Flexible Covariance Estimation in Gaussian Graphical Models

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Refreshments following the seminar in Eckhart 110.

ABSTRACT

Covariance estimation is known to be a challenging problem, especially for high-dimensional data. In this context, graphical models can act as a tool for regularization and have proven to be excellent tools for the analysis of high dimensional data. Graphical models are statistical models where dependencies between variables are represented by means of a graph. Both frequentist and Bayesian inferential procedures for graphical models have recently received much attention in the statistics literature. The hyper-inverse Wishart distribution is a commonly used prior for Bayesian inference on covariance matrices in Gaussian Graphical models. This prior has the distinct advantage that it is a conjugate prior for this model but it suffers from lack of flexibility in high dimensional problems due to its single shape parameter. In this talk, for posterior inference on covariance matrices in decomposable Gaussian graphical models, we use a flexible class of conjugate prior distributions defined on the cone of positive-definite matrices with fixed zeros according to a graph $G$. This class includes the hyper inverse Wishart distribution and allows for up to $k + 1$ shape parameters where $k$ denotes the number of cliques in the graph. We first add to this class of priors, a reference prior, which can be viewed as an improper member of this class. We then derive the general form of the Bayes estimators under traditional loss functions adapted to graphical models and exploit the conjugacy relationship in these models to express these estimators in closed form. The closed form solutions allow us to avoid heavy computational costs that are usually incurred in these high-dimensional problems. We also investigate decision-theoretic properties of the standard frequentist estimator, which is the maximum likelihood estimator, in these problems. Furthermore, we illustrate the performance of our estimators by exploring frequentist risk properties and the efficacy of graphs in the estimation of high-dimensional covariance structures. We demonstrate that our estimators yield substantial risk reductions over the maximum likelihood estimator in the graphical model.