

Math 474 Quiz 3

(Time: 2 hours Closed Textbook Due: Dec 3rd)

Name: _____ Student ID: _____

1.(20 points) The average life of a bread-making machine is 7 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find

- (a) the probability that the mean life of a random sample of 9 such machines falls between 6.4 and 7.2 years;
- (b) the values of x to the right of which 15% of the means computed from random samples of size 9 would fall.

2.(20 points) A manufacturing firm claims that the batteries used in their electronic games will last an average of 30 hours. To maintain this average, 16 batteries are tested each month. If the computed t-value falls between $-t_{0.025}$ and $t_{0.025}$, the firm is satisfied with its claim. What conclusion should the firm draw from a sample that has a mean $\bar{x} = 27.5$ hours and a standard deviation $s = 5$ hours? Assume the distribution of battery lives to be approximately normal.

3.(20 points) IF the number of hurricanes that hit a certain area of Texas per year is a random variable having a Poisson distribution with $\mu = 6$, find the probability that this area will be hit by

- (a) exactly 15 hurricanes in 2 years;
- (b) at most 9 hurricanes in 2 years.

4.(20 points) Let X be a random variable with probability distribution

$$f(x) = \begin{cases} \frac{1+x}{2}, & -1 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

Find the probability distribution of the random variable $Y = X^2$.

5.(20 points) Show that the moment generating function of the random variable X having a normal probability distribution with mean μ and variance σ^2 is given by $M_X(t) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2)$.