The first model deals with the percolation on $\mathbb{Z}^d \times [N]$, where edges are between all pairs $(x,i)$ and $(y,j)$ such that $||x-y||_1 = 1$. For each edge, we retain it with probability $p_N = 1/(2dN)$ and remove it otherwise. In the remaining graph, we are interested in how the largest connected clusters behave when $N$ is large and the scaling limits of their joint distribution. I will show this problem is closely related to a spatial SIR epidemic model, whose scaling limit is a measure-valued process ("superprocess). One result of this type was obtained by Aldous (1997) for the critical Erdős-Rényi random graphs, where the scaling limits of the largest connected components are given by the excursion lengths of a Brownian motion with time inhomogeneous drift.

The second model, called the first passage percolation (FPP), was originally introduced to describe the fluid flow through a random medium. On $\mathbb{Z}^d$, each edge $e$ is assigned an i.i.d. nonnegative random variable $\tau_e$, which represents the time needed to traverse the edge $e$. The passage time between two lattice points $x$ and $y$, $T(x,y)$, is the shortest time needed to go from $x$ to $y$. The limit $\lim_{n \to \infty} T(0,nx)/n = \mu(x)$ exists due to sub-additivity, and it is called the time constant in the $x$ direction. However, little is known about $T(0,nx)$ or the time constant $\mu(x)$. I will discuss about our recent result on the time constant in high dimensional FPP and its implications on the limit shape. A few open problems will be introduced at the end.