The Shub-Smale tau-Conjecture is a hitherto unproven statement (on integer roots of polynomials in one variable) whose truth would resolve two variants of the $P$ vs. $NP$ Problem. We give a simpler statement, potentially easier to prove, whose truth implies the hardness of the permanent. Along the way, we discuss new upper bounds on the number of $p$-adic valuations of roots of certain sparse polynomial systems, culminating in a purely tropical geometric statement that implies the hardness of the permanent.

Our framework also yields new complexity lower bounds for the permanent, even if only weaker versions of our conjectures are proved. Some of the results presented are joint work with Pascal Koiran and Natacha Portier. We assume no background in complexity theory.