THE UNIVERSITY OF CHICAGO

Department of Statistics

MASTER'S THESIS PRESENTATION

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Robust Estimation of Covariance Matrix and Precision Matrix

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ABSTRACT

Given n i.i.d. observations of a random vector $\mathbf{X} \in \mathbb{R}^p$, we consider the estimation of its covariance matrix $\boldsymbol{\Sigma}$ and its precision matrix $\boldsymbol{\Omega}$ when $\mathbf{X}$ is heavy-tailed distributed. To deal with the heavy-tailed issue, we propose a robust estimator for $\boldsymbol{\Sigma}$ using Huber loss and a modified robust Dentizig type estimator for $\boldsymbol{\Omega}$. We show that, with proper choice of truncating parameter in Huber loss, the robust estimator is consistent under the infinite norm, and dimension p is allowed to grow exponentially with the sample size n. We derive the explicit exponential-type tail bound for it in both cases with existence of only the fourth moment and only the second moment. When the true precision matrix is sparse, we derive the explicit tail bound for the modified robust Dentizig type estimators under the spectral norm. We also establish the rates of convergence under different norms for the estimator. Numerical performance of these two estimators is investigated by simulation studies. We compare our robust estimator for $\boldsymbol{\Sigma}$ with sample covariance matrix and the modified robust Dentizig type estimator for $\boldsymbol{\Omega}$ with existing regularized estimator (**CLIME**). The procedure of robust estimation is easy to implement and we propose a data driven method to choose truncating parameter in Huber loss. Simulation results show that the robust estimators have comparable performance with existing estimators when distribution of $\mathbf{X}$ is Gaussian. When $\mathbf{X}$ is heavy-tail distributed, for example t_3 distribution, the robust estimators have better performance and the advantage become more significant as p increases.
