Put a random – i.e. uniformly distributed – point \( X_1 \) in the unit interval \((0,1)\). Choose the longest of the resulting two subintervals \((0, X_1)\) and \((X_1,1)\) and put a random point \( X_2 \) in this interval. Continue in this way, choosing \( X_k \) randomly in the longest of the \( k \) intervals into which \( X_1, X_2, \ldots, X_{k-1} \) subdivide \((0,1)\). Kakutani asked whether in the long run the points become evenly – i.e. uniformly - distributed in \((0,1)\). There are obvious reasons why this should be true, but the proof turned out to be a different matter altogether.

Once we have a proof, we can resolve some related matters. For instance, one can ask how the speed of convergence compares with well-studied classical case where the random variables \( X_1, X_2, \ldots \) are independent and uniformly distributed on \((0,1)\). While answering this question we come across some interesting and unexpected phenomena. All of this is based on joint work with Ronald Pyke.