ABSTRACT

Linear structural equation models (SEMs) are widely used in economics and the social sciences when the variables of interest are related through noisy, linear functional relationships. The increasing popularity of these models in the journal literature can be largely attributed to their natural causal interpretability. Despite a large and growing body of work on the SEM paradigm, many theoretical properties of these models remain unknown. We review some of the features of these models and present progress in addressing several important gaps in our understanding of linear SEMs.

Mixed graphs, containing sets of both directed and bidirected edges, are commonly used to represent linear SEMs. We examine their causal interpretability; a mixed graph is said to have a strict Gaussian causal interpretation if there exists a directed acyclic graph on a superset of the variables such that the corresponding Gaussian models are equal. We restrict attention to the sub-class of chain graphs and present a graphical characterization of precisely which chain graphs lend themselves to this type of interpretability.

Second, we look at exploiting the structure of mixed graphs for the purpose of simplifying likelihood inference. In particular, we introduce a simple graphical criterion for determining the exact minimum number of observations \( n \) required for the likelihood function to be bounded over a given acyclic mixed graph model on \( p \) nodes. This criterion makes use of a graph decomposition technique from Tian (2005) and in many cases, can offer a considerable reduction over the classical \( n \geq p \) requirement necessary if no graphical structure were known. We also present a detailed discussion of the directed cyclic graph on three nodes as an example of the difficulty in expanding this characterization to cyclic models.

Finally, we discuss the issue of fitting linear structural equation models, particularly for graphs with cycles. We extend an iterative, likelihood-based algorithm from Drton, Eichler, and Richardson (2009) to handle cyclic models, and show that the generalized algorithm enjoys some of the nice properties of its predecessor.