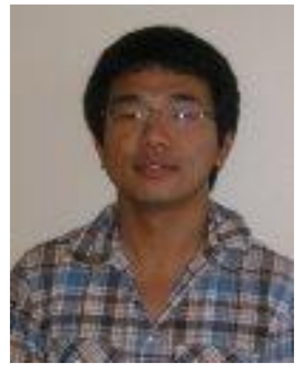
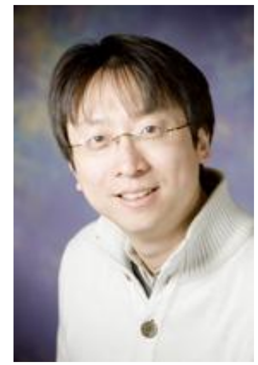
Robust Principal Component Analysis?

John Wright
Microsoft Research

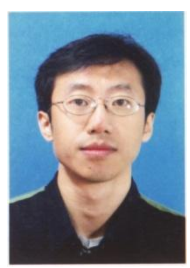
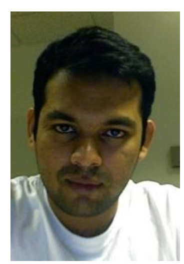
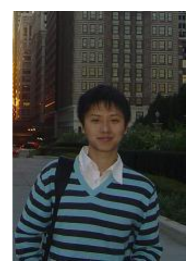
Modern Massive Data Sets, Stanford, June 18, 2010

ROBUST PRINCIPAL COMPONENT ANALYSIS?

Joint work with **Emmanuel Candès**, **Xiaodong Li** and **Yi Ma**

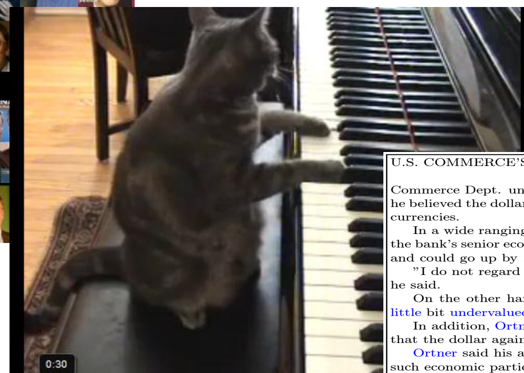
Contributions from Zhouchen Lin, Yang Xu, Arvind Ganesh, Zihan Zhou, many MSR interns…

CONTEXT – data increasingly *massive, high-dimensional…*

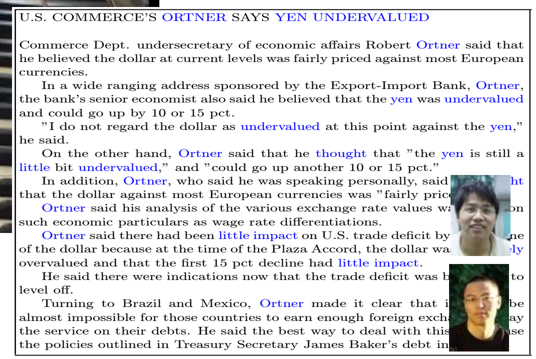
Images

> 1M dimensions



Videos

> 1B dimensions



U.S. COMMERCE'S ORTNER SAYS YEN UNDERVALUED

Commerce Dept. undersecretary of economic affairs Robert Ortner said that he believed the dollar at current levels was fairly priced against most European currencies.

In a wide ranging address sponsored by the Export-Import Bank, Ortner, the bank's senior economist also said he believed that the yen was undervalued and could go up by 10 or 15 pct.

"I do not regard the dollar as undervalued at this point against the yen," he said.

On the other hand, Ortner said that he thought that "the yen is still a little bit undervalued," and "could go up another 10 or 15 pct."

In addition, Ortner, who said he was speaking personally, said ht that the dollar against most European currencies was "fairly pric

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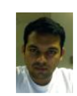
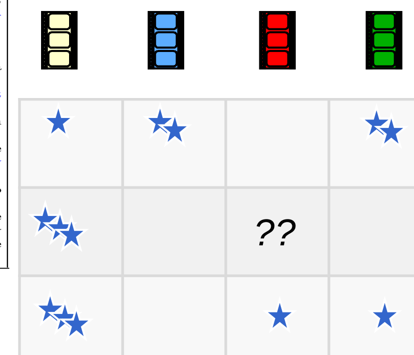
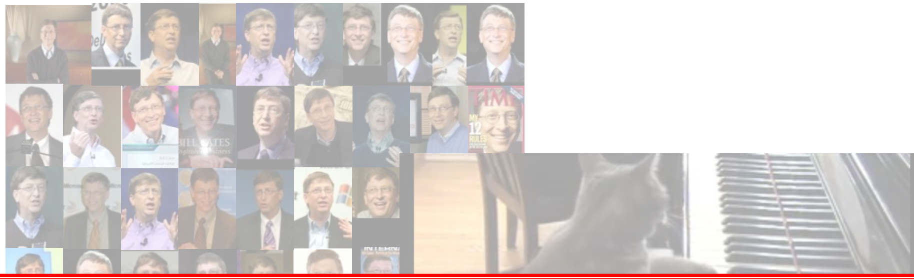


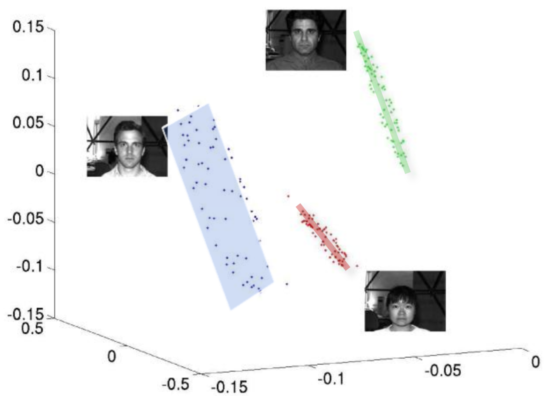


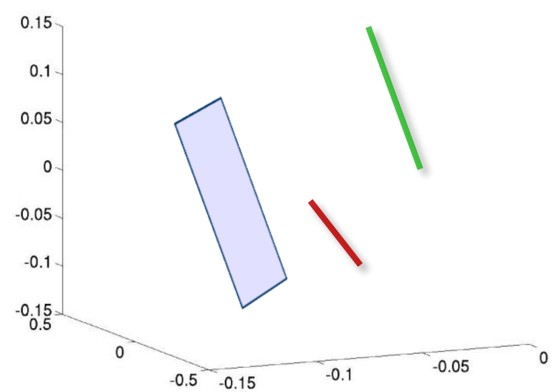
Web data

> 10B+ dimensions?

… but intrinsic structures are low-dimensional.







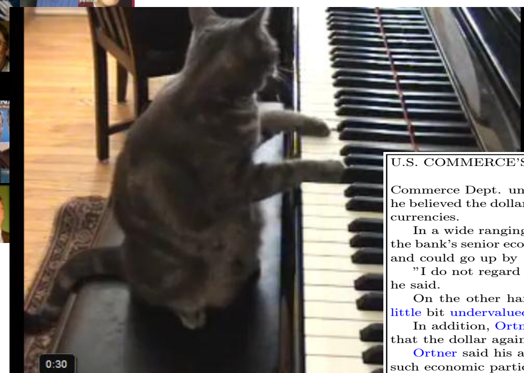
*How can we exploit **low-dimensional structure** in **high-dimensional data?***

CONTEXT – *Good solutions impact many applications*

Images

⇓

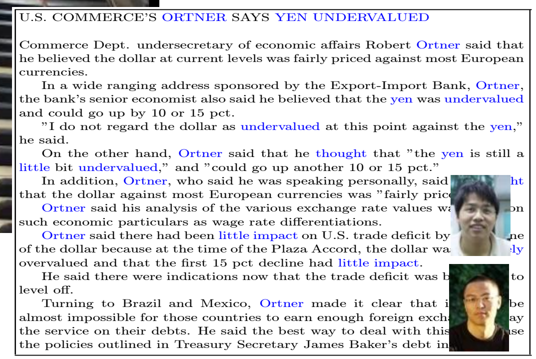
Recognition
Inpainting
Denoising



Videos

⇓

Compression
Transmission
Stabilization
Repair



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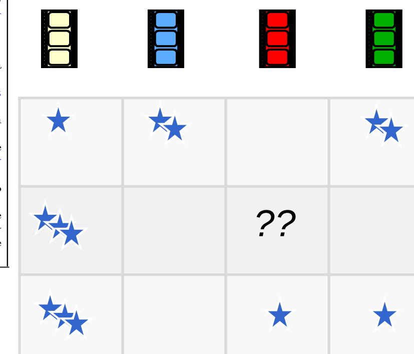


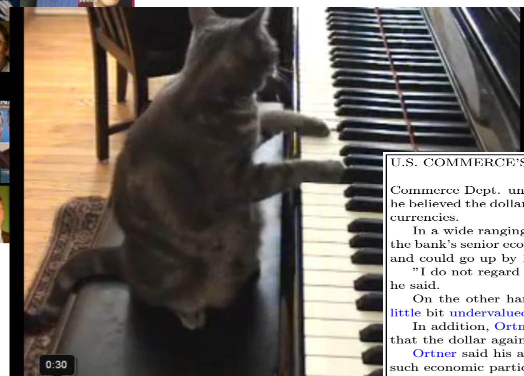
Web data

⇓

Indexing
Ranking
Search
Collaborative filtering…

But … its NOT EASY

Images



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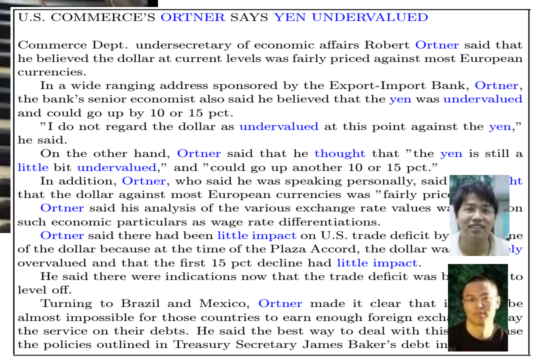

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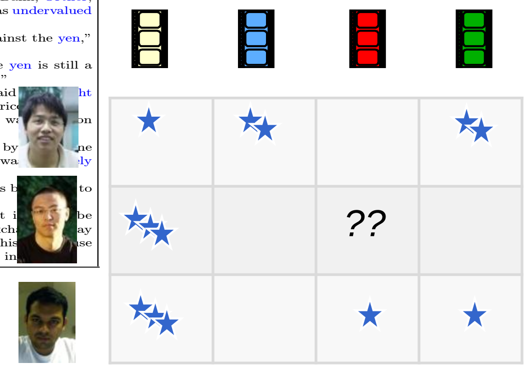
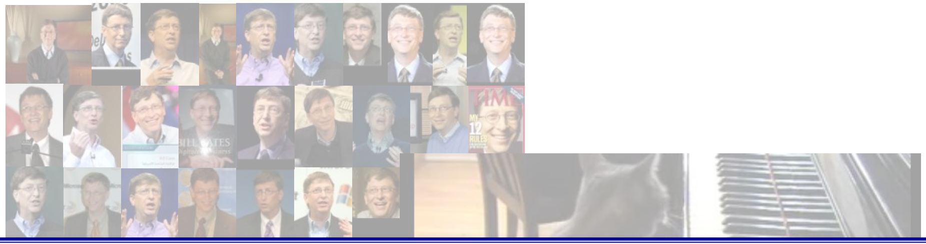


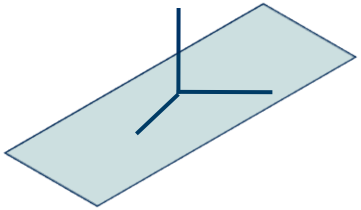
*Real application data often contain **missing observations**, **corruption** or even **malicious errors** and **noise.***

Classical algorithms (e.g., least squares, PCA) break down …

THIS TALK – Robust Principal Component Analysis?



How do we develop ***provably correct*** *and* ***efficient*** *algorithms for recovering* ***low-dimensional linear structure*** *from* ***corrupted high-dimensional observations****?*



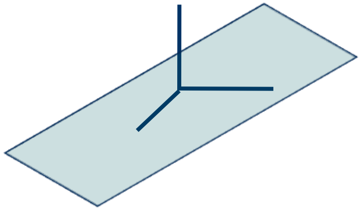
THIS TALK - *Outline*

- **Robust PCA via Convex Programming**
- **Main Result: Exact Recovery from Gross Errors**
- **Implications on Matrix Completion**
- **Algorithms, Simulations, and Experiments**
- **Open Problems and Future Directions**

CLASSICAL PCA – *Fitting data with a subspace*

If degenerate observations are stacked as columns of a matrix

$$X = [\mathbf{x}_1 \mid \cdots \mid \mathbf{x}_n] \in \mathbb{R}^{m \times n}$$



then

$$r \doteq \operatorname{rank}(X) \ll m.$$

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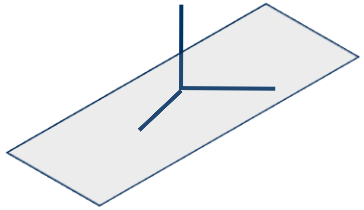

Principal Component Analysis (PCA) via singular value decomposition (SVD):

- Stable, efficient computation
- Optimal estimate of X under iid Gaussian noise $Y = X + Z$
- Fundamental statistical tool, huge impact in image processing, vision, search, bioinformatics…

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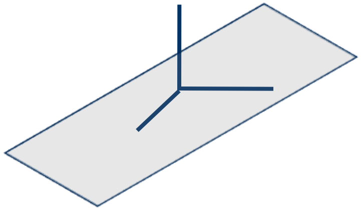

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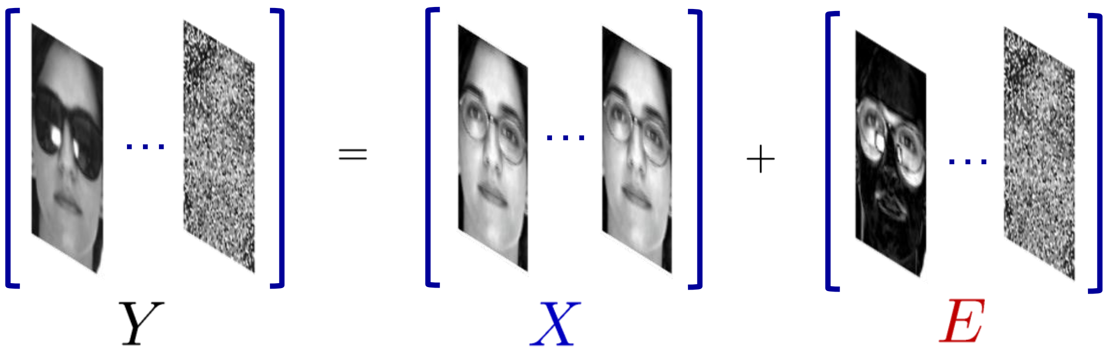
But… **PCA breaks down under even a single corrupted observation.**

PROBLEM FORMULATION - *Robust PCA?*



Given $Y = X + E$, with X low-rank, E sparse, recover X.

Numerous approaches to Robust PCA in the literature:

- Multivariate trimming *[Gnanadeskian + Kettering '72]*
- Random sampling *[Fischler + Bolles '81]*
- Alternating minimization *[Ke + Kanade '03]*
- Influence functions *[de la Torre + Black '03]*

No polynomial-time algorithm with strong performance guarantees…

Some related solutions…

Classical PCA/SVD – low rank + noise *[Hotelling '35, Karhunen+Loeve '72,…]*

From $Y = X + Z$, recover X.

Stable, efficient algorithm, theoretically optimal $\rightarrow$ huge impact

Matrix Completion – low rank, missing data *[Candes + Recht '08, Candes + Tao '09, Keshevan, Oh, Montanari '09]*

From $Y = \mathcal{P}_\Omega[X]$, recover X.

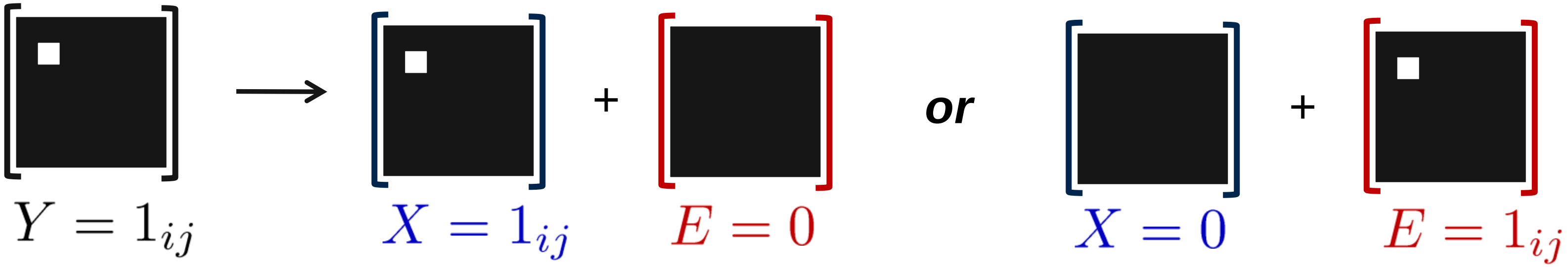
Increasingly well-understood; solvable if X is low rank and Ω large enough:

E.g., $|\Omega| \geq C\mu^2 nr(\log^2 n)$ suffices.

Our problem, with $Y = X + E$***, looks more difficult…***

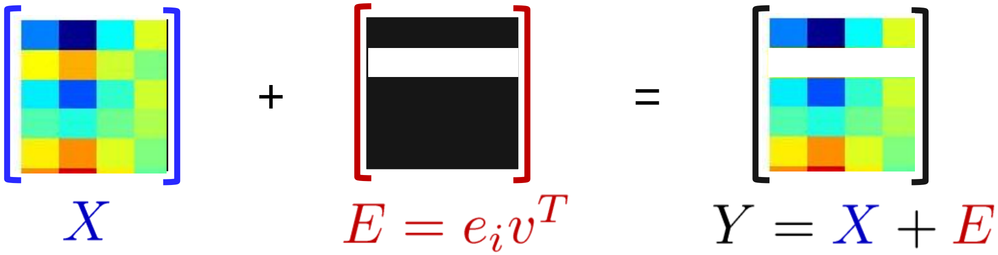
Why is the problem with Y = X + E difficult?

Some very sparse matrices are also low-rank:



*Can we recover X that are **incoherent** with the standard basis?*

Certain sparse error patterns E make recovering X impossible:



*Can we correct E whose support is not **adversarial**?*

When is there hope?

*Can we recover X that are **incoherent** with the standard basis from **almost all** errors E?*

Incoherence condition on singular vectors, **singular values arbitrary:**

Singular vectors of X not too sparse: $\begin{cases} \max_i \|U_i\|^2 \leq \mu r/m. \\ \max_i \|V_i\|^2 \leq \mu r/n. \end{cases}$

not too cross-correlated: $\|UV^*\|_\infty \leq \sqrt{\mu r/mn}$

Uniform model on error support, **signs and magnitudes arbitrary:**

$$\mathrm{supp}(E) \sim \mathrm{uni}\binom{[m]\times[n]}{\rho mn} \qquad \Omega \sim \mathrm{uni}\binom{[m]\times[n]}{\rho' mn}$$

Incoherence condition: [Candes + Recht ‘08]

… and how might we solve it?

Naïve optimization approach

Look for a low-rank *X* that agrees with the data up to some sparse error *E*:

$$\min \ \operatorname{rank}(X) + \gamma \|E\|_0 \quad \text{subj} \quad X + E = Y.$$

$$\operatorname{rank}(X) = \#\{\sigma_i(X) \neq 0\}. \qquad \|E\|_0 = \#\{E_{ij} \neq 0\}.$$

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INTRACTABLE

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Convex relaxation

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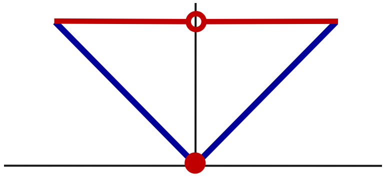
$$\downarrow\downarrow \qquad\qquad \downarrow\downarrow$$

$$\|X\|_* = \sum_i \sigma_i(X). \qquad \|E\|_1 = \sum_{ij} |E_{ij}|.$$



Nuclear norm heuristic: [Fazel, Hindi, Boyd ‘01], see also [Recht, Fazel, Parillo ‘08]

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Convex relaxation

$$\min \ \|X\|_* + \lambda \|E\|_1 \quad \mathrm{subj} \quad X + E = Y.$$

Semidefinite program, solvable in polynomial time – **"efficient"** algorithm.

Practical thanks to steady advances in large-scale convex programming

Does this actually work?

Key question

Does this practical surrogate actually solve the problem?

$$\min \ \operatorname{rank}(X) + \gamma \|E\|_0 \ \ \text{subj} \ \ X + E = Y.$$

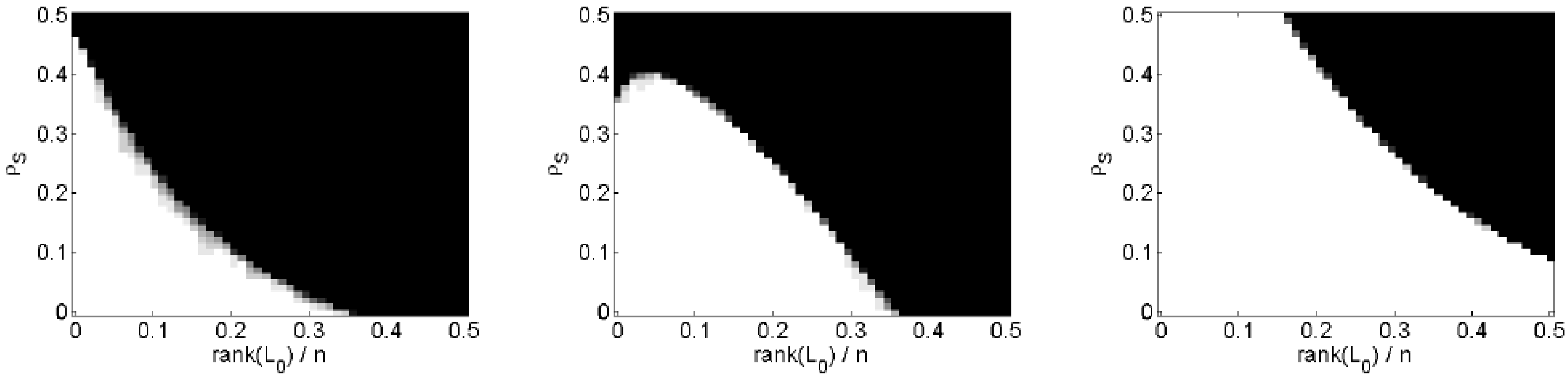
$$\Updownarrow \qquad \textit{EQUIVALENT?} \qquad \Updownarrow$$

$$\min \ \|X\|_* + \lambda \|E\|_1 \ \ \text{subj} \ \ X + E = Y.$$

Not always – original problem is NP-hard.

But maybe it **succeeds for the cases we care about?**

Does this actually work?



(a) Robust PCA, Random Signs (b) Robust PCA, Coherent Signs (c) Matrix Completion

$$Y = X + E$$

$$Y = \mathcal{P}_\Omega[\, X \,]$$

Apparently yes… white regions are problems with perfect recovery.

Correct recovery when *X* is indeed **low-rank** and *E* is indeed **sparse**?

MAIN RESULTS – *Exact Solution by Convex Optimization*

Theorem 1 (Principal Component Pursuit). *If $X_0 \in \mathbb{R}^{m \times n}$, $m \geq n$ has rank*

$$r \;\leq\; \rho_r \, \frac{n}{\mu \log^2(m)}$$

and E_0 has Bernoulli support with error probability $\rho \leq \rho_s^\star$, then with very high probability

$$(X_0, E_0) \;=\; \arg\min \|X\|_* + \frac{1}{\sqrt{m}} \|E\|_1 \quad \text{subj} \quad X + E = X_0 + E_0,$$

and the minimizer is unique.

"Convex optimization recovers matrices of rank $O\left(\frac{n}{\log^2 m}\right)$ from errors corrupting $O\left(mn\right)$ entries"

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Non-adaptive weight factor

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"Convex optimization recovers matrices of rank $O\left(\frac{n}{\log^2 m}\right)$ ***from errors corrupting*** $O(mn)$ ***entries"***

ROBUST PCA – *Comparison to existing results*

Chandrasekaran et. al. 2009 *give an incoherence condition for correct recovery. Set:*

$$T = \{UW^* + QV^*\},\ X_0 = USV^*,$$

$$\xi \doteq \sup_{M \in T, \|M\| \leq 1} \|M\|_\infty,$$

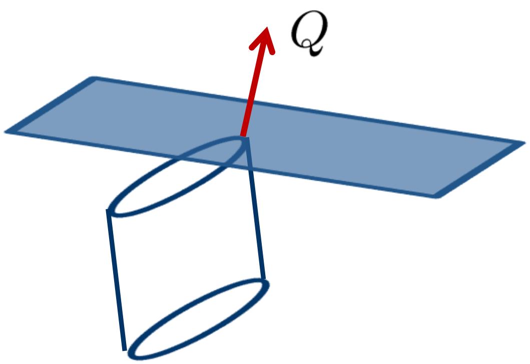
$$\mu \doteq \sup_{M \in \Omega, \|M\|_\infty \leq 1} \|M\|,$$

Correct recovery occurs if $\xi \times \mu < \frac{1}{6}$.

- Strong point: **deterministic condition**
- For random problems, success when $\|E\|_0 \leq Cm^{1.5}/r^{.5} \log m$.

MAIN IDEAS OF THE PROOF

As in the vector case, construct a **dual certificate:**



Clever iterative construction due to **D. Gross** – the "**golfing scheme**":

$$R_{k+1} = R_k + q^{-1}\mathcal{P}_{\Omega_{k+1}}\mathcal{P}_T(UV^* - R_k), \quad Q_L = \mathcal{P}_{T^\perp} R_{k^\star}.$$

Showing this construction succeeds requires a detailed analysis of a certain **random operator:**

$$\mathcal{P}_T\mathcal{P}_\Omega\mathcal{P}_T \approx \rho\mathcal{P}_T$$

Builds on results by **E. Candes + B. Recht.**

MAIN RESULTS – *Corrupted, Incomplete Matrix*

$$Y = \mathcal{P}_\Omega[\ X_0\ +\ E_0\ ], \qquad \Omega \sim \mathrm{uni}\left(\binom{[m]\times[n]}{mn}\right)$$

Theorem 2 (Matrix Completion and Recovery). *If $X_0, E_0 \in \mathbb{R}^{m\times n}, m \geq n$, with*

$$\mathrm{rank}(X_0)\ \leq\ C\,\frac{n}{\mu \log^2(m)}, \quad \textit{and} \quad \|E_0\|_0\ \leq\ \rho^\star mn,$$

and we observe only a random subset of size $|\Omega| = mn/10$ entries, then with very high probability, solving the convex program

$$\min \|X\|_* + \tfrac{1}{\sqrt{m}}\|E\|_1 \quad \mathrm{subj} \quad P_\Omega[X+E] = Y,$$

uniquely recovers (X_0, E_0).

"Convex optimization succeeds with large fractions of errors and missing entries."

Candes, Li, Ma, and Wright, submitted to JACM, 2009.

MAIN RESULTS – *Dense Random Errors*

Theorem 3 (Dense Error Correction). *If* X_0 *has rank* $r \leq \rho_r \frac{m}{\mu \log^2(n)}$ *and* E_0 *has random signs and Bernoulli support with error probability* $\rho < 1$, *then with very high probability*

$$(X_0, E_0) \;=\; \arg\min \|X\|_* + \lambda \|E\|_1 \quad \text{subj} \quad X + E = X_0 + E_0,$$

and the minimizer is unique.

"If the error sign pattern is random, large fractions of errors can be corrected"

MAIN RESULTS – *Stable recovery with noise*

Theorem 4 (Robust PCA with Noise). *Given* $Y = X_0 + E_0 + Z$ *for any* $\|Z\|_F \leq \eta$, *if* X_0 *has rank* $r \leq \rho_r \frac{m}{\mu \log^2(n)}$ *and* E_0 *has Bernoulli support with error probability* $\rho \leq \rho_s^\star$, *then with very high probability*

$$(\hat{X}, \hat{E}) \;=\; \arg\min \|X\|_* + \frac{1}{\sqrt{m}}\|E\|_1 \quad \text{subj} \quad \|Y - X - E\|_F \leq \eta,$$

sastisfies $\|(\hat{X}, \hat{E}) - (X_0, E_0)\| \leq C\eta$ *for some* $C > 0$ *independent of* η.

"When exact recovery occurs, the recovery is also stable under noise"

Rapid development in fast algorithms…

For a 1000x1000 matrix of rank 50, with 10% (100,000) entries randomly corrupted: $\min \; \|X\|_* + \lambda \|E\|_1 \quad \text{subj} \quad X + E = Y.$

Algorithms	Accuracy	Rank	||E||_0	# iterations	time (sec)
IT	5.99e-006	50	101,268	8,550	119,370.3
DUAL	8.65e-006	50	100,024	822	1,855.4
APG	5.85e-006	50	100,347	134	1,468.9
APG_P	5.91e-006	50	100,347	134	82.7
$EALM_P$	2.07e-007	50	100,014	34	37.5
$IALM_P$	3.83e-007	50	99,996	23	11.8

10,000 times speedup!

Provably Robust PCA with only ~20 times more computation than SVD.

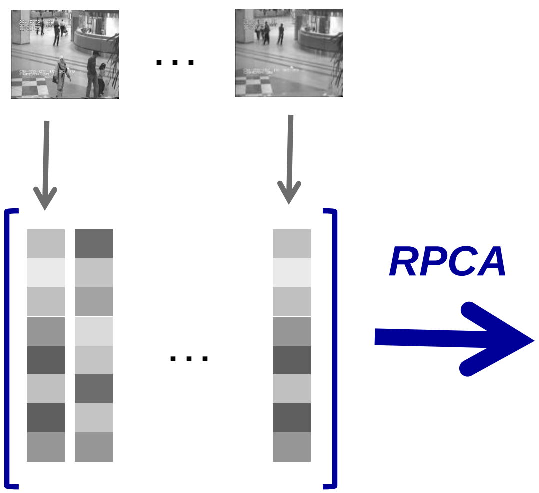
Work by Z. Lin, M. Chen … X. Yuan

APPLICATIONS – *Background modeling from video*

Static camera
surveillance video

200 frames,
144 x 172 pixels,

Significant foreground
motion



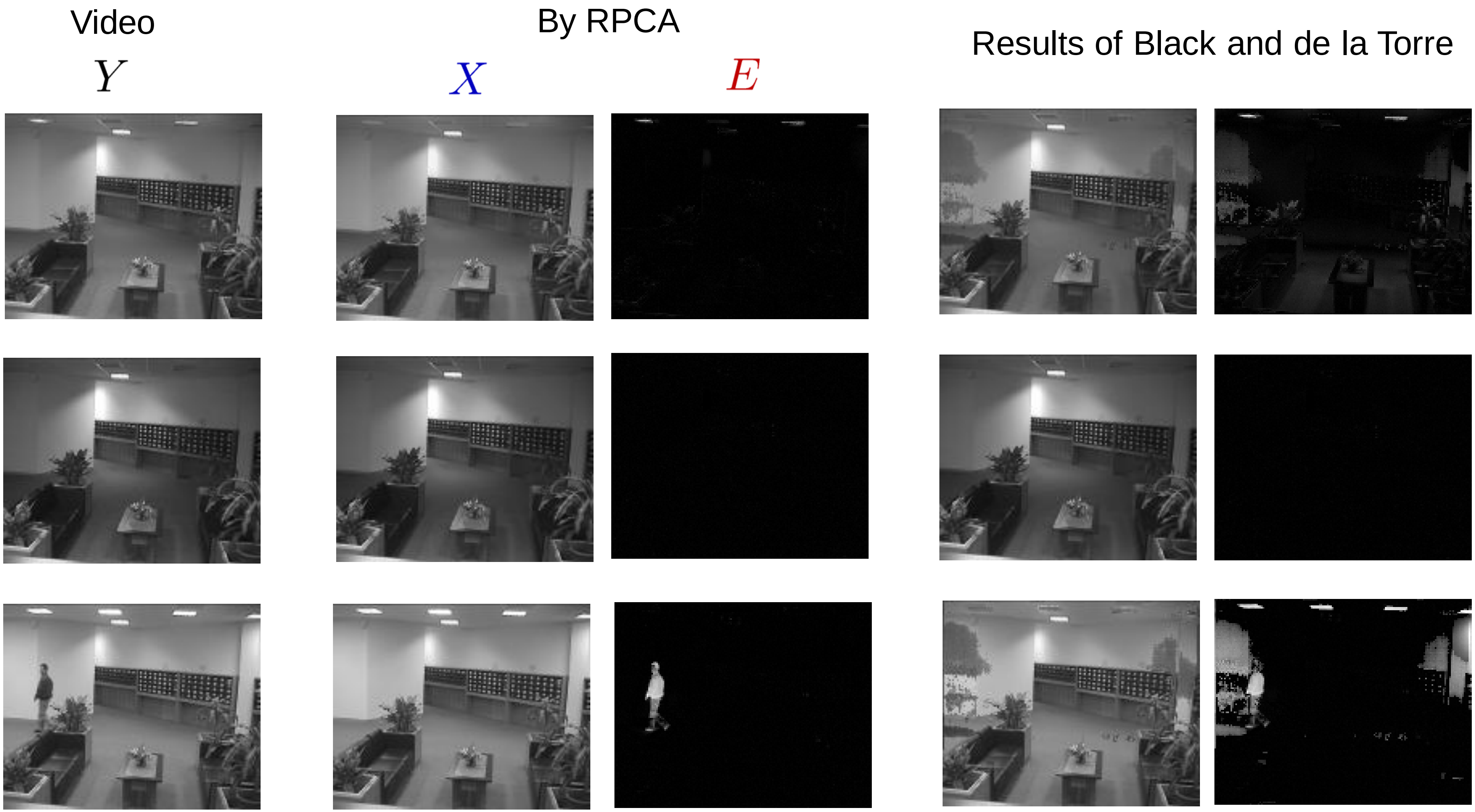
Y

Video Y = Low-rank appx. X + Sparse error E

Candes, Li, Ma, and Wright, submitted to JACM, 2009.

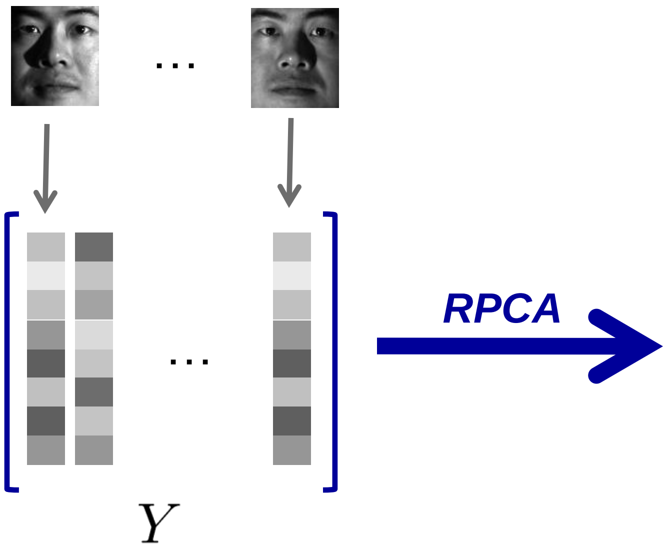
APPLICATIONS – *Background modeling from video*

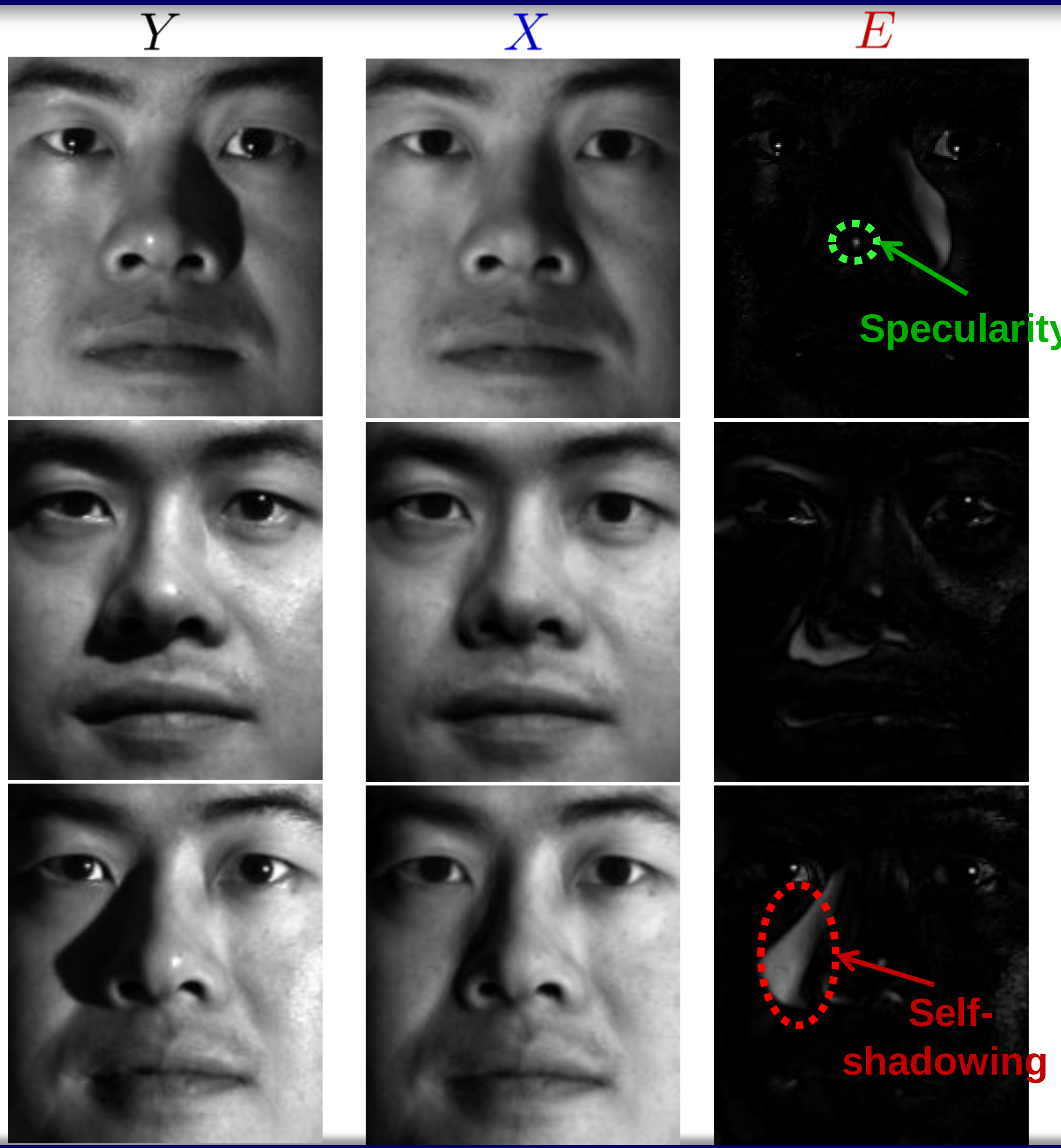
Surveillance video: 250 frames, 128 x 160 pixels, with significant illumination variation



APPLICATIONS – *Faces under varying illumination*

58 images of one person under varying lighting:



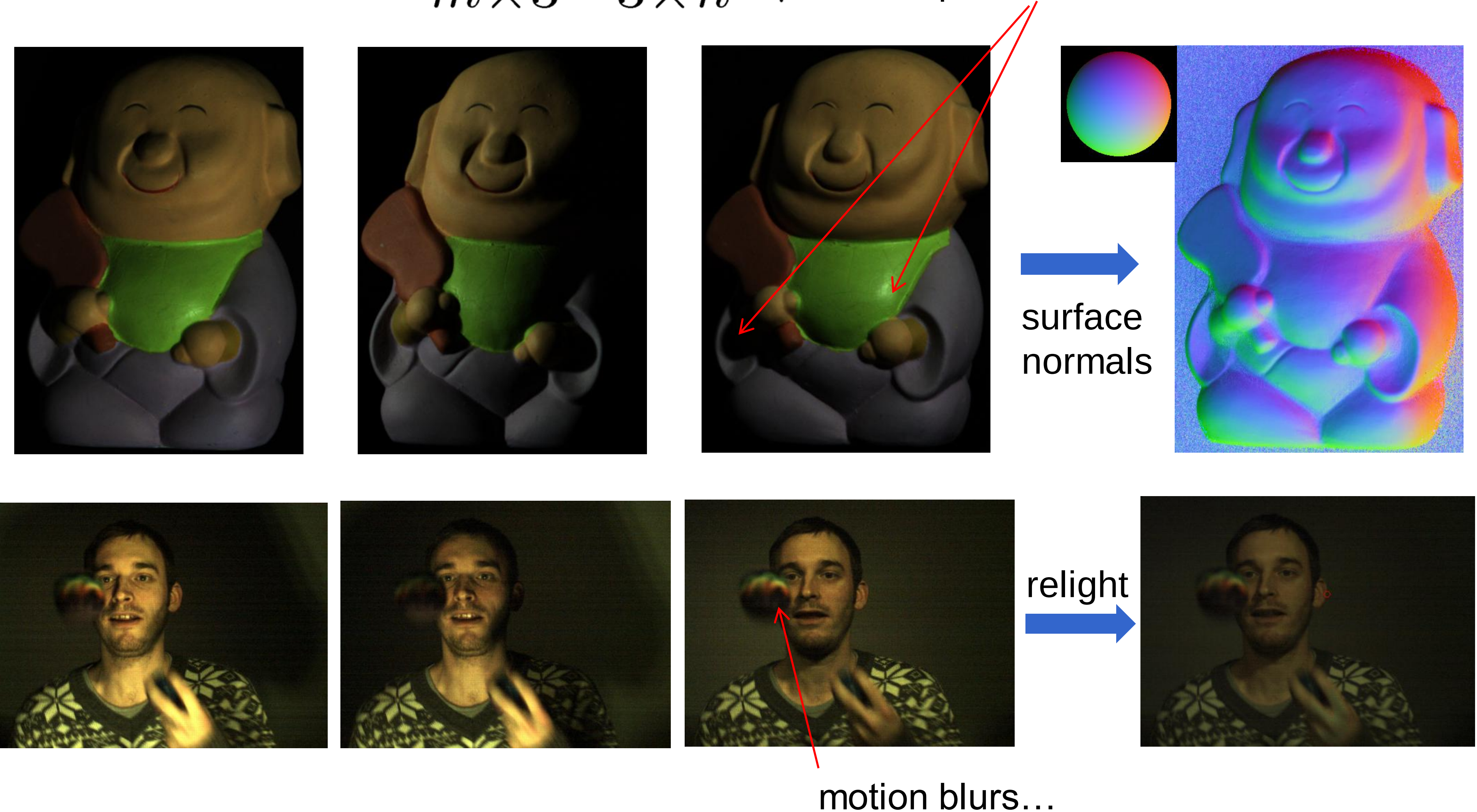


Candes, Li, Ma, and Wright, submitted to JACM, 2009.

High-quality photometric stereo

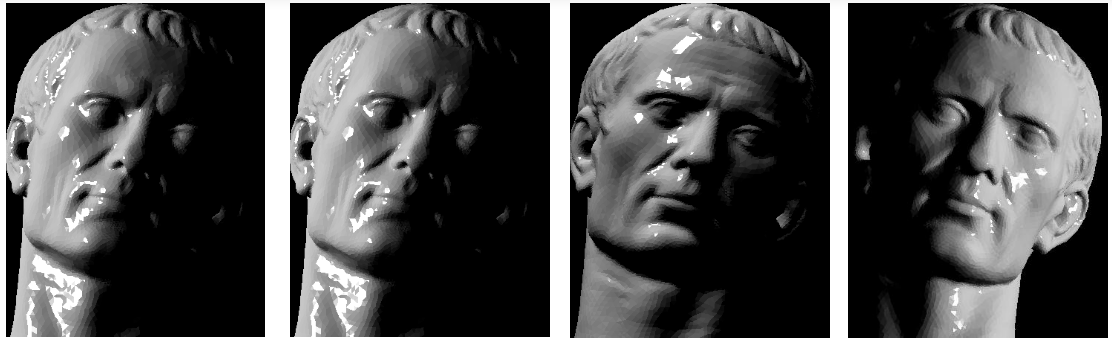
$$D = N_{m\times 3} L_{3\times n} + E$$

specularities, shadows…

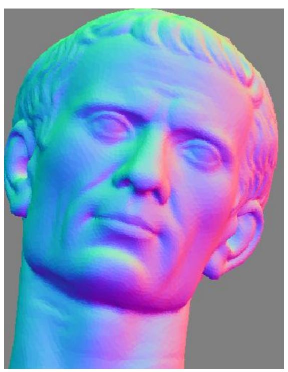


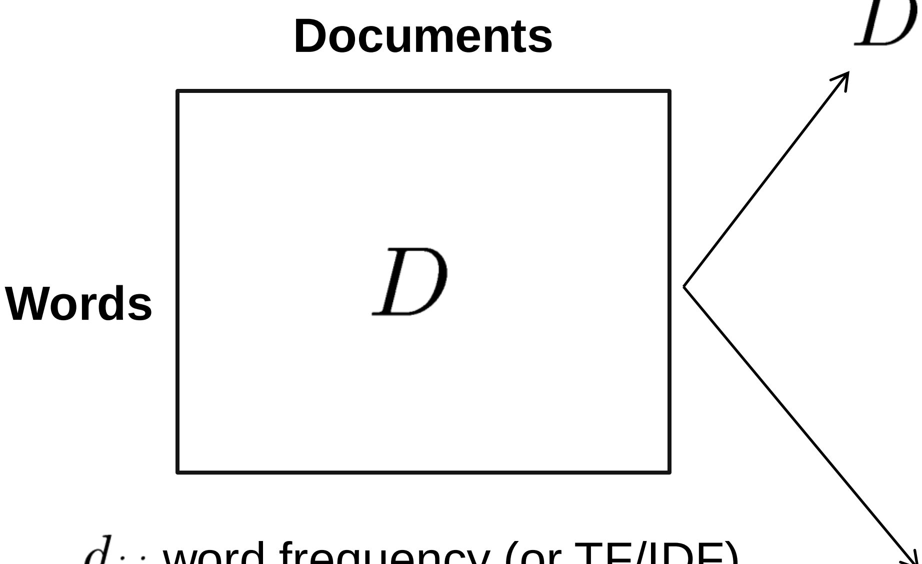
High-quality photometric stereo

Input images



$$\min \ \|A\|_* + \lambda \|E\|_1 \quad \text{subj} \quad D = \mathcal{P}_\Omega(A + E). \qquad \Omega^c \sim \text{shadow}(20.7\%)$$
$$E \sim \text{specularities}(13.6\%)$$

	Ground truth	Robust PCA	Robust LS
			
Mean error		**0.014^o**	0.96^o
Max error		**0.20^o**	8.0^o



d_{ij} word frequency (or TF/IDF)

Classical solution (LSI)

$$\begin{aligned} D &= A + N \\ &= U_1\Sigma_1V_1^T + \underline{U_2\Sigma_2V_2^T} \end{aligned}$$

Dense, difficult to interpret

A better model/solution

$$D = A + \underline{E}$$

Low-rank "background" topic model

Informative, discriminative "keywords"

APPLICATIONS: Document retrieved by title words

U.S. COMMERCE'S ORTNER SAYS YEN UNDERVALUED

Commerce Dept. undersecretary of economic affairs Robert Ortner said that he believed the dollar at current levels was fairly priced against most European currencies.

In a wide ranging address sponsored by the Export-Import Bank, Ortner, the bank's senior economist also said he believed that the yen was undervalued and could go up by 10 or 15 pct.

"I do not regard the dollar as undervalued at this point against the yen," he said.

On the other hand, Ortner said that he thought that "the yen is still a little bit undervalued," and "could go up another 10 or 15 pct."

In addition, Ortner, who said he was speaking personally, said he thought that the dollar against most European currencies was "fairly priced."

Ortner said his analysis of the various exchange rate values was based on such economic particulars as wage rate differentiations.

Ortner said there had been little impact on U.S. trade deficit by the decline of the dollar because at the time of the Plaza Accord, the dollar was extremely overvalued and that the first 15 pct decline had little impact.

He said there were indications now that the trade deficit was beginning to level off.

Turning to Brazil and Mexico, Ortner made it clear that it would be almost impossible for those countries to earn enough foreign exchange to pay the service on their debts. He said the best way to deal with this was to use the policies outlined in Treasury Secretary James Baker's debt initiative.

APPLICATIONS: Web document corpus analysis

Reuters-21578 dataset: 1,000 longest documents; 3,000 most frequent words

CHRYSLER SETS STOCK SPLIT, HIGHER DIVIDEND

Chrysler Corp said its board declared a three-for-two stock split in the form of a 50 pct stock dividend and raised the quarterly dividend by seven pct.

The company said the dividend was raised to 37.5 cts a share from 35 cts on a pre-split basis, equal to a 25 ct dividend on a post-split basis.

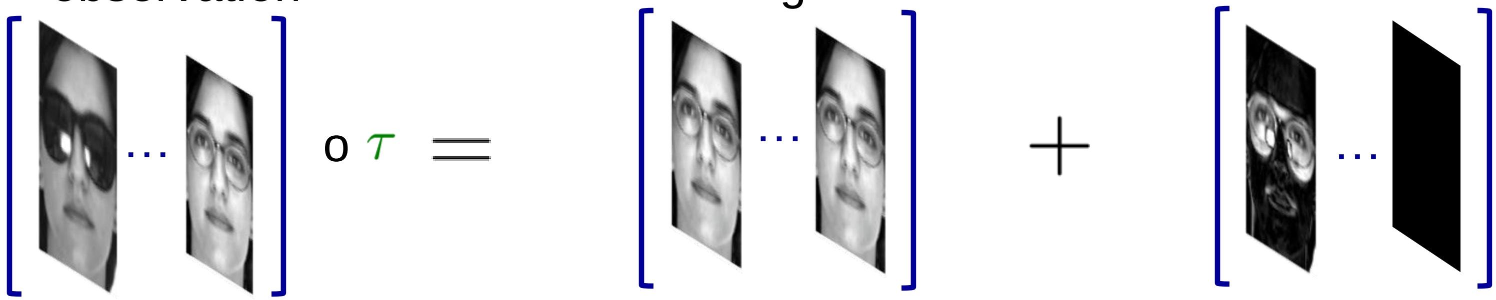
Chrysler said the stock dividend is payable April 13 to holders of record March 23 while the cash dividend is payable April 15 to holders of record March 23. It said cash will be paid in lieu of fractional shares.

With the split, Chrysler said 13.2 mln shares remain to be purchased in its stock repurchase program that began in late 1984. That program now has a target of 56.3 mln shares with the latest stock split.

Chrysler said in a statement the actions "reflect not only our outstanding performance over the past few years but also our optimism about the company's future."

Robust Alignment via Sparse and Low-rank Decomposition

Y – corrupted & misaligned observation X – aligned low-rank signals E – sparse errors



Problem: Given $Y \circ \tau = X_0 + E_0$, recover τ, X_0 and E_0.

Parametric deformations (rigid, affine, projective…)

Low-rank component

Sparse component

Solution: Robust Alignment via Low-rank and Sparse (**RASL**) Decomposition

Iterate:

$$\min \ \|X\|_* + \lambda \|E\|_1 \quad \text{subj} \quad A + E = Y \circ \tau_k + J \Delta \tau$$

Peng, Ganesh, Wright, Ma. *CVPR 2010*

APPLICATIONS – *Aligning Bill Gates faces from the Internet*

*48 images collected from internet

Peng, Ganesh, Wright, Ma, submitted to CVPR'10.

APPLICAITONS – *Bill Gates faces detected*

Input: faces detected by a face detector (D)

Average

APPLICATIONS – *Bill Gates faces aligned*

Output: aligned faces ($D \circ \tau$)

Average

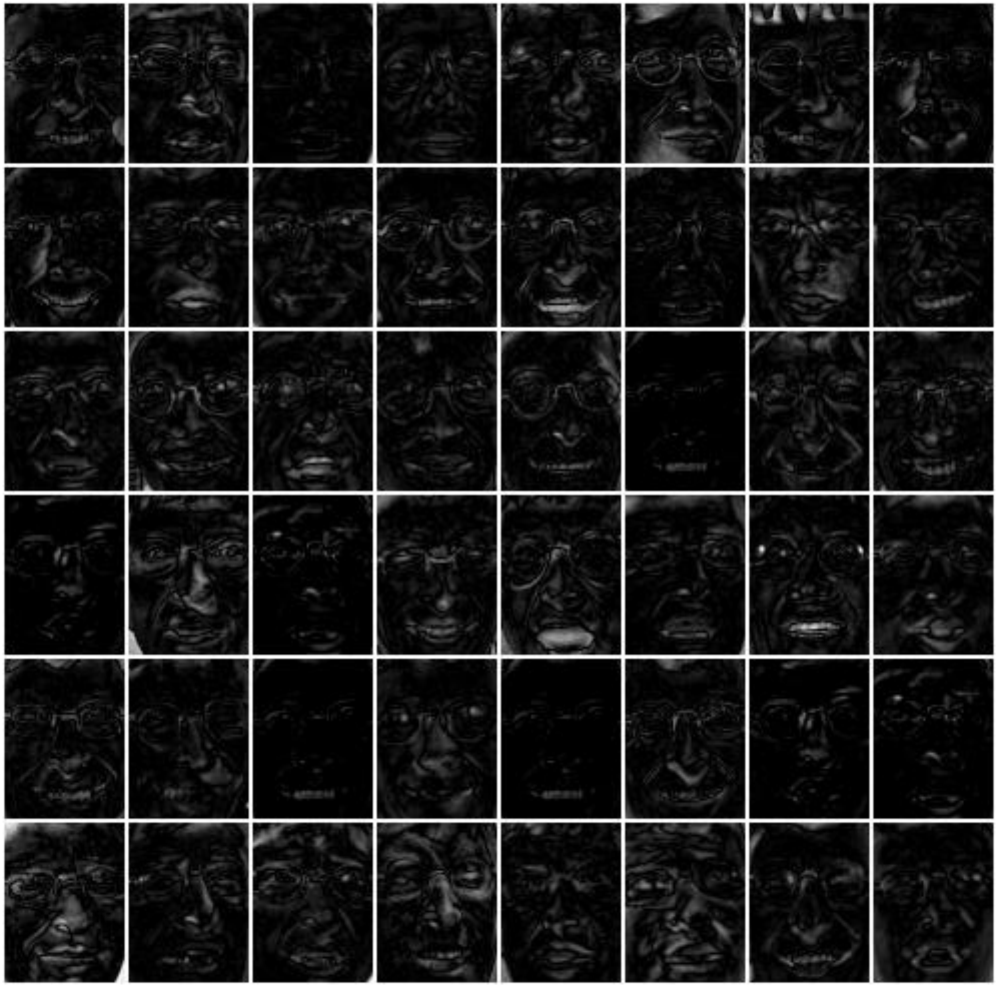
APPLICAITONS – *Bill Gates faces cleaned*

Output: clean low-rank faces (A)

Average

APPLICATIONS – *Sparse errors of Bill Gates face images*

Output: sparse error images (E)



APPLICATIONS – *Celebrity images from the Internet*

Average face before alignment

Gloria Macapagal Arroyo
Jennifer Capriati
Laura Bush
Serena Williams
Barack Obama
Ariel Sharon
Arnold Schwarzenegger
Colin Powell
Donald Rumsfeld
George W Bush
Gerhard Schroeder
Hugo Chavez
Jacques Chirac
Jean Chretien
John Ashcroft
Junichiro Koizumi
Lleyton Hewitt
Luiz Inacio Lula da Silva
Tony Blair
Vladimir Putin

APPLICATIONS – *Face recognition with less controlled data?*

Average face after alignment

Gloria Macapagal Arroyo
Jennifer Capriati
Laura Bush
Serena Williams
Barack Obama
Ariel Sharon
Arnold Schwarzenegger
Colin Powell
Donald Rumsfeld
George W Bush
Gerhard Schroeder
Hugo Chavez
Jacques Chirac
Jean Chretien
John Ashcroft
Junichiro Koizumi
Lleyton Hewitt
Luiz Inacio Lula da Silva
Tony Blair
Vladimir Putin

REFERENCES + ACKNOWLEDGEMENT

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- *Decomposing Background Topics from Keywords by Principal Component Pursuit*, Min, Zhang, Wright, and Ma, submitted to SIGIR'10, 2010.

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THANK YOU!

Questions, please?