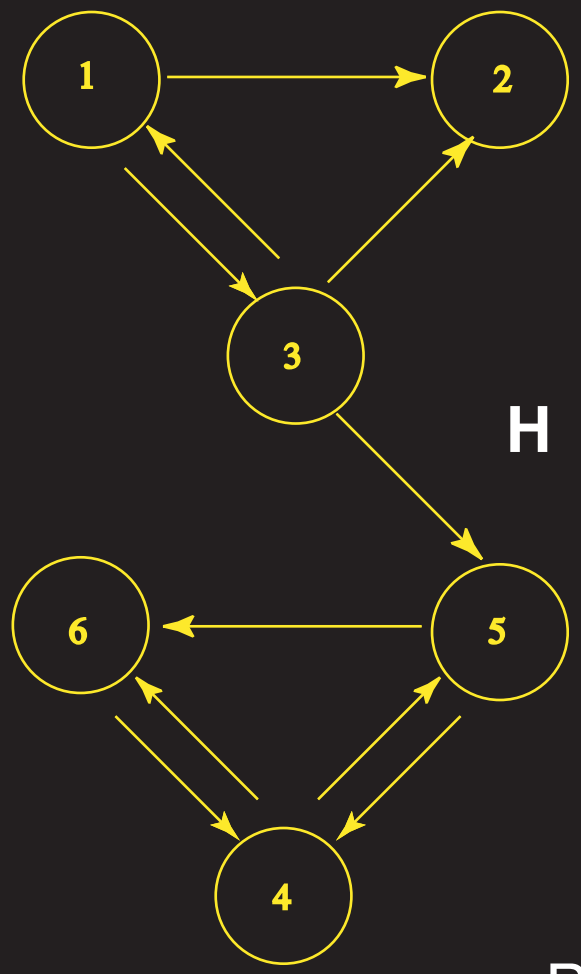
- A random walk on the Web Graph

Tiny Web



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- PageRank = π_i = amount of time spent at P_i

Ranking with a Random Surfer

- Rank each page corresponding to a search term by number and *quality* of votes cast for that page.

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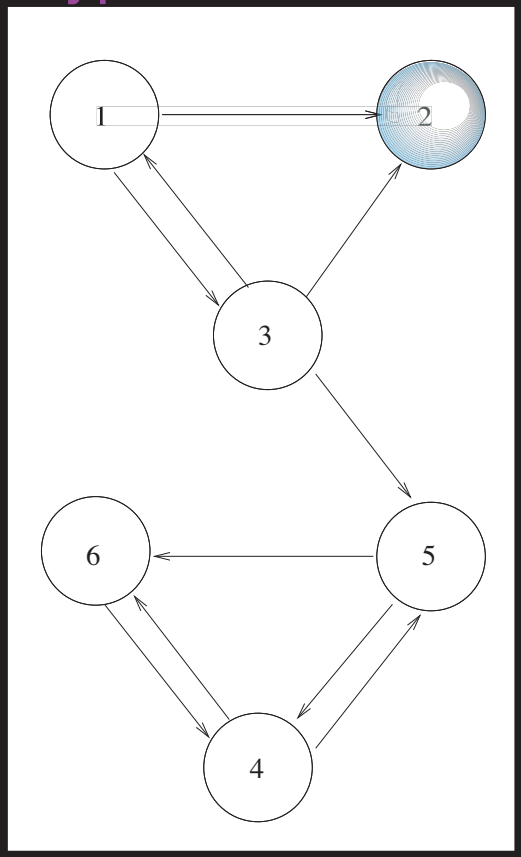

Markov chain

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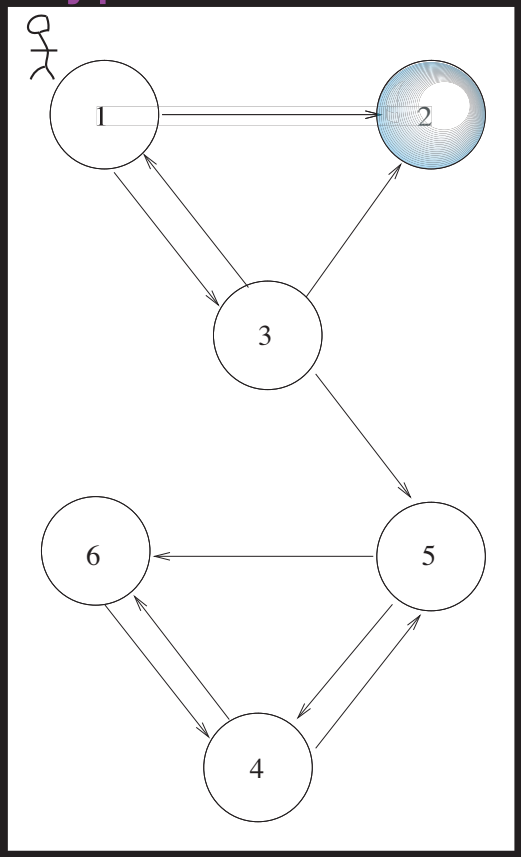

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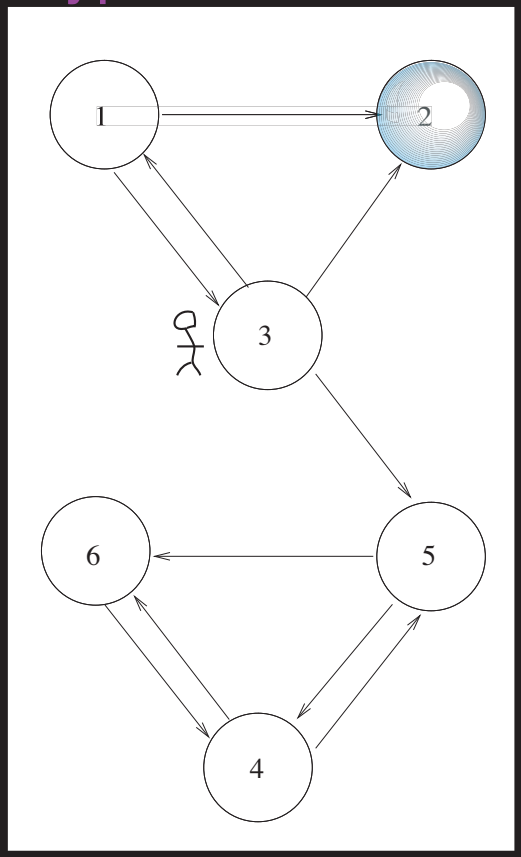

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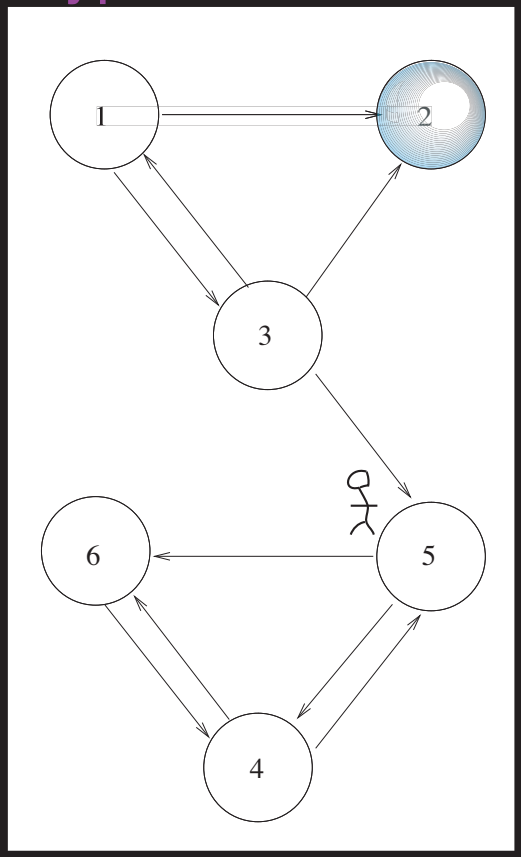

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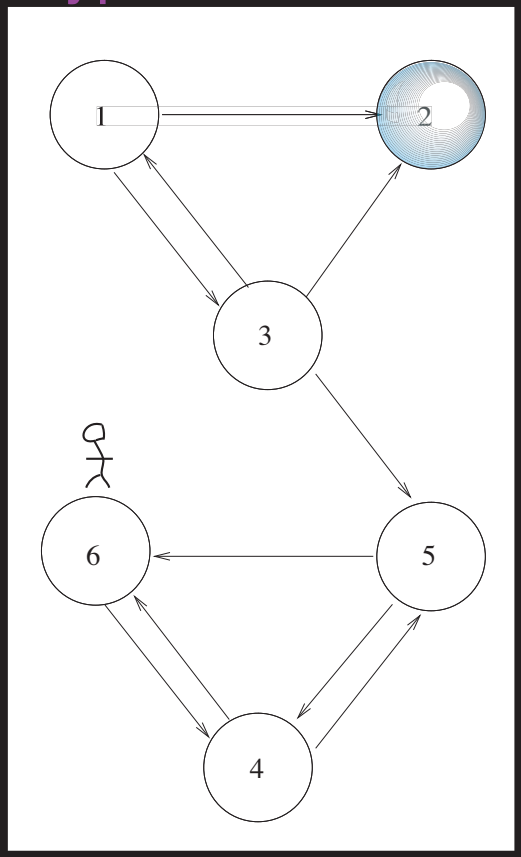

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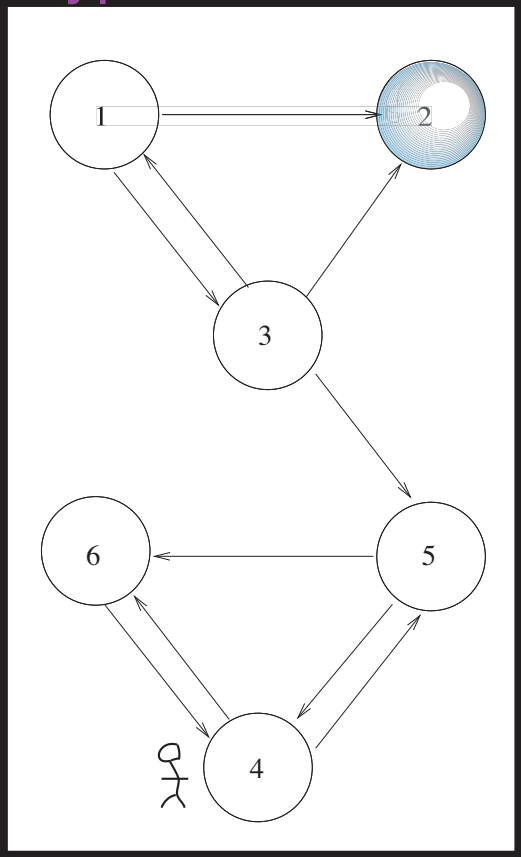

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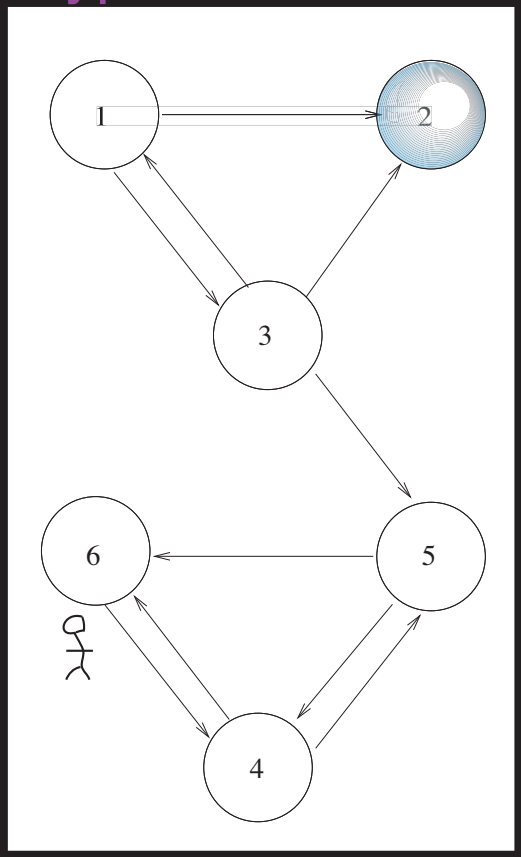

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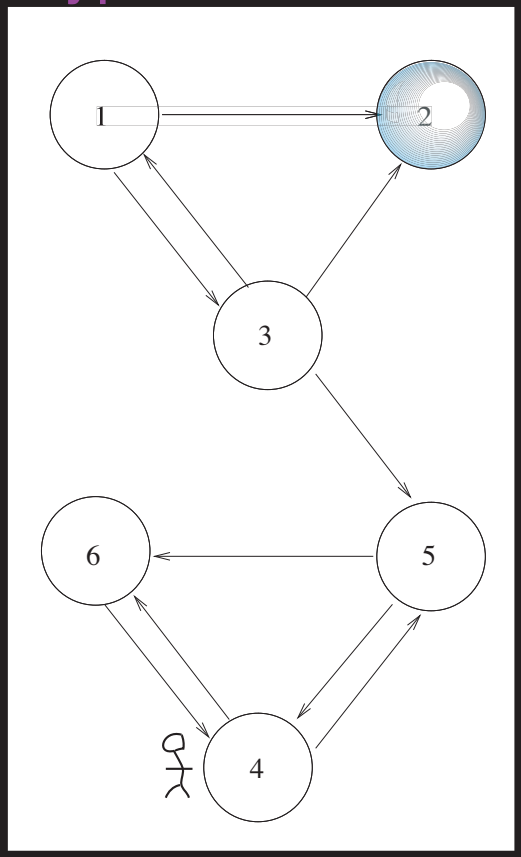

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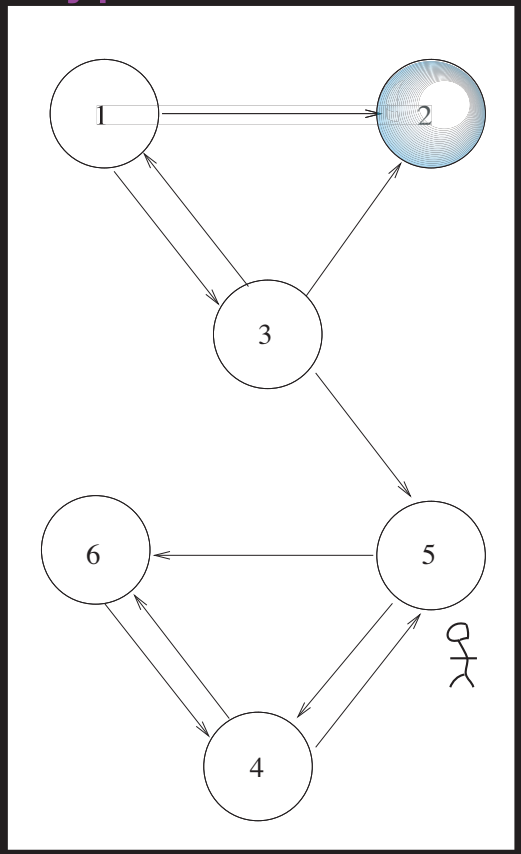

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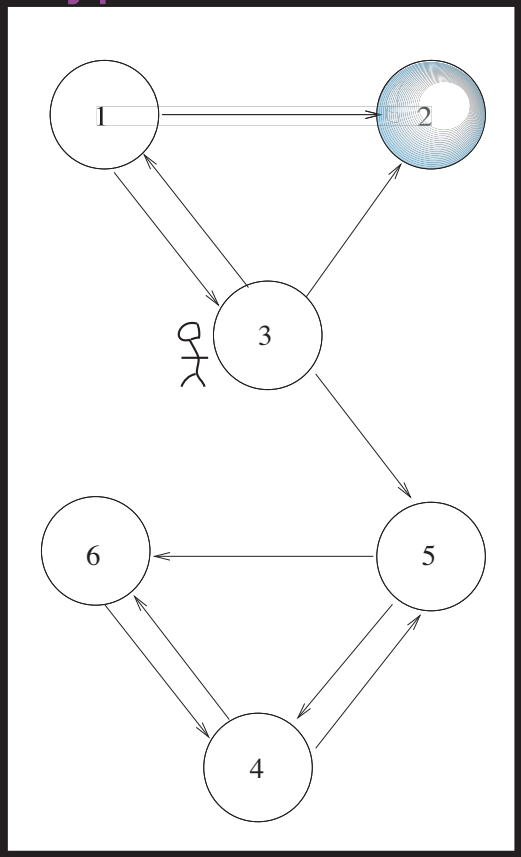

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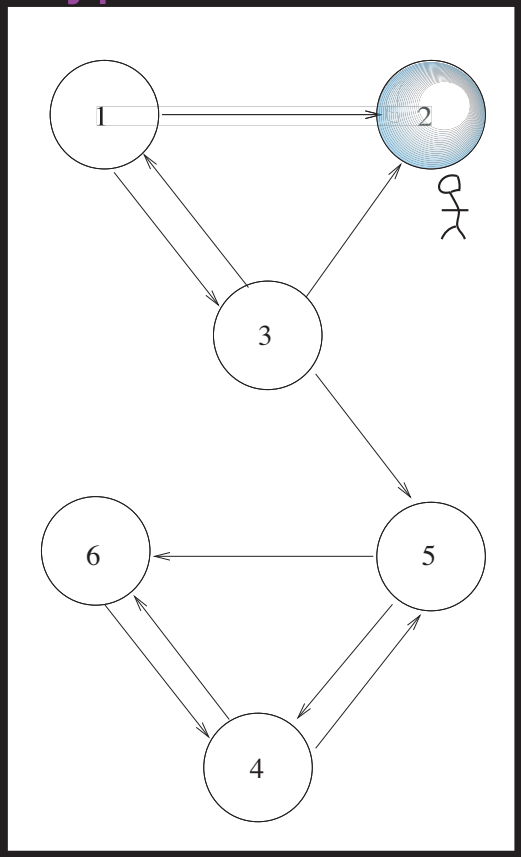
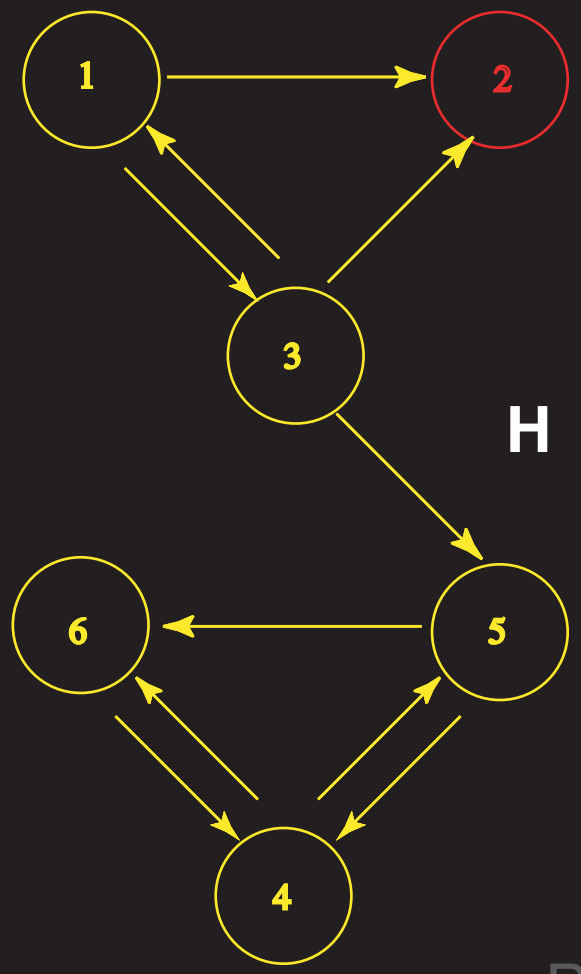


page 2 is a dangling node

Tiny Web



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- Dead end page (nothing to click on) — a “dangling node”

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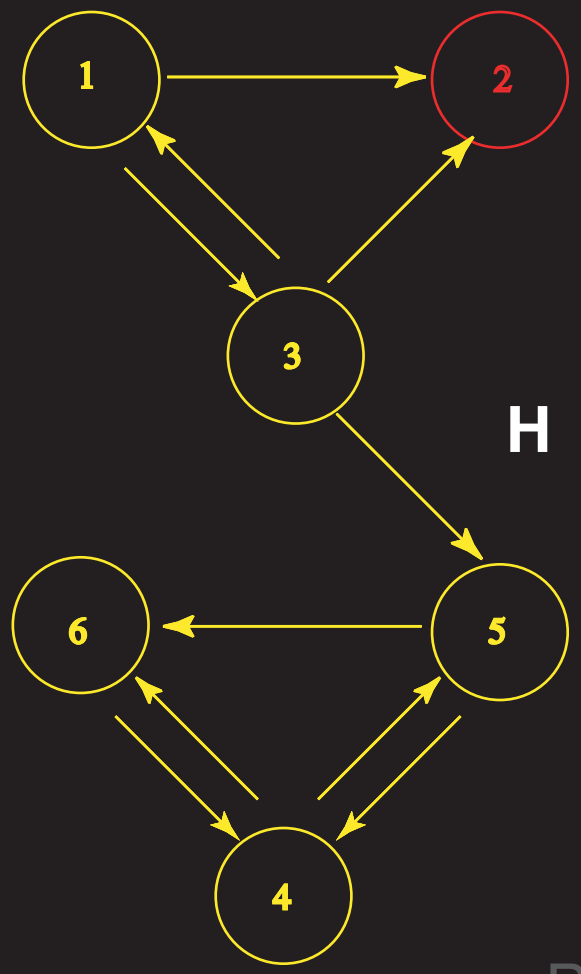

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- ▷ A random walk on the Web Graph
- ▷ PageRank = π_i = amount of time spent at P_i
- ▷ Dead end page (nothing to click on) — a "dangling node"
- ▷ $\boldsymbol{\pi}^T = (0, 1, 0, 0, 0, 0)$ = e-vector $\Longrightarrow$ Page P_2 is a "rank sink"

The Fix

Allow Web Surfers To Make Random Jumps

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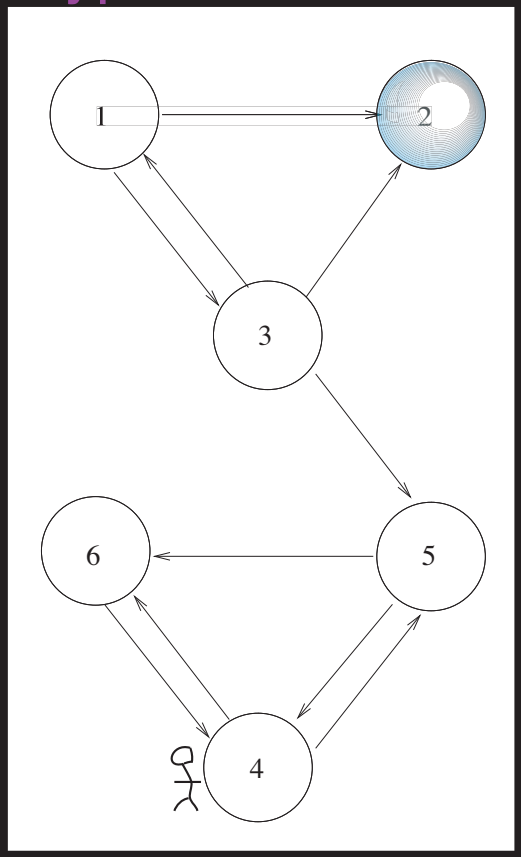

surfer “teleports”

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- $\mathbf{S} = \mathbf{H} + \frac{\mathbf{a}\,\mathbf{e}^T}{6}$ is now row stochastic $\Longrightarrow$ $\rho(\mathbf{S}) = 1$
- Perron says $\exists$ $\boldsymbol{\pi}^T \geq 0$ s.t. $\boldsymbol{\pi}^T = \boldsymbol{\pi}^T\mathbf{S}$ with $\sum_i \pi_i = 1$

Nasty Problem

The Web Is Not Strongly Connected

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- S is reducible

$$
\mathbf{S} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{array}
\begin{array}{c}
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\left(\begin{array}{ccc|ccc}
0 & 1/2 & 1/2 & 0 & 0 & 0 \\
1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\
1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\
\hline
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0 & 0 & 0 & 1 & 0 & 0
\end{array}\right)
\end{array}
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- Reducible $\Longrightarrow$ PageRank vector is not well defined
- Frobenius says $\mathbf{S}$ needs to be *irreducible* to ensure a unique $\pi^T > 0$ s.t. $\pi^T = \pi^T \mathbf{S}$ with $\sum_i \pi_i = 1$

Irreducibility Is Not Enough

Could Get Trapped Into A Cycle $(P_i \rightarrow P_j \rightarrow P_i)$

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The Google Fix

Allow A Random Jump From Any Page

- $\mathbf{G} = \alpha\mathbf{S} + (1-\alpha)\mathbf{E} > 0, \quad \mathbf{E} = \mathbf{e}\mathbf{e}^T/n, \quad 0 < \alpha < 1$

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- If a page is “important,” it gets lots of votes from other important pages, which means the random surfer visits it often.
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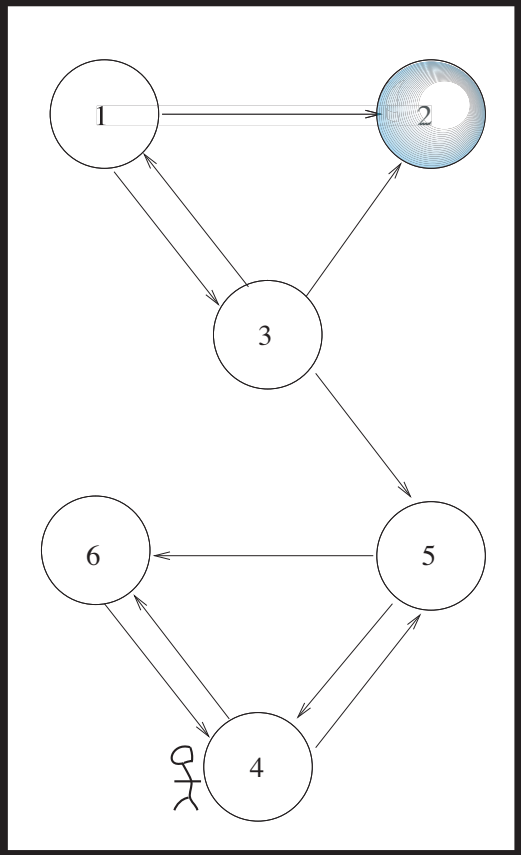
Proportion of Time

Page 1 = .04
Page 2 = .05
Page 3 = .04
Page 4 = .38
Page 5 = .20
Page 6 = .29



Ranked List of Pages

Page 4
Page 6
Page 5
Page 2
Page 1
Page 3

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- $\mathbf{x}^T\mathbf{G} = \alpha\mathbf{x}^T\mathbf{H} + \beta\mathbf{v}^T$ Sparse computations with the original link structure

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- The choice of $\mathbf{v}^T$ allows for personalization

PageRank Issues

- Computing PageRank: simulation, eigensystem, linear system; accuracy
- power law distribution: sensitivity, spamming
- link strategies
- overuse