The behavior of a variogram at the origin is important as it determines the smoothness of a random process. The power-law variogram at the origin is capable of capturing the local behaviors of many known parametric covariance functions and self-similar processes. In addition, the fractal dimension, a scale invariant measure of surface roughness, is a function of the exponent in the power-law model. The estimation of the fractal dimension using the empirical variogram has been well studied for a stationary Gaussian process, observed at evenly spaced sampling locations. This paper extends the past work to observations at unevenly spaced sites and attempts to fully specify the variogram around the origin. We propose and compare two approaches: maximum local restricted likelihood and weighted least squares. The local restricted likelihood is the composite conditional likelihood which approximates the full restricted likelihood by conditioning on observations at neighboring sites only. The weighted least squares method minimizes the difference between squared increments and their expected values. Finite sample performances of two estimators are compared numerically through mean square errors and Godambe’s information measure. Both approaches are computationally inexpensive, especially for large data collected at irregular observation locations, while the maximum local likelihood estimates outperform the weighted least squares estimates in all cases, sometimes by a large margin. The asymptotic properties of the least squares estimator are relatively easier to derive than the local maximum likelihood estimators. Under fixed domain asymptotics, the least squares estimator is consistent and achieves the same rates of convergence to either a Gaussian or a Rosenblatt distributions as for observations located on a grid.

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