

Collaborative Hierarchical Sparse Modeling

Guillermo Sapiro

With P. Sprechmann, I. Ramirez, and Y. Eldar

arXiv 2010



Collaborative source separation



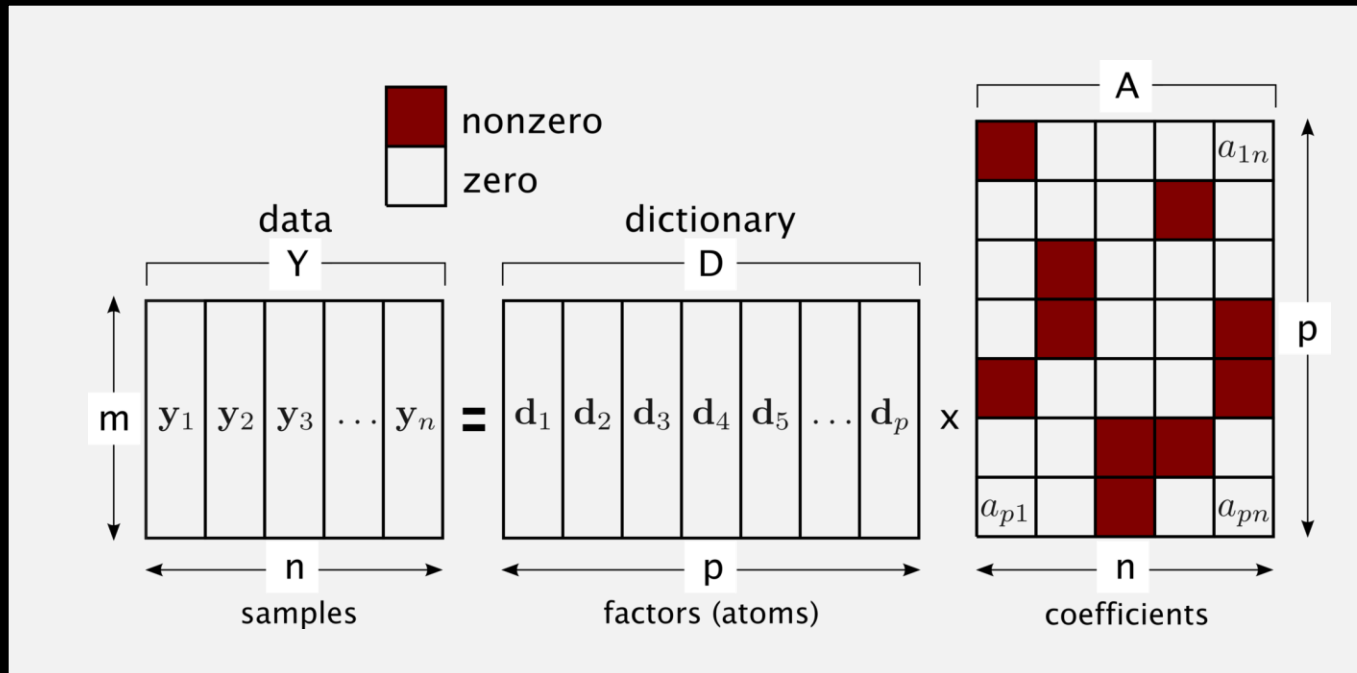
Collaborative source separation



Collaborative source separation



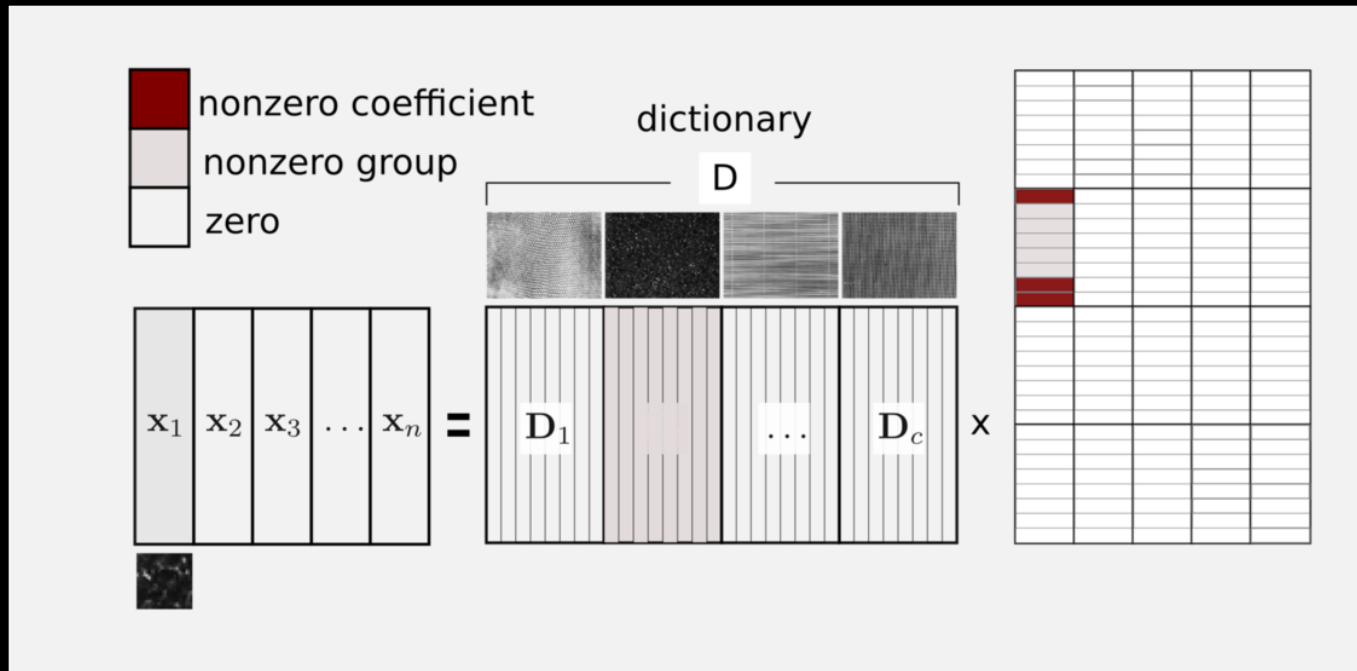
Classical Sparse Coding



$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_1 \|\mathbf{a}\|_1, j = 1, \dots, n$$



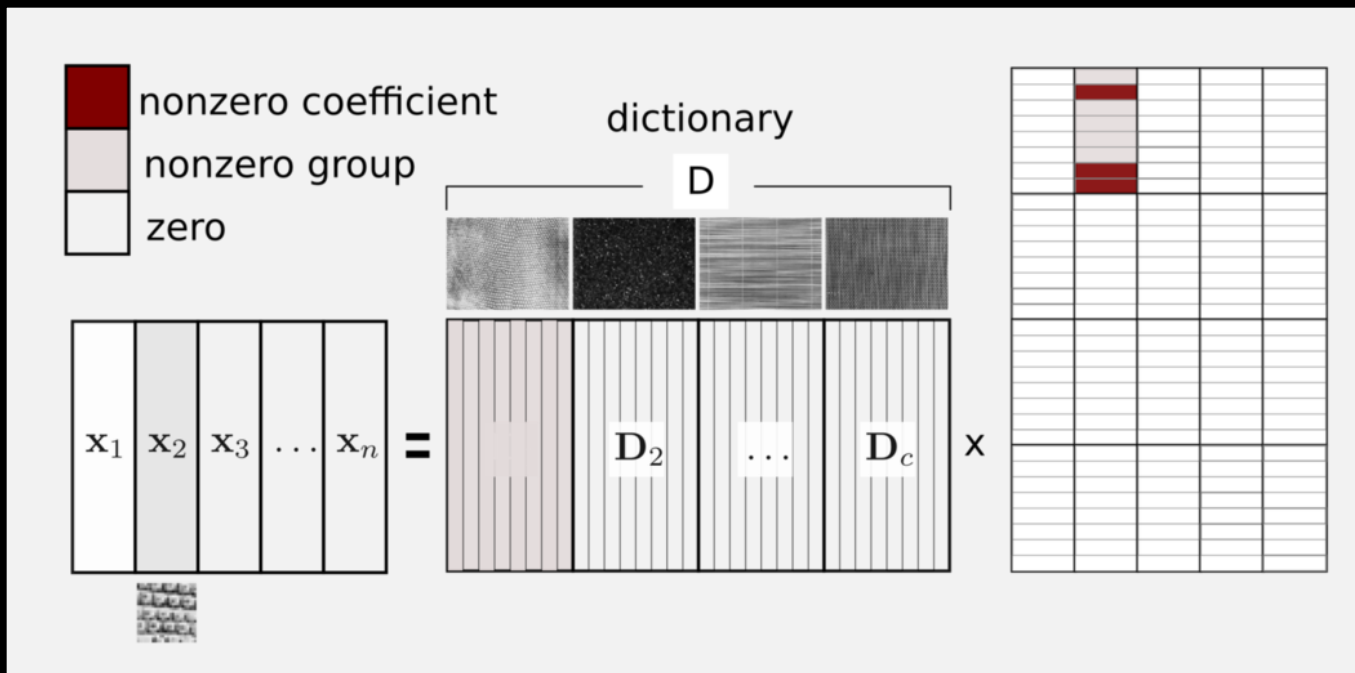
Hierarchical Sparse Model



$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{a}_g\|_2 + \lambda_1 \|\mathbf{a}\|_1$$

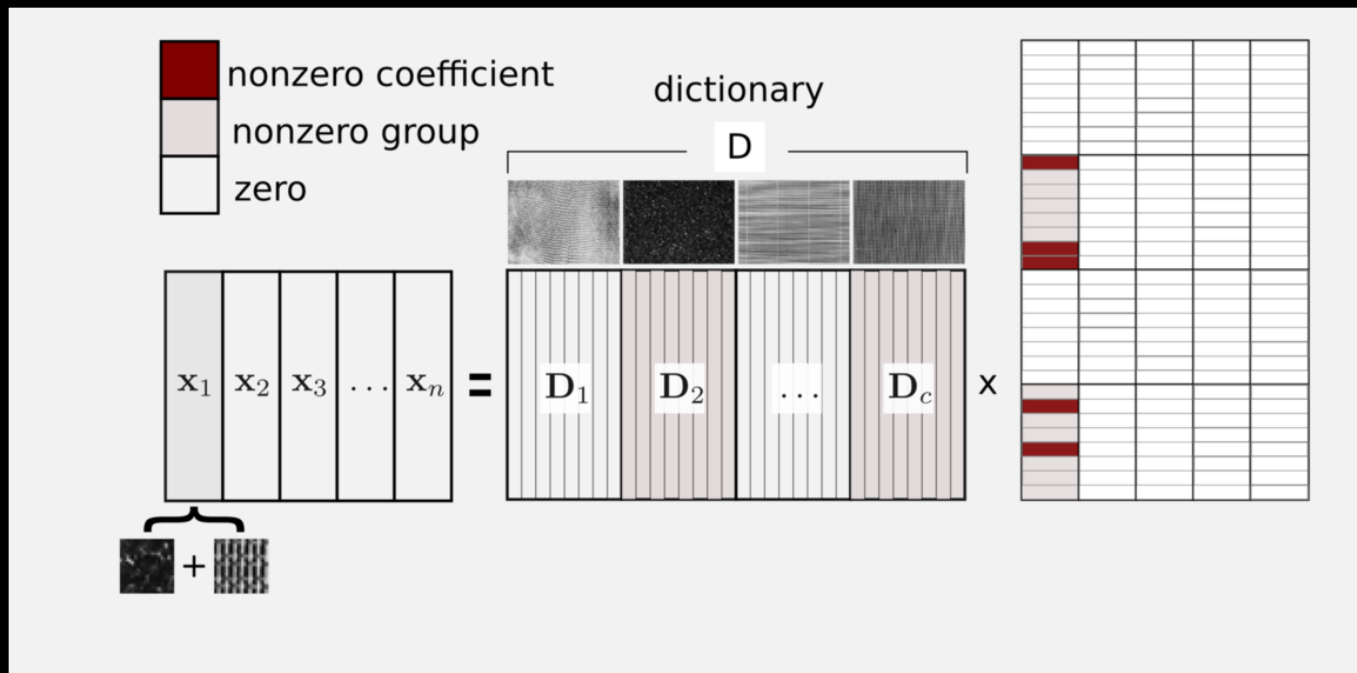
See also Jenatton et al. 2009 , Friedman et al. 2010.

Hierarchical Sparse Model



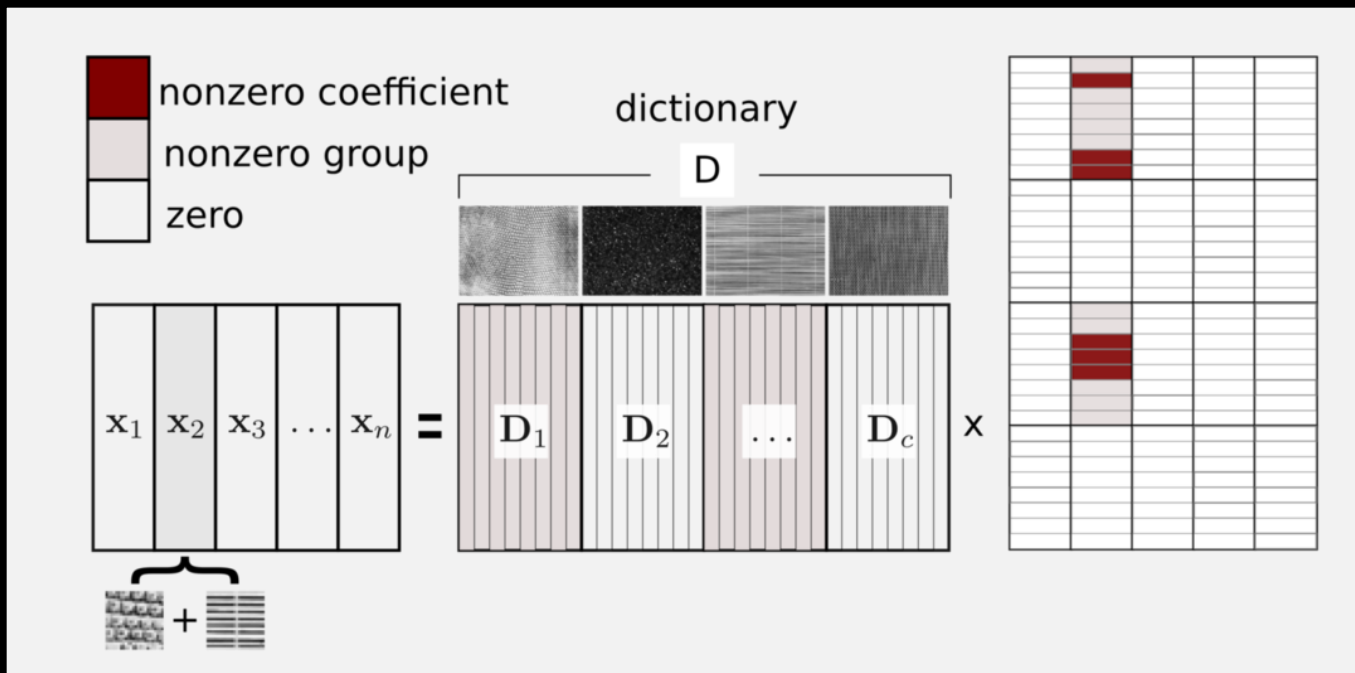
$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{a}_g\|_2 + \lambda_1 \|\mathbf{a}\|_1$$

Hierarchical Sparse Model



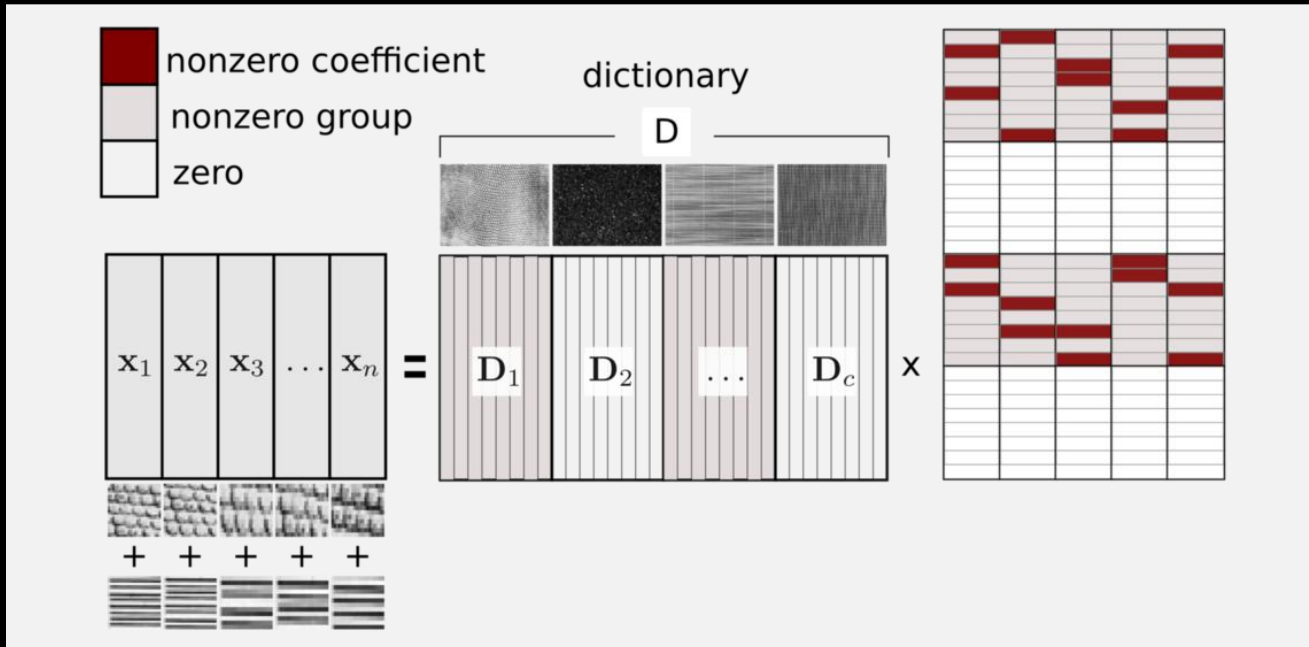
$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{a}_g\|_2 + \lambda_1 \|\mathbf{a}\|_1$$

Hierarchical Sparse Model



$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{a}_g\|_2 + \lambda_1 \|\mathbf{a}\|_1$$

Collaborative Hierarchical Sparse Model



$$\min_{\mathbf{A} \in \mathbb{R}^{p \times n}} \frac{1}{2} \|\mathbf{X} - \mathbf{DA}\|_F^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{A}_g\|_F + \lambda_1 \sum_{j=1, k=1}^{n,p} |a_{kj}|$$

See also Peng et al. 2009

Models Evolution

$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_1 \|\mathbf{a}\|_1, j = 1, \dots, n \quad (\text{Lasso})$$

$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{a}_g\|_2 \quad (\text{Group Lasso})$$

$$\min_{\mathbf{a} \in \mathbb{R}^p} \frac{1}{2} \|\mathbf{x}_j - \mathbf{D}\mathbf{a}\|_2^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{a}_g\|_2 + \lambda_1 \|\mathbf{a}\|_1 \quad (\text{HiLasso})$$

$$\min_{\mathbf{A} \in \mathbb{R}^{p \times n}} \frac{1}{2} \|\mathbf{X} - \mathbf{D}\mathbf{A}\|_F^2 + \lambda_2 \sum_{g=1}^c \|\mathbf{A}_g\|_F + \lambda_1 \sum_{j=1, k=1}^{n, p} |a_{kj}| \quad (\text{C-HiLasso})$$

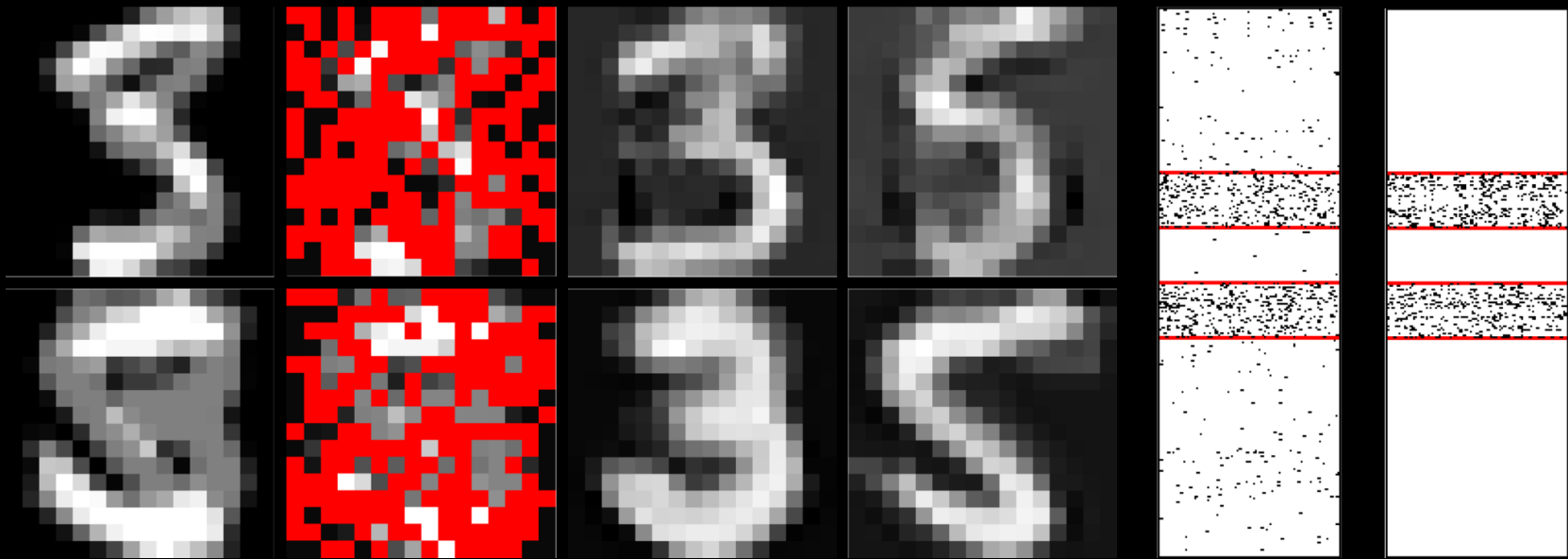
- Optimization

- **SpaRSA** (Wright, Nowak, and Figueiredo)

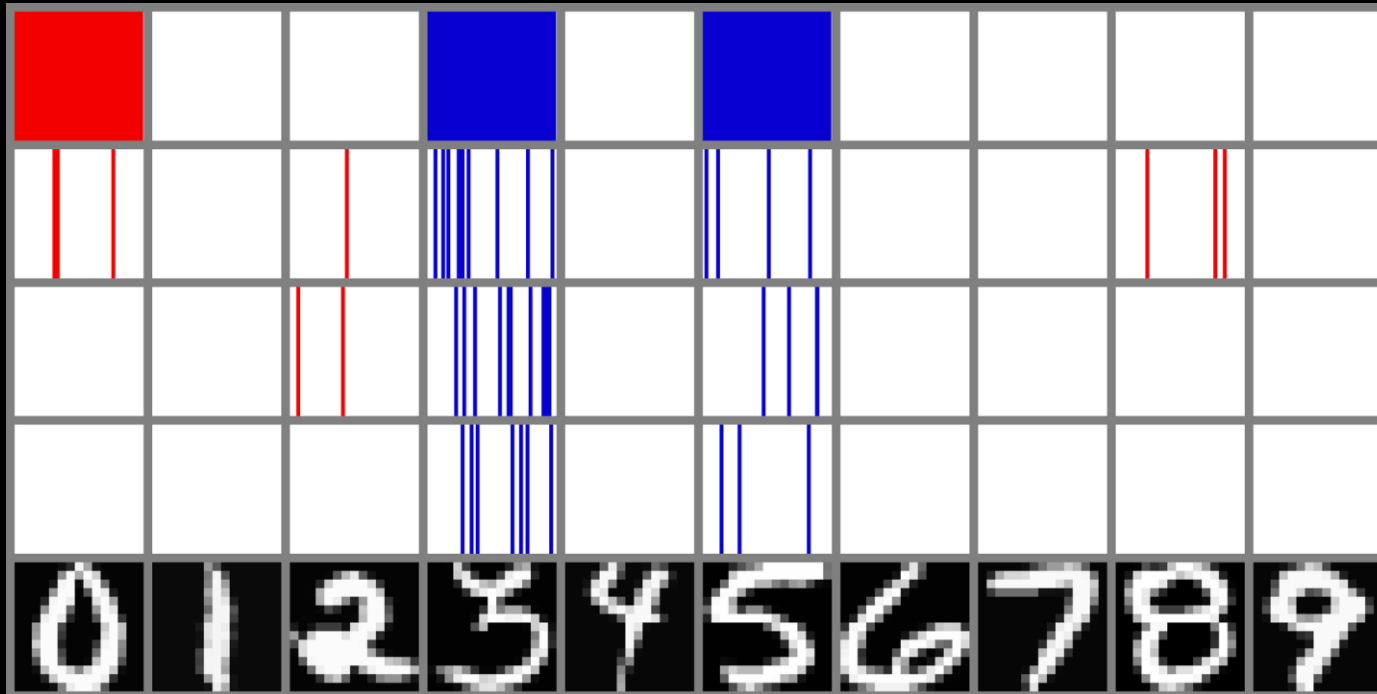
- **ADMOM** (Bertsekas and Tsitsiklis)



Example



Example



Summary C-HiLasso

- New model with new applications
- Efficient optimization
- Theoretical guarantees
- Add dictionary learning
 - Ramirez, Sprechman, Sapiro, CVPR 2010



Thanks

