We consider an $L^2$ norm based test for mean vectors of high-dimensional data. An invariance principle for the $L^2$ norm can be derived under Lyapunov-type conditions that involve a delicate interplay between the dimension $p$, the sample size $n$ and the moment assumption. Under proper normalization, central and non-central limit theorems are obtained. To obtain cutoff values of our tests, we consider a plug-in Gaussian multiplier calibration method. We also propose a subsampling procedure to approximate the distributions of the $L^2$ norms that bypasses the estimation of the covariance matrix.

Future work concerns the subsample size selection for the subsampling procedure. We propose to take the subsample size to be $n/2$, which is called a Half-sampling procedure. Simulation results show that half-sampling is capable in estimating the sampling distribution.