In the first part of this work, we investigate properties of an existing class of models for Gaussian processes on the sphere that are invariant to shifts in longitude. The class is obtained by applying first-order differential operators to an isotropic process and potentially adding an independent isotropic term. For a particular choice of the operators, we derive explicit forms for the spherical harmonic representation of these processes’ covariance functions. Because the spherical harmonic representation is a spectral one, these forms allow us to draw conclusions about the local properties of the processes. For one, the coefficients in the spherical harmonic representation relate to the equivalence and orthogonality of the measures induced by the models. It turns out that under certain conditions the models will lack consistent parameter estimability even when the process is observed everywhere on the sphere. We also consider the ability of the models to capture isotropic tendencies on the local level, a phenomenon observed in some data.

The second part of this work relates to the fitting of a spherical harmonic basis model to 60 days of MODIS Aerosol Optical Depth data. We introduce the data and present a model that takes into account local variability, arguing that such a model allows for the assumption of independence across days and invariance in longitude. We develop techniques based around two approximations to the log-likelihood that take advantage of the ability to parallelize computations to allow us to fit models with a large number of basis functions. The relevant approximations themselves depend on properties of the locations of data observations, namely having close to uniform and global longitudinal coverage and similar areas of observation from day to day. We detail some of the fits along with some diagnostic results, but conclude that for data as locally variable as this AOD data, the spherical harmonic basis models we are able to fit are too smooth. The resulting inability to capture the local behavior provides for poor fits at both the local and global level and difficulty in spatial interpolation.