THE UNIVERSITY OF CHICAGO

Departments of Computer Science, Mathematics, Statistics, and the Computation Institute

SCIENTIFIC AND STATISTICAL COMPUTING SEMINAR

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Approximation Rates for the Hierarchical Tensor Format in Periodic Sobolev Spaces

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Eckhart 133, 5734 S. University Avenue

ABSTRACT

The best approximation of a bivariate L_2 function on a product domain by a sum of products of univariate L_2 function is, as in the finite-dimensional case, given by the truncated singular value decomposition (Schmidt expansion) of the associated integral operator. The rate of convergence is therefore determined by the decay of the singular numbers, and the study of this decay in dependence of the regularity of the kernel has a long history. Quite exhaustive results for bilinear approximation have been obtained by Temlyakov in the 80s of the last century.

The decay rate with respect to the number of terms (the rank) is of course better for the best bilinear approximation than for any linear tensor structured approximation method. However, for spaces of bounded mixed derivatives Griebel and Harbrecht recently showed that the asymptotic storage requirement (degrees of freedom) to achieve a precision ϵ can be worse than with the linear sparse grid approach due to the need of storing the initially unknown parameters of the decomposition. (This does not come as a surprise since these spaces are tailored to sparse grids.)

Using the hierarchical Tucker format proposed by Kühn and Hackbusch these considerations can be repeated in the d-variate case thanks to the quasi-optimality of the high-order SVD projection which is based on successive bilinear approximation. We obtain the approximation rates in terms of the hierarchical rank and the asymptotic storage complexity for periodic functions from isotropic and mixed Sobolev classes. In view of the black-box character of the hierarchical tensor product approximation, which is not natively related to regularity, the negative results in comparison to linear approximation methods in these particular spaces should not be interpreted to pessimistically.

This is joint work with Reinhold Schneider from TU Berlin, a preprint with the same title is available.

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