Spectral Ranking

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Dipartimento di Scienze dell'Informazione
Università degli Studi di Milano

Advertising

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- LAW (Laboratory for Web Algorithmics) @ Università degli Studi di Milano

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- Some Matlab stuff by David Gleich
- *Stanford Matrix Considered Harmful* [V.]

A Historical Talk

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- This talk is about *spectral ranking*
- PageRank is just the currently trendy incarnation of spectral ranking
- The main ideas were developed in the late forties and in the early fifties
- However, the connection between these ideas emerged during the study of PageRank

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- Nonetheless, we have no scientific, reproducible proof that it works (quite the opposite)...
- ...and it's likely to be of minuscule importance in today's ranking
- Nonetheless, the idea is useful in several applications

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- *i* likes *j*, *j* likes *k*, but *i* does not like *k*, or...

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- *M* might contain "contradictory" information, as in...
- *i* likes *j*, *j* likes *k*, but *i* does not like *k*, or...
- *i* is better than *j*, *j* is better than *k*, but *i* is *not* better than *k*

The Basic Solution

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- Given M containing 0 or 1 depending on whether i likes j...
- Seeley argues that the rank of a child should be the sum of the ranks of the children that like him...
- ...and here we are! Seeley computes the dominant left eigenvector of M (normalised by row)

How it works

x_0	x_1	x_2	x_3	x_4

0	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$
0	0	0	1	0
0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0
$\frac{1}{2}$	0	0	$\frac{1}{2}$	0

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$\frac{1}{3} x_3 + \frac{1}{2} x_4$	$\frac{1}{3} x_0 + \frac{1}{3} x_3$	$\frac{1}{3} x_0 + \frac{1}{2} x_2 + \frac{1}{3} x_3$	$x_1 + \frac{1}{2} x_2 + \frac{1}{2} x_4$	$\frac{1}{3} x_0$

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- *P* express the probability that we try to meet child *j* after meeting child *i*...
- ...or, if you want, that we visit page *j* after visiting page *i*.
- The dominant left eigenvector is the *stable state* or *stationary* distribution

Perron–Frobenius

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Perron–Frobenius

- If M is nonnegative, the spectral radius is a dominant eigenvalue and there's a nonnegative dominant eigenvector
- If M is *irreducible* iff it is unique and strictly positive
- If M is *unichain* iff it is unique
- Otherwise, many possible solutions (Markovianly speaking, depending on the initial distribution)

The Dual View

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- Wei argues that the score of a team should be the sum of the scores of the teams it defeated, plus half the sum of the scores of the teams with which there was a tie...
- ...and here we are! Wei computes the dominant *right* eigenvector of *M* (no normalisation!)

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- Given a matrix *M* with a real, positive, strictly dominant eigenvalue
- The (left) spectral ranking of *M* is its (left) dominant eigenvector
- Left eigenvectors are good for endorsement; right eigenvectors for "better than" relationships (or you can just transpose your matrix, of course)

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- Katz claims that the importance of *i* depends not only on the number of the voters, but on the number of the voters' voters, etc., with suitable *attenuation* α
- He computes $\mathbf{1}\sum_{n=0}^{\infty}\alpha^n M^n = \mathbf{1}(1-\alpha M)^{-1}$

How it works

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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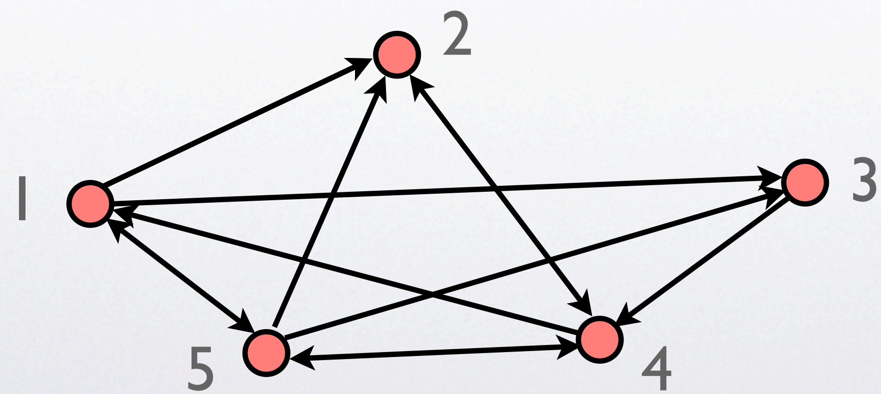

How it works

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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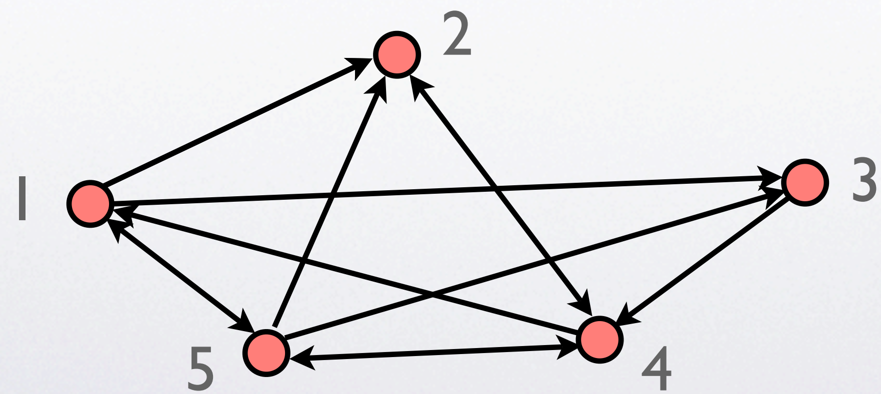

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0	0	0	1	0
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1	1	0	0	1
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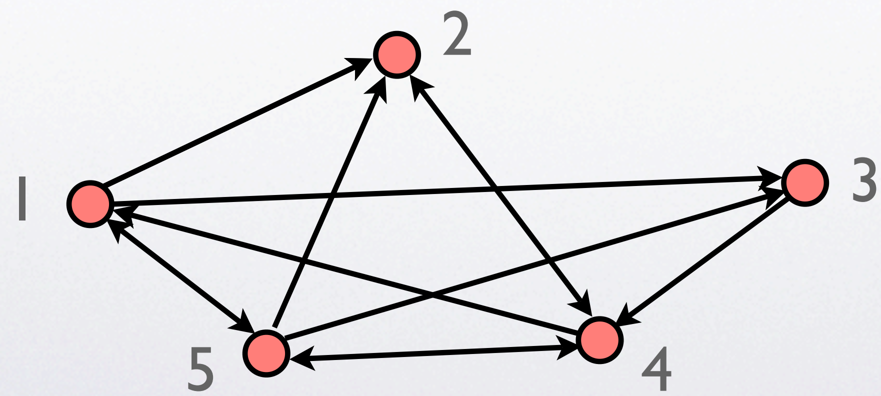

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0	1	1	0	1
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0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
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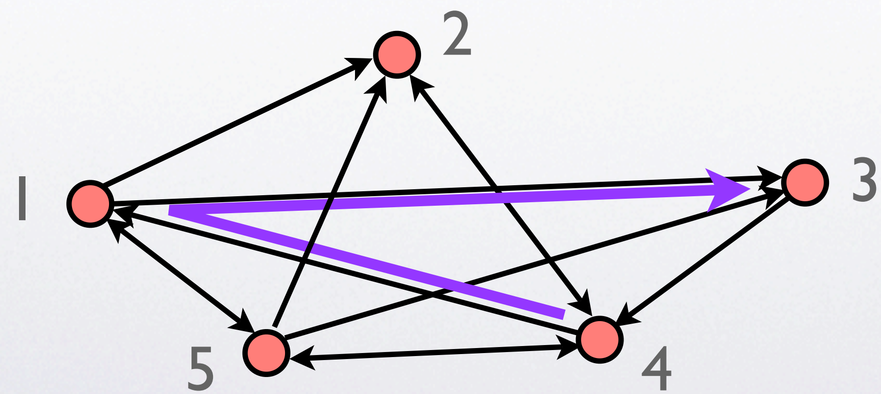

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0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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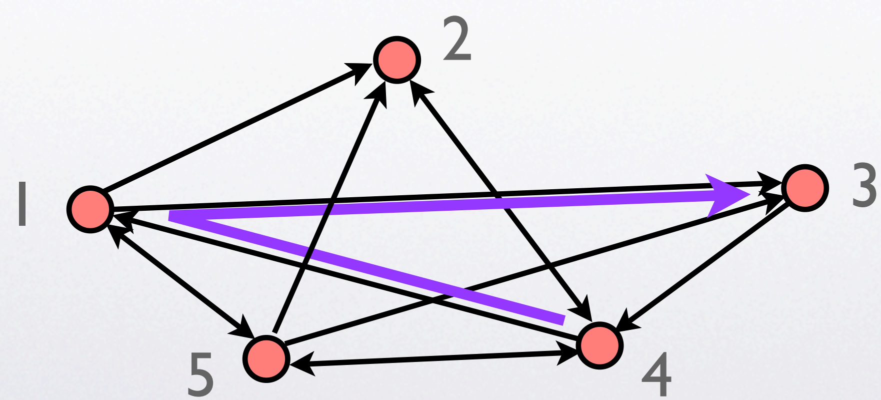

How it works

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0	0	0	1	0
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1	1	0	0	1
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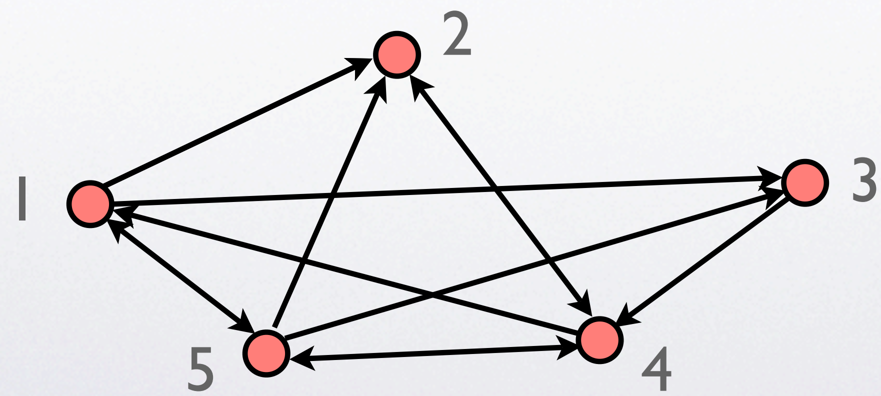

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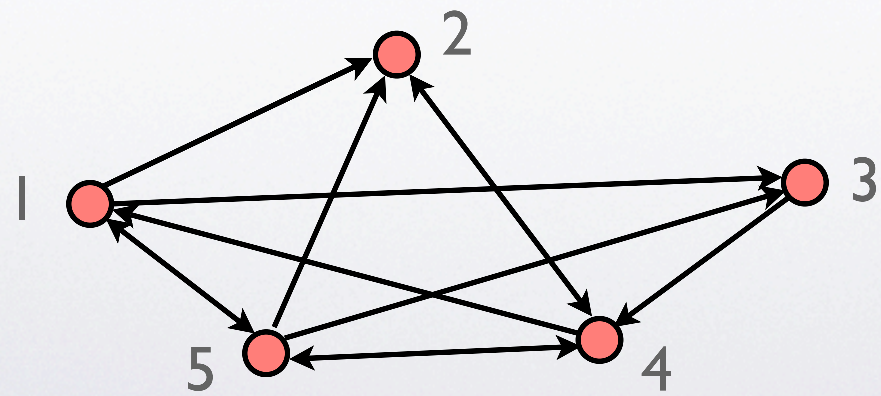

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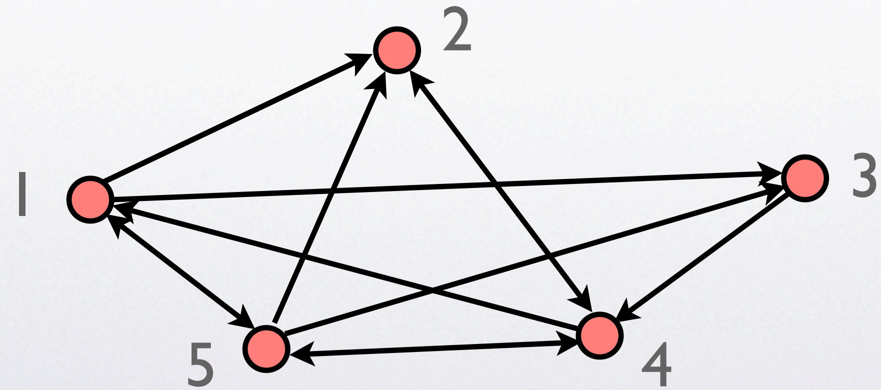

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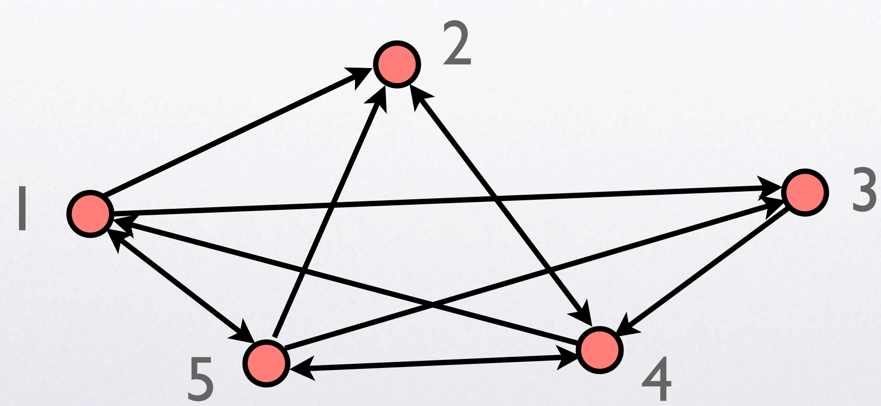

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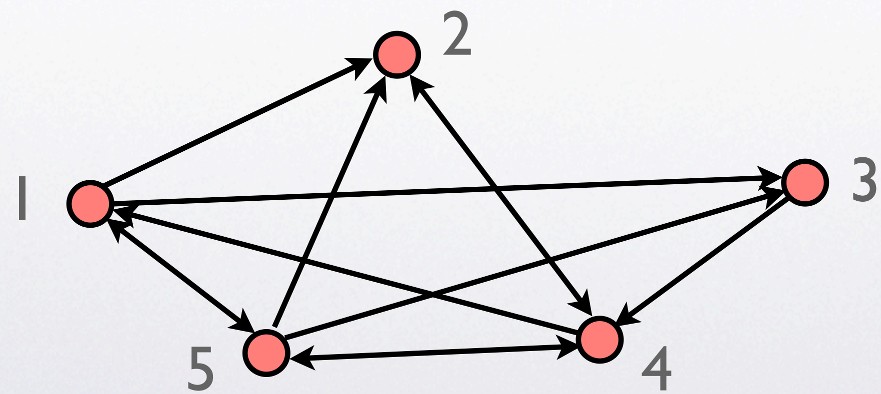

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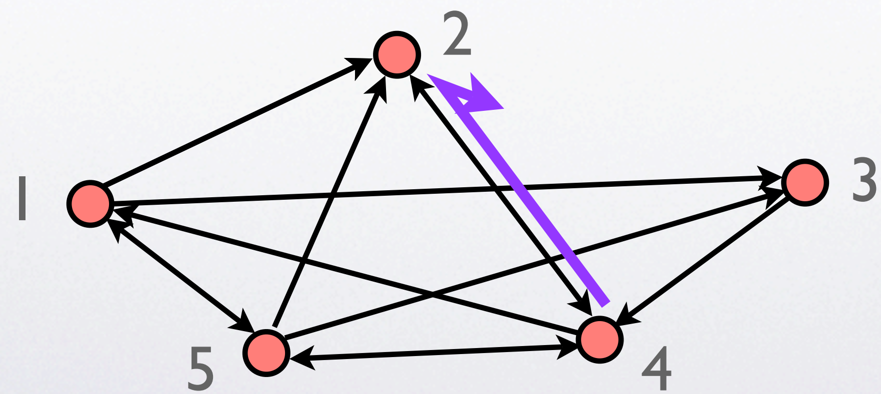

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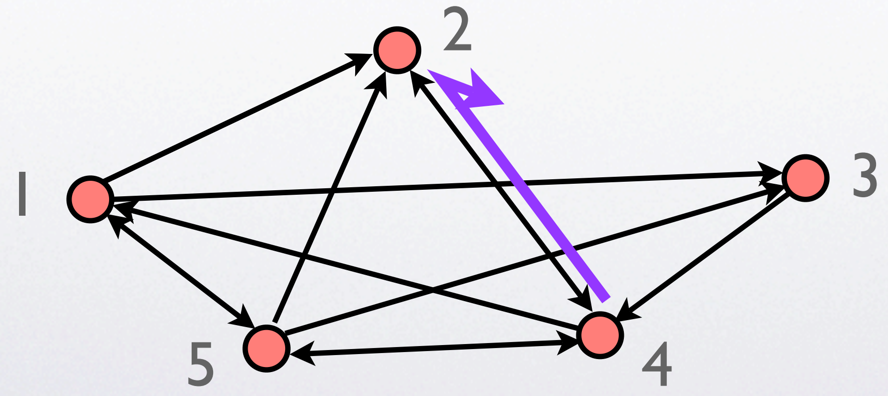

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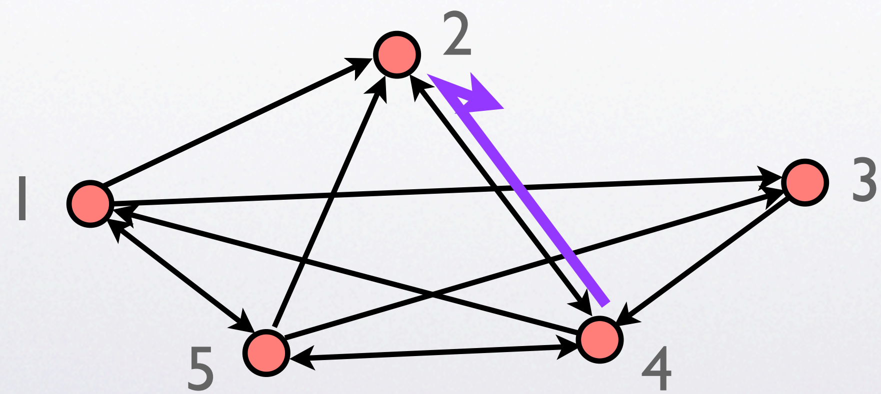

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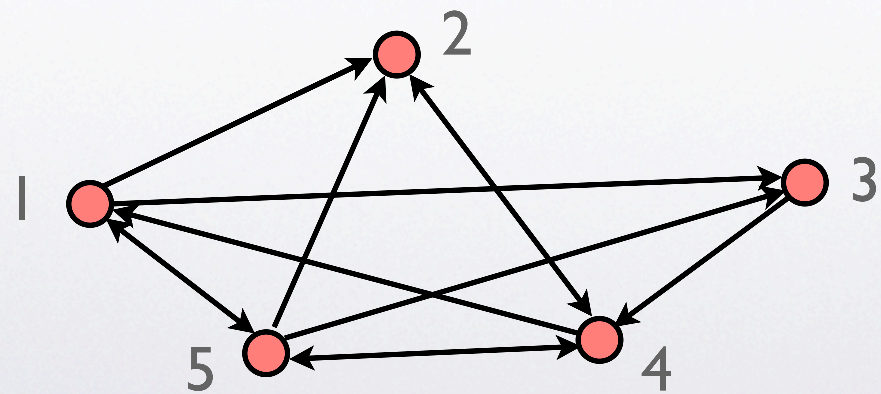

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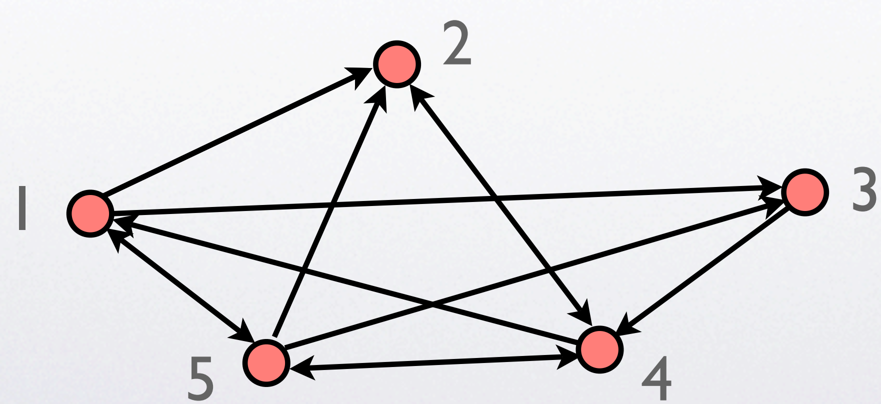

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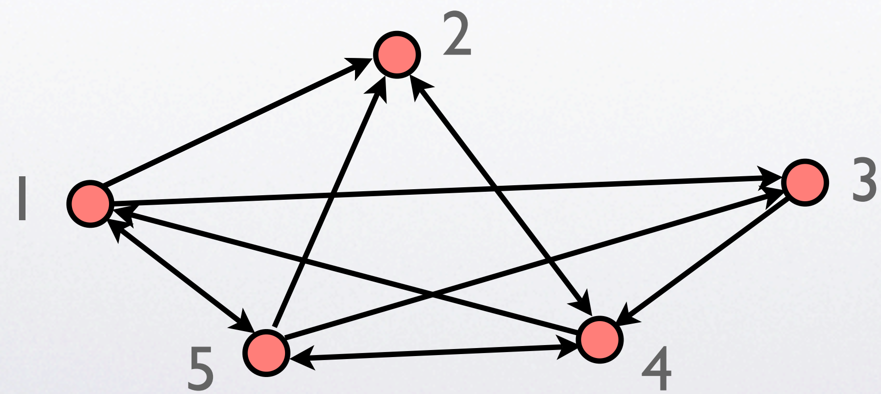

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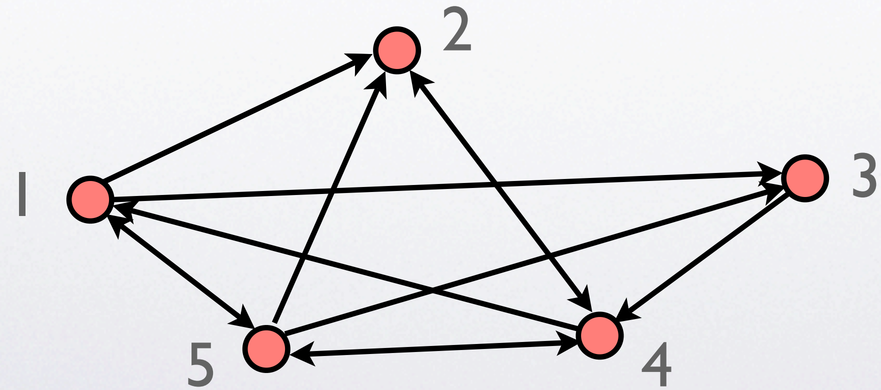

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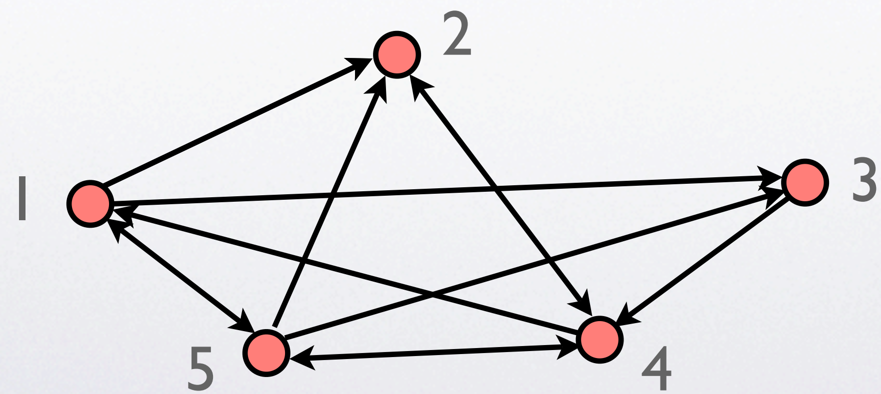

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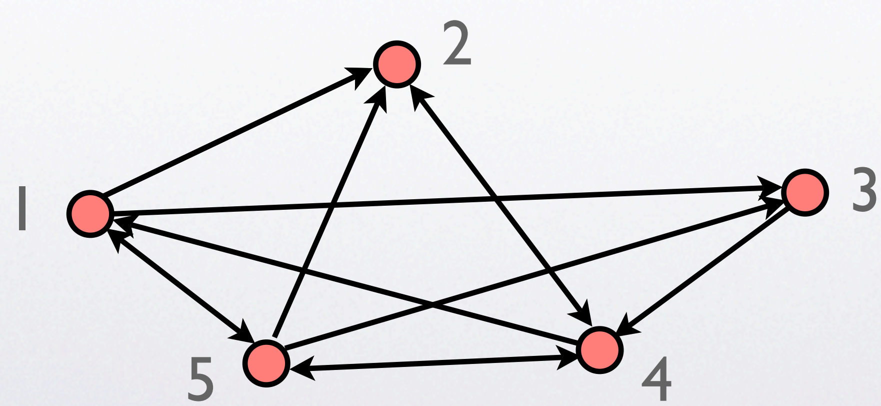

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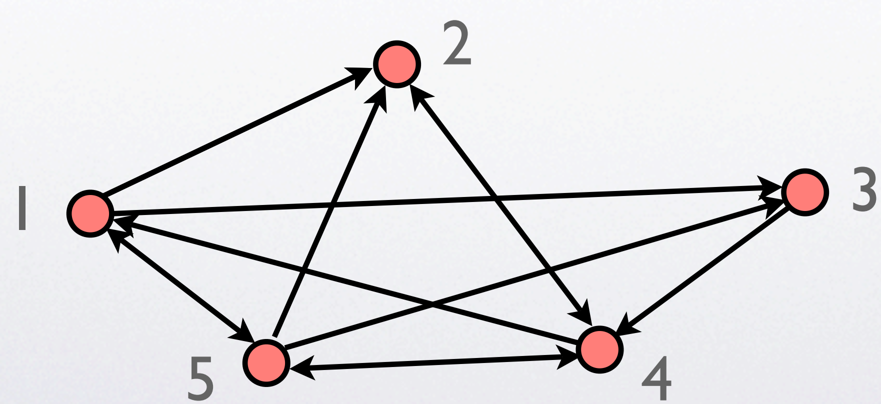

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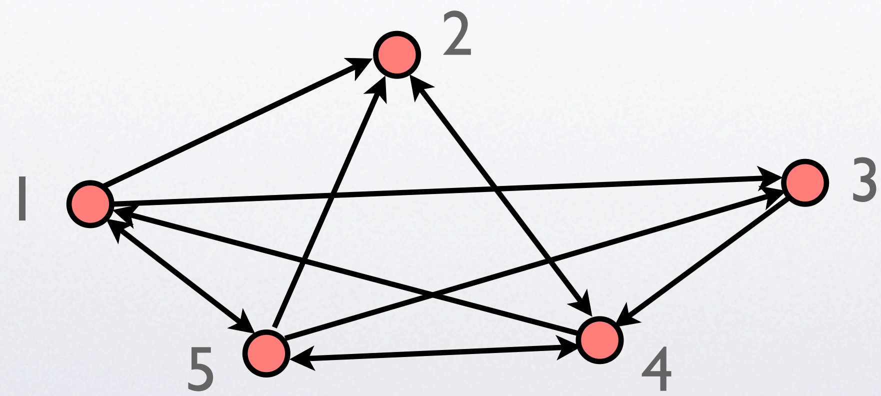

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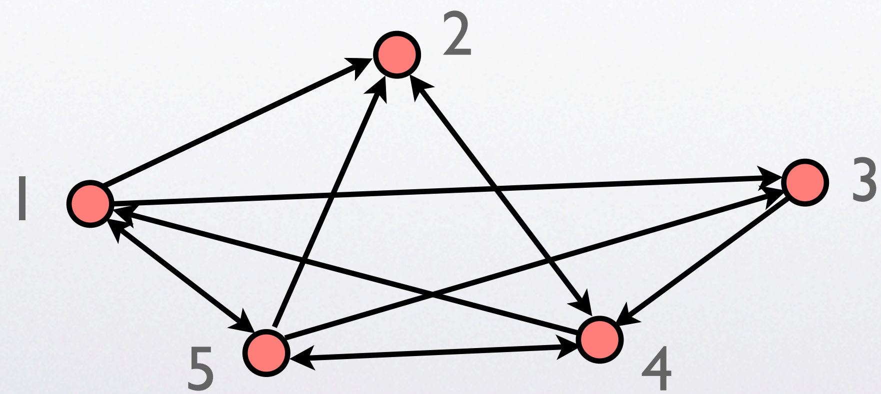

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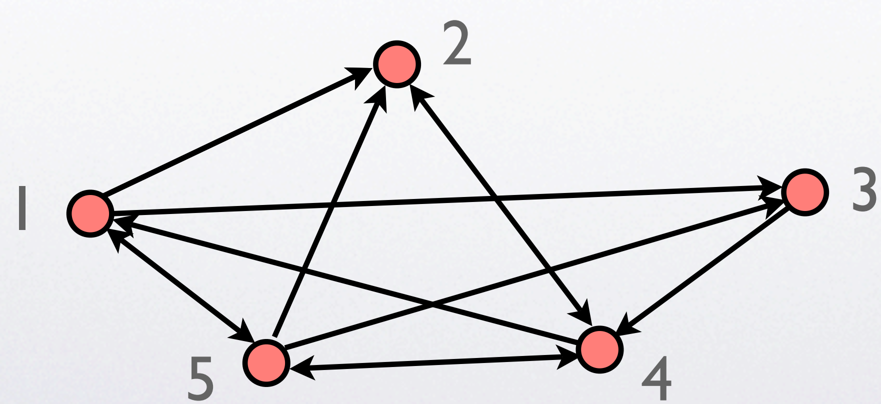

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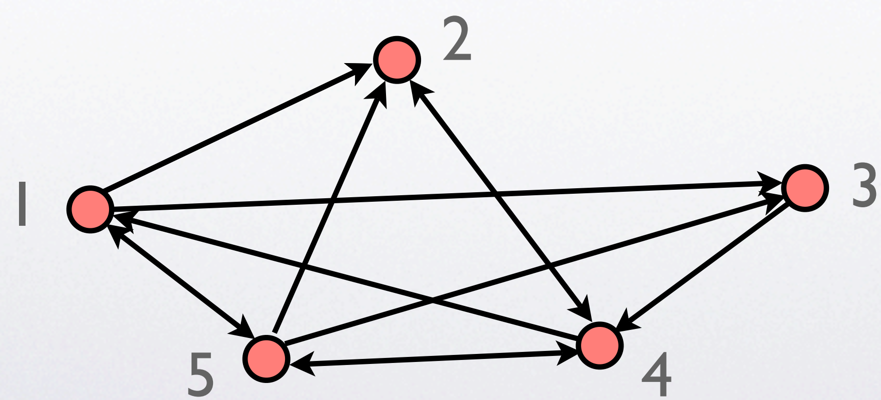

How it works

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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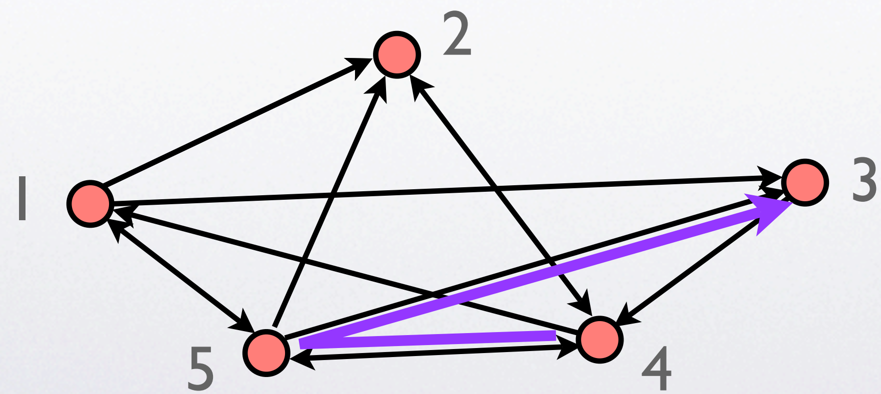

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0	0	0	1	0
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1	1	0	0	1
1	0	1	1	0

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0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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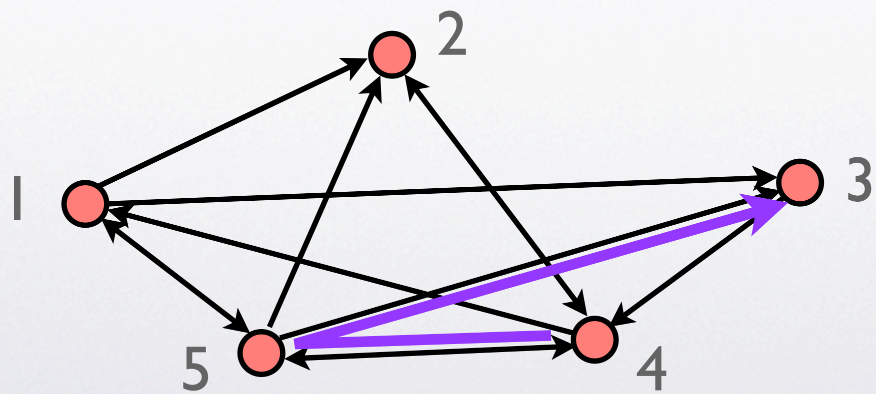

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0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

0	1	1	0	1
0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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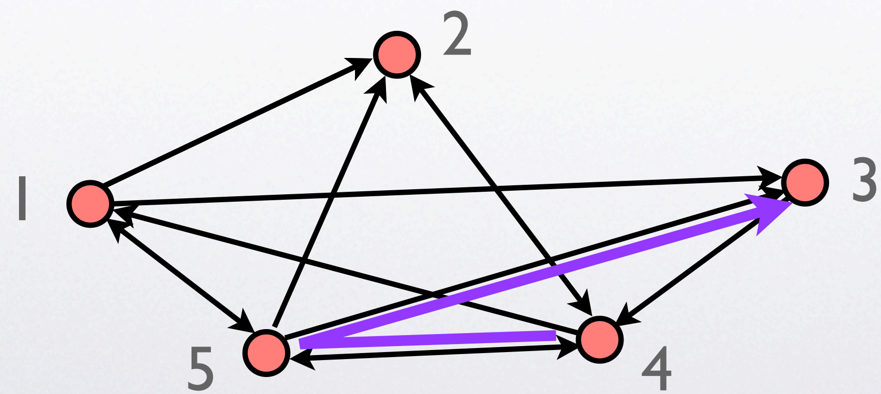
		2		



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0	0	0	1	0
0	0	0	1	0
1	1	0	0	1
1	0	1	1	0

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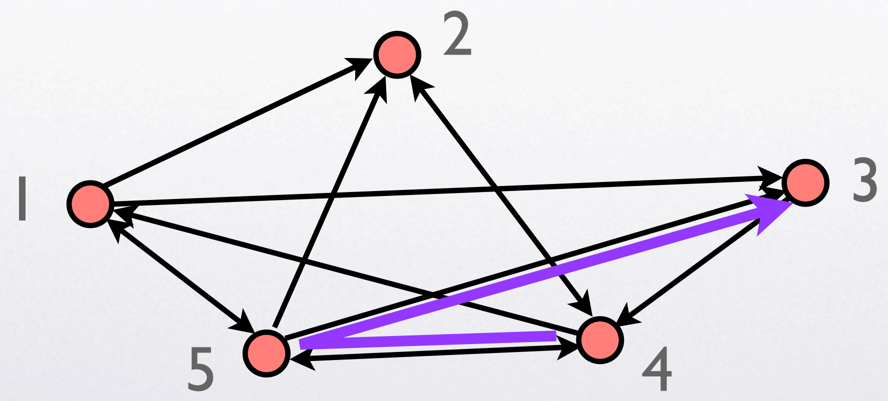
		2		



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0	1	1	0	1
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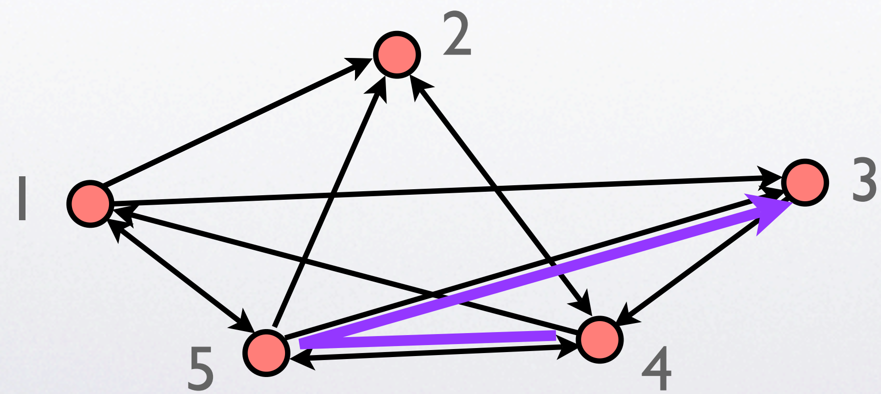
		2		



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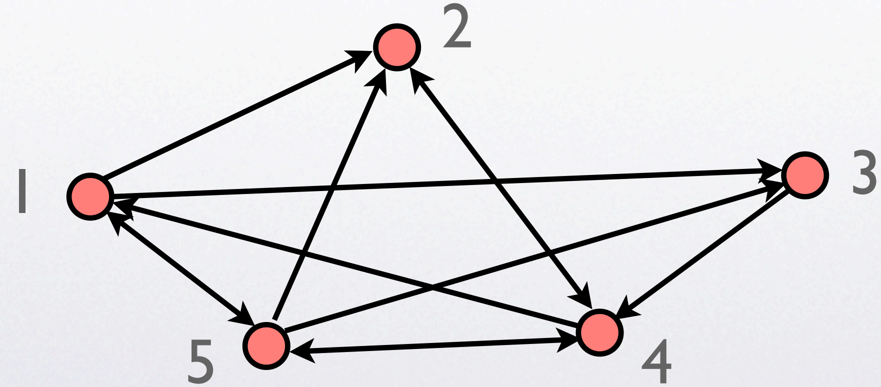

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$$\boldsymbol{v}\sum_{n=0}^{\infty} M^n = \boldsymbol{v}(1 - M)^{-1}$$

Back and Forth

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- This matrix has the same dominant eigenvalue of M, but the separation is at least α

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- Katz–Hubbell's index! It's the spectral ranking of a *perturbed* matrix

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- Functional analysis (resolvent theory) has the answer: it goes to $\boldsymbol{v}(M/\lambda_0)^*$, where X^* denotes Cesàro's limit of X^n
- But $\boldsymbol{v}(M/\lambda_0)^* M/\lambda_0 = \boldsymbol{v}(M/\lambda_0)^*$, so $\boldsymbol{v}(M/\lambda_0)^*$ is a left dominant eigenvector of M. Spectral ranking, again!

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- However, it is always relevant in the damped case

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- The *damped Markovian* spectral ranking of M with border condition $\boldsymbol{v}$ is $(1 - \alpha)\boldsymbol{v}(1 - \alpha S)^{-1}$ [PageRank]

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- Saaty ['70s] uses *right* spectral ranking on a matrix indexed by alternative decisions to identify the best alternatives

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- Bonacich [1987] proposes to extend Katz's index to negative α's
- Kandola *et al.* [2003] propose a *von Neumann kernel* for learning semantic similarity; given an original kernel matrix K, the new kernel is $K(1 - \alpha K)^{-1}$

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- See *Spectral Ranking* [V.] (at `vigna.dsi.unimi.it`)

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- Justify your existence!
- But nobody does :(
- Note: the same happens for the web