Coupling the Monopoly Sampler: An Illustration of Convergence Assessment Techniques for Markov Chain Monte Carlo

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ABSTRACT

While MCMC is one of the most widely used tools in advanced applied statistics, there is a consensus in the literature that the existing battery of convergence diagnostics is large but inadequate, especially for complicated chains. Methods based on exact results are often unwieldy in applications or badly overconservative; certain existing ad-hoc techniques are not sufficiently rigorous. Still other methods are well-tested but restricted to small classes of MCMC algorithms.

As the quintessential American board game, Monopoly has attracted the attention of several academics, including mathematicians. Ash and Bishop (1972) and Abbott and Richey (1997) formally (albeit briefly) modeled the movements in a typical game as the transitions of an ergodic Markov chain on a finite state space. None of these authors built a complete transition matrix for the chain, nor did they discuss its convergence properties.

I will use the Monopoly sampler as an illustrative tool in the application of several MCMC convergence methods. In particular, I will explain why two coupling schemes due to Johnson (1996; 1998) fail in this instance and may likewise for more general chains. This will lead to a foray into the notion of maximal coupling. The exact mixing speed and convergence rate to stationarity for Monopoly (both derived from the transition matrix itself) will also be discussed. Finally, if time permits, I will demonstrate an ad-hoc simulation method due to Giakoumatos et al (1999) based on taking subsamples of the iterates and employing a graphical diagnostic.