STATISTICS 244 **Problem Set 4** **9:30-10:50 TTh**

Due Tuesday February 5

1. Evaluating Normal distribution probabilities. Let Z have a standard normal distribution (N(0,1)) and let X be $N(\mu, \sigma^2)$. Let $\Phi(z)$ be the cdf of Z. Then probabilities $P(a < X < b) = P(X< b) - P(X < a)$ can be found for any a and b from any table of $\Phi(z)$ for $z > 0$ using $\Phi(z) = 1 - \Phi(-z)$ and the fact that $P(X < x) = \Phi((x-\mu)/\sigma)$. One such table is given in Rice, page A7. There are also tables on the web; here are some examples:
http://math2.org/math/stat/distributions/z-dist.htm
http://itl.nist.gov/div898/handbook/eda/section3/eda3671.htm
Suppose that X has a N(–2, 9) distribution; find (a) $P(X > 2)$, (b) $P(0 < X < 2)$, (c) $P(|X + 3| \geq 1.5)$, (d) $P(X \leq -1 \text{ or } X \geq 1)$

2. A "psychic" uses a five-card deck of cards to demonstrate psychic ability (ESP), and claims to be able to guess a card correctly with probability .5 (ordinary guessing would be right with probability 1/5 = .2). A single experiment consists of making five guesses, reshuffling the deck after each guess. The experiment is tried and the "psychic" guesses correctly 3 times out of five. Assuming the only two possibilities are "ESP" and "ordinary guessing", how high must the a priori probability be that the "psychic" really has ESP, in order that the a posteriori probability that the "psychic" has ESP is at least .7?

3. Suppose that a Bayesian statistician has a Beta (2,1) prior distribution on the cure rate θ (= Probability of cure) for an experimental drug. The drug is tried (independently) on three subjects, and X are cured. Compute $P(\theta \leq .2|X=k)$ and $E(\theta|X=k)$ for k = 0, 1, 2, 3.

4. Laplace's rule of succession. What is the a posteriori expectation of the probability that the sun will rise tomorrow given that it has risen n days in a row and that before those n days began we had an a priori uniform distribution for the probability the sun would rise? [This is a classical problem.]

5. Suppose $X_1, X_2, X_3, ..., X_n$ are independent random variables, each with a standard normal distribution. We define the Chi-square distribution with n degrees of freedom as follows: it is the distribution of $Z_n = X_1^2 + X_2^2 + X_3^2 + ...+ X_n^2$. (For n = 1 this agrees with our earlier definition.) Show that $E(Z_n) = n$ and $Var(Z_n) = 2n$ by first verifying these for n = 1 by direct calculation, and then using the formulas for the expectation and variance of a sum of independent random variables. [The only hard part is to show this for n=1; that is, that if X has a standard normal density, $E(X^2)=1$ and $Var(X^2)=2$. You may use the density of X^2 we found earlier (see Chapter 1) and a table of integrals.]