Problems

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1. **A simple graphical model**

   (a) Consider the following 6 random variables:

   - R = Is it raining?
   - U = Do you have an umbrella with you?
   - C = Is it cloudy outside?
   - G = Is the grass wet?
   - S = Are the sprinklers on?
   - F = Is there a football game today?

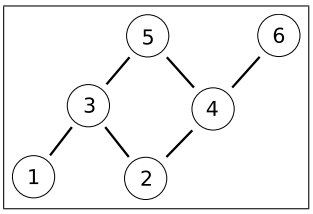
   What do you think is the graphical model that you think best explains the relationships between the events? Write down both a network structure and rough conditional probabilities for each event.

   (b) Suppose we observe that the grass is wet ($G = 1$). Compute the probabilities that

   i. You have your umbrella ($U = 1$).
   ii. It is raining ($R = 1$).
   iii. The sprinklers are on ($S = 1$).

   Which is the best explanation?

2. **Posets and distributive lattices.**
   Let P be the poset



   (a) How many chains with $0, 1, 2$, and 3 elements does P have? (A chain is a totally ordered subset.)

   (b) What is the distributive lattice of P?

3. **Conjunctive Bayesian networks**

   (a) Given the following data, construct the maximum likelihood conjunctive Bayesian network.

$$\begin{array}{ll} u_{0000} = 4 & u_{1000} = 15 \\ u_{0100} = 20 & u_{1100} = 10 \\ u_{1010} = 10 & u_{1110} = 12 \\ u_{1111} = 5 & \end{array}$$

   (b) The ML parameters θ_e are "the number of times an event occured, divided by the number of times it could have (or did) occur."
   Compute $\theta_1, \theta_2, \theta_3, \theta_4$.

(c) What happens to the model structure if, in addition, we also see the data $u_{0101} = 1$?

4. **An EM algorithm**

The poisson distribution with rate λ, written Pois(λ), has probability mass function

$$\Pr(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

for $k = 0, 1, \ldots$.

This is the probability that an event which happens with rate λ will happen k times over a fixed time.

(a) What is the expected value of a random variable X which is distributed according to Pois(λ)?

(b) Given counts u_x for $x = 0, 1, \ldots$, what is the maximum-likelihood estimate of λ?

(c) We observe data that we think is a mixture of two different Poisson distributions with parameters λ and μ.

If we could classify each data point according to which distribution it comes from, we could estimate λ and μ (how?).

We will apply an EM algorithm: the hidden data is the classification of the data according to which distribution it comes from: $u_{0,x}$ and $u_{1,x}$, with $u_{0,x} + u_{1,x} = u_x$.

Write an EM algorithm to estimate λ and μ.

E-step: Given $\hat{\lambda}$ and $\hat{\mu}$, compute $u_{0,x}$ and $u_{1,x}$.

M-step: Given $u_{0,x}$ and $u_{1,x}$, compute $\hat{\lambda}$ and $\hat{\mu}$

5. **Disjunctive Bayesian networks**

An event can happen if *any* of its parents have happened. The conditional probabilities look like

$$\begin{array}{r} \\ \mathrm{pa}(X) = (0, \ldots, 0) \\ \mathrm{pa}(X) = (0, \ldots, 1) \\ \\ \mathrm{pa}(X) = (1, \ldots, 1) \end{array} \begin{array}{c} \begin{array}{cc} X = 0 & X = 1 \end{array} \\ \begin{pmatrix} 1 & 0 \\ 1 - \theta_e & \theta_e \\ \vdots & \vdots \\ 1 - \theta_e & \theta_e \end{pmatrix} \end{array}$$

Pick a directed graph with 3-4 vertices.

- What subsets of events have positive probability under this graph structure?
- What are the maximum likelihood parameters θ_e as a function of the data u_g for this graph?

Open question

Given data, what is the maximum likelihood network structure?