

## MathFinance 345/Stat390

### Homework 2

Due October 10

In the following problems, all markets are assumed to be arbitrage-free, and to contain a riskless asset (called BOND), with riskless rate of return  $r$ .

1. **Numeraire Invariance.** Prove Proposition 1 in the notes.

2. **Put-Call Parity.** Consider a single-period market with a risky asset STOCK, a CALL option with strike  $K$ , and a PUT option with strike  $K$ . Assume that the rate of return on the riskless asset BOND is  $r = 0$ .

(a) Show that there is a replicating portfolio in the assets STOCK, CALL, and BOND for the asset PUT.

(b) Deduce a formula for the  $t = 0$  market price of PUT in terms of the  $t = 0$  market prices of STOCK and CALL.

(c) Explain how your answers to parts (a) and (b) must be modified if  $r > 0$ .

3. **Stocks with Dividends.** Consider a  $T$ -period market with a riskless asset BOND whose rate of return is  $r = 0$ . Let STOCK be a risky asset that, at each  $t = 1, 2, \dots, T$ , pays a dividend of  $\delta S_t$  shares of BOND, where  $1 > \delta > 0$  is a fixed constant and  $S_t$  is the current share price of STOCK. Let  $\pi$  be an equilibrium distribution for the market.

(a) Show that

$$S_0 = (1 + \delta)^T \sum_{\omega \in \Omega} \pi(\omega) S_T(\omega),$$

where the sum is over all possible market scenarios  $\omega$ .

(b) What is the forward price  $F_0$  of asset STOCK? (NOTE: The forward contract is an agreement made at  $t = 0$  for a BUYER to pay  $F_0$  shares of BOND at  $t = T$  in exchange for one share of STOCK.)

4. Let  $\mathcal{M}$  be a homogeneous,  $T$ -period binary market with a risky asset STOCK whose share price follows equations (26)-(27) of the notes. Assume that the market  $\mathcal{M}$  has a riskless asset BOND with rate of return  $r = 0$ . Consider a contract FLOOR that pays the BUYER one share of BOND at every time  $t = 1, 2, \dots, T$  when the share price of STOCK is below its initial value  $S_0$ . What is the arbitrage price of one FLOOR at time  $t = 0$ ?