Accessibility percolation is a problem motivated by evolutionary biology that has recently received much attention in probability. Let $H_N = \{0,1\}^N$ be an $N$ dimensional binary hypercube where each vertex represents a genome consisting of $N$ alleles. A directed path on $H_N$ therefore represents successive genetic mutations where each mutation is a single allele substitution. Assign to each vertex on $H_N$ a real number (called the fitness value) by some probability distribution. We are interested in the question that whether one vertex (say $v$) can be connected from another (say $u$) by a directed path along which the fitness values only increase, i.e., whether a “selectively accessible” evolutionary path exists. We show that if the underlying distribution of the fitness landscape is simply i.i.d. uniform $[0,1]$ and the Hamming distance between $u$ and $v$ is $[\beta N]$ for fixed $\beta \in (0,1]$, then there is a sharp phase transition of the accessible probability from $u$ to $v$, depending on the difference of their fitness values.

The $NK$ fitness model is a more general model of the fitness landscape on $H_N$, defined with another tunable parameter $K$ ($K=N$ corresponds to the i.i.d. fitness case discussed above). This model has also received much attention in probability, and will be the primary subject of our future research. Some possible directions are (1) to study the accessibility percolation problem in the $NK$ fitness landscape (2) to derive the asymptotics of the global maximum of the $NK$ fitness landscape as $K \to \infty$ and (3) to consider an adaptive walk (a more general evolutionary process than that considered in accessibility percolation) in the $NK$ fitness landscape.