ABSTRACT

We define a new family of cone programming problems that interpolates linear programming, second-order cone programming, and semi-definite programming. We show that there is a natural family of $k$th order cones that may be realized either as cones of $n \times n$ symmetric matrices or as cones of $n$-variate even degree polynomials. The cases $k = 1, 2, n$, correspond to the nonnegative orthant, the Lorentz cone, and the semidefinite cone respectively. Linear optimization over these cones then correspond to LP, SOCP, SDP; alternatively, in the language of polynomial optimization, they correspond to DSOS, SDSOS, SOS programming. For general values of $k$ between 3 and $n - 1$, we obtain new cone programming problems that we call $k$OCP’s (which are unrelated to the occasional generalization of SOCP with $p$-norm in place of 2-norm). We will discuss the properties of $k$OCP and see how the assembly of LP, SOCP, and SDP in such a hierarchy shed light on their relative computational efficiencies.