ABSTRACT

Because of their tractability and their natural interpretations in term of market quantities, Hawkes processes are nowadays widely used in high frequency finance. However, in practice, the statistical estimation results seem to show that very often, only nearly unstable Hawkes processes are able to fit the data properly. By nearly unstable, we mean that the $L_1$ norm of their kernel is close to unity. We study in this work such processes for which the stability condition is almost violated. Our main result states that after suitable rescaling, they asymptotically behave like integrated Cox Ingersoll Ross models. Thus, modeling financial order flows as nearly unstable Hawkes processes may be a good way to reproduce both their high and low frequency stylized facts. We then extend this result to the Hawkes based price model introduced by Bacry et al. We show that under a similar criticality condition, this process converges to a Heston model. Again, we recover well known stylized facts of prices, both at the microstructure level and at the macroscopic scale.

Joint work with Thibault Jaisson (Ecole Polytechnique Paris).