ABSTRACT

Precise limit theorems are developed for Markov chains and Markov processes on general state spaces, under natural and often minimal assumptions. Assuming only that the chain is geometrically ergodic, we obtain Edgeworth expansions, a large deviations principle, and exact large deviations asymptotics for the partial sums of a real-valued functional of the chain. In particular, we give the first complete description of these results for Doeblin chains, as well as for important models in applications like reflected Brownian motion. In some special cases it also possible to obtain non-asymptotic bounds with computable constants.

The gist of the present approach is the development of a multiplicative ergodic theory of Markov chains, in close analogy to the standard additive theory. The main technical step is the study of an eigenvalue problem, the "multiplicative Poisson equation." This is an exponential version of the classical Poisson equation in Markov process theory. These results are motivated, in part, by potential applications in stochastic networks, linear systems, and simulation.