ABSTRACT

In this talk I will consider nonparametric estimation of an unknown density function \( g \) under shape constraints from a mixture model perspective. Let \( k \) be a non-negative integer and let \( G \) be a distribution function on \((0, \infty)\). Then

\[
f(x) = \int_0^\infty \frac{k}{y^k} (y-x)^{k-1} 1_{[0,y]}(x) dG(y)
\]

is monotone (decreasing) when \( k = 1 \), \( g \) is convex and decreasing when \( k = 2 \), and higher values of \( k \) correspond to densities which are \( k \) times differentiable with derivatives of alternating sign. I will discuss what is known concerning estimation of \( f \) and the mixing distribution \( G \) when \( k = 1 \) and \( k = 2 \), and then discuss current work connected with the cases \( 3 \leq k < \infty \). Splines and a particular Hermite interpolation problem begin to play a role.