Statistical mechanics models are ubiquitous at the interface of probability theory, information theory, and inference problems in high dimensions. In this talk we will focus on sparse graphs, and polymers on lattices; two canonical models in natural sciences. The study of large deviations is intimately related to the understanding of such models. We will consider the rare events that a sparse random network has an atypical number of certain local structures and that a polymer in random media has atypical weight. Conditioning on such events can produce different, ranging from local to more global, geometric effects. We will discuss some such results obtained, relying on a variety of entropy theoretic, combinatorial, and analytic tools.