1. We need only try the divisors of $\phi(37) = 36$, starting from the smallest.
   (a) $34 \equiv -3 \pmod{37}$, $34^2 \equiv 9 \pmod{37}$, $34^3 \equiv 10 \pmod{37}$, $34^4 \equiv 7 \pmod{37}$, $34^6 \equiv 26
   \pmod{37}$, $34^9 \equiv 1 \pmod{37}$. So $\text{ord}(34) = 9$.
   (b) $2^{12} \equiv -11 \pmod{37}$, $(2^{12})^2 \equiv 10 \pmod{37}$, $(2^{12})^3 \equiv 1 \pmod{37}$. So $\text{ord}(2^{12}) = 3$.

3. Given that $(a, m) = 1$ and $\text{ord}(a) = m - 1$. Since $\text{ord}(a) \mid \phi(m)$ by (6.2), we have $m - 1 \mid \phi(m)$ and so
   $m - 1 \leq \phi(m)$. On the other hand, we always have $\phi(m) \leq m - 1$. Hence $\phi(m) = m$ and $m$ is prime
   by 3-59.

4. Given $\text{ord}(9) = 4$ and $\text{ord}(10) = 5$. Observe that $9 \times 10 = 90 \equiv 8 \pmod{41}$. So by (6.4),
   
   $\text{ord}(8) = [\text{ord}(9), \text{ord}(10)] = [4, 5] = 4 \times 5 = 20$
   
   where $[a, b]$ denotes the least common multiple of $a$ and $b$.

5. By (6.3)(i),
   
   $\text{ord}(a^{15}) = \frac{\text{ord}(a)}{(15, \text{ord}(1))} = \frac{42}{(15, 42)} = \frac{42}{3} = 14$.

3. Observe that $2^{12} \equiv -4 \equiv -2^2 \pmod{100}$. Using this repeatedly, we get
   
   $2^{11212} = 2^{934 \times 12 + 4} = (2^{12})^{934} \times 2^4 \equiv (-2^2)^{934} \times 2^4 = 2^{1872}$
   
   $= 2^{12 \times 156} = (2^{12})^{156} \equiv (-2^2)^{156} = 2^{312} = 2^{12 \times 26} = (2^{12})^{26} \equiv (-2^2)^{26} = 2^{52}$
   
   $= 2^{12 \times 4 + 4} = (2^{12})^4 \times 2^4 \equiv (-2^2)^4 \times 2^4 = 2^{12} \equiv -2^2 = -4 \pmod{100}$.

   So
   
   $2^{11212}(2^{11213} - 1) \equiv -4(-4 \times 2 - 1) \equiv 36 \pmod{100}$.

5. Let $n$ be a perfect number such that $n \equiv 8 \pmod{10}$. By 7-9, $n = 2^{p-1}(2^p - 1)$ where $p = 4k + 3$ for
   some $k \in \mathbb{N}$. Notice that the sequences become periodic when taken modulo 100:
   
   $k = 1, 2, 3, 4, 5, 6, 7, \ldots,$
   
   $4k + 2 = 2, 6, 10, 14, 18, 22, \ldots,$
   
   $2^{4k+2} \equiv 4, 64, 24, 84, 44, 64 \ldots \pmod{100},$
   
   $2^{4k+3} - 1 \equiv 7, 27, 47, 87, 7, 27, \ldots \pmod{100},$
   
   $n = 2^{4k+2}(2^{4k+3} - 1) \equiv 28, 28, 28, 28, 28, \ldots \pmod{100}$.

11. $2^{26} + 1 = 4(2^6)^4 + 1 = (2 \times 2^{12} + 2 \times 2^6 + 1)(2 \times 2^{12} - 2 \times 2^6 + 1) = 8,321 \times 8,065 = 5 \times 53 \times 157 \times 1613.$
   
   $2^{34} + 1 = 4(2^8)^4 + 1 = (2 \times 2^{16} + 2 \times 2^8 + 1)(2 \times 2^{16} - 2 \times 2^8 + 1) = 131,585 \times 130,561 = 5 \times 137 \times 953 \times 26,317.$

13. Notice that the sequences become periodic when taken modulo 11:
   
   $n = 1, 2, 3, 4, 5, 6, \ldots,$
   
   $2^{2n} = 4, 5, 3, 9, 4, 5, \ldots \pmod{11},$
   
   $2^{2n} + 7 \equiv 0, 1, -1, 5, 0, 1, \ldots \pmod{11}.$

   Hence $2^{2n} + 7 \equiv 0 \pmod{11}$ whenever $n \equiv 1 \pmod{4}.$