



Analysis of Latent Relationships in Semantic Graphs using DEDICOM

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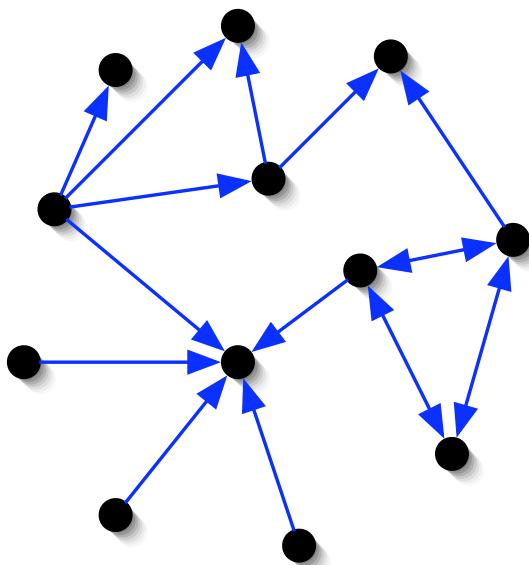
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Workshop for Algorithms on Modern Massive Data Sets

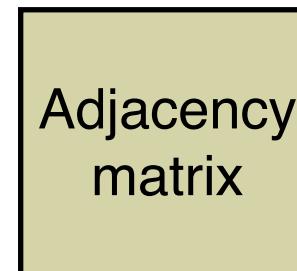
June 24, 2006

Common Graph Analysis Technique



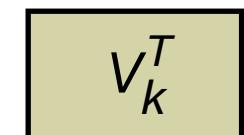
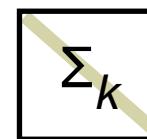
For example:

Web search - HITS (Kleinberg, 1998)



Truncated SVD

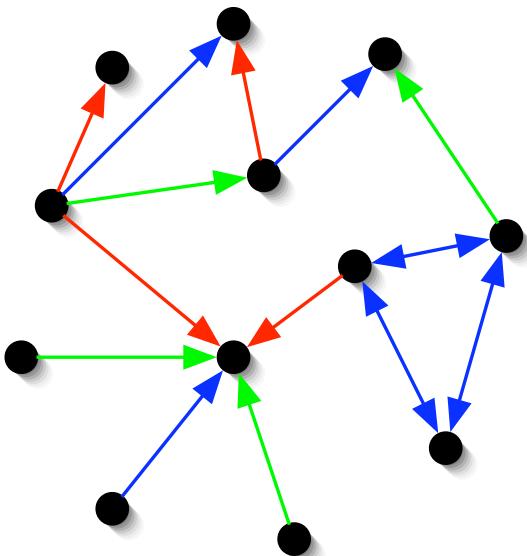
$$A_k = U_k \Sigma_k V_k^T = \sum_{i=1}^k \sigma_i u_i v_i^T$$



Best rank- k matrix filters out noise and captures “latent” information, which improves certain data mining tasks

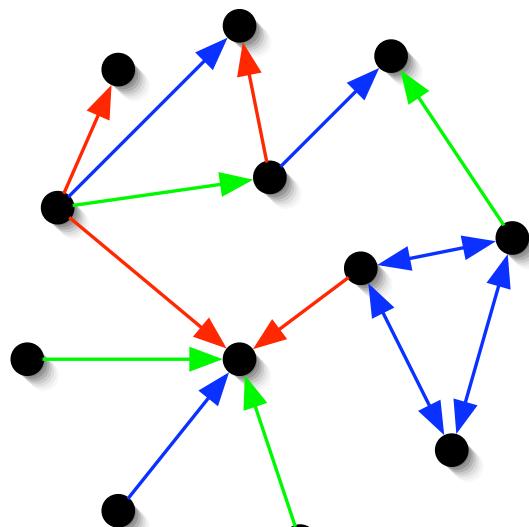
But we may have ignored critical information by not considering edge metadata!

Semantic Graphs

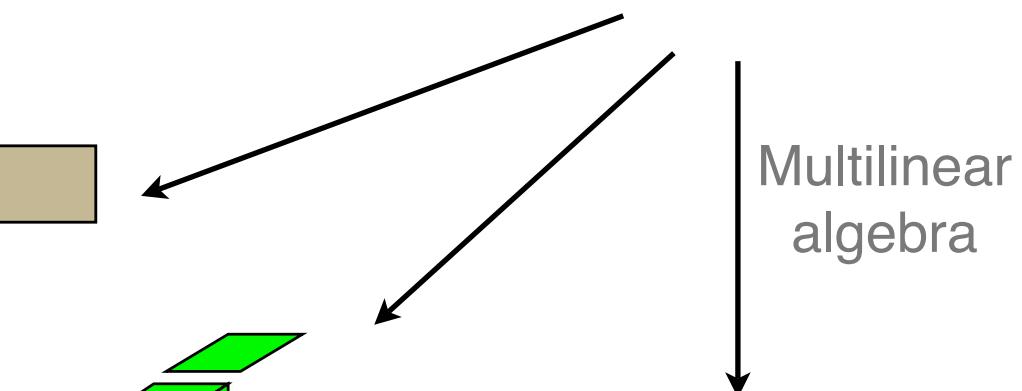
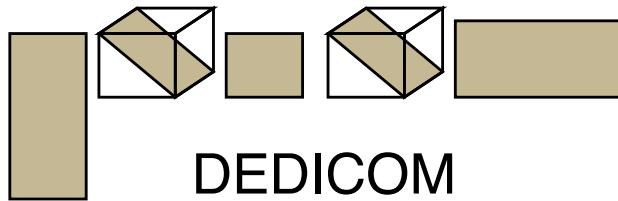
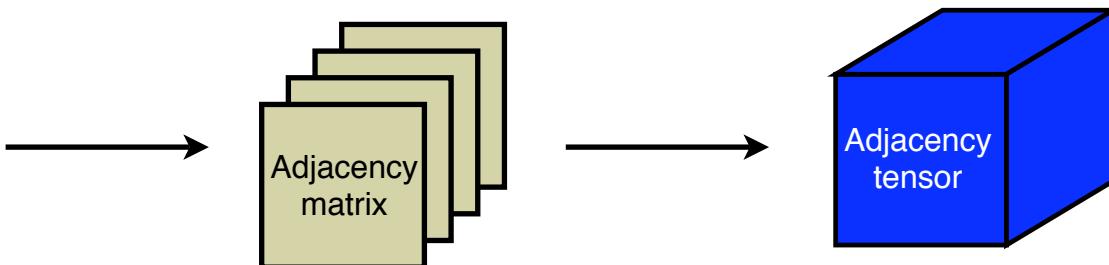


- Different types of edges
- Examples
 - WWW (anchor text)
 - Subway map [thanks Orly!]
 - Email communications (time stamp, to/cc)

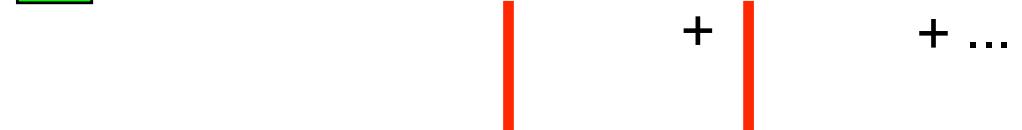
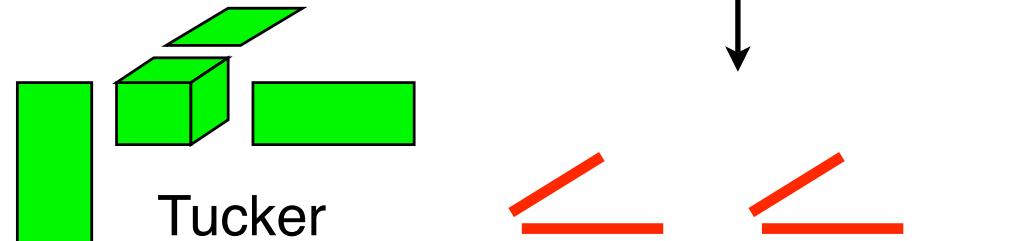
New Paradigm: “Multidimensional Data Mining”



Build an “adjacency tensor” such that there is an adjacency matrix for each edge type.

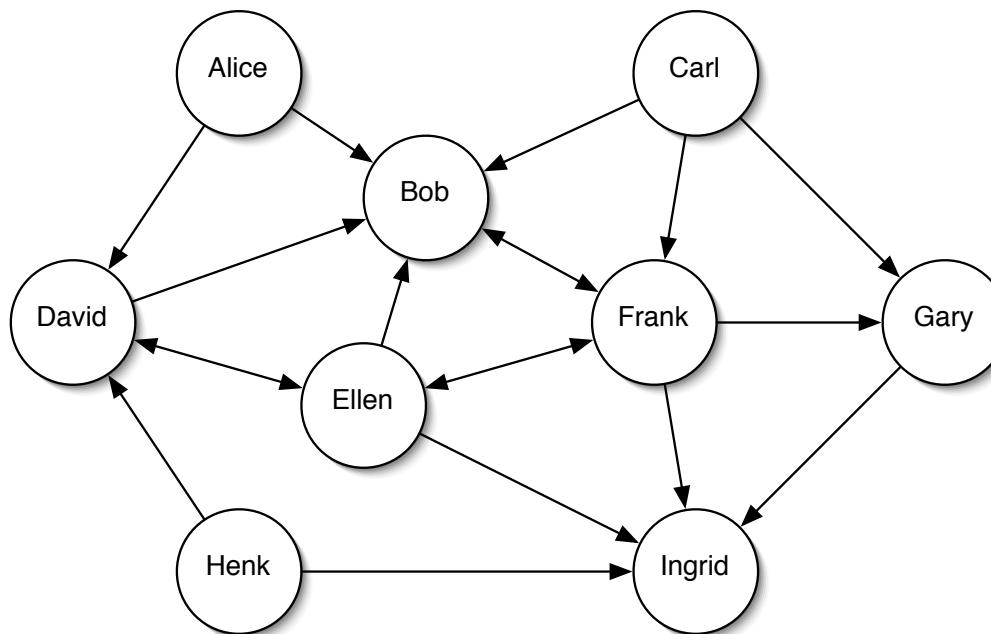
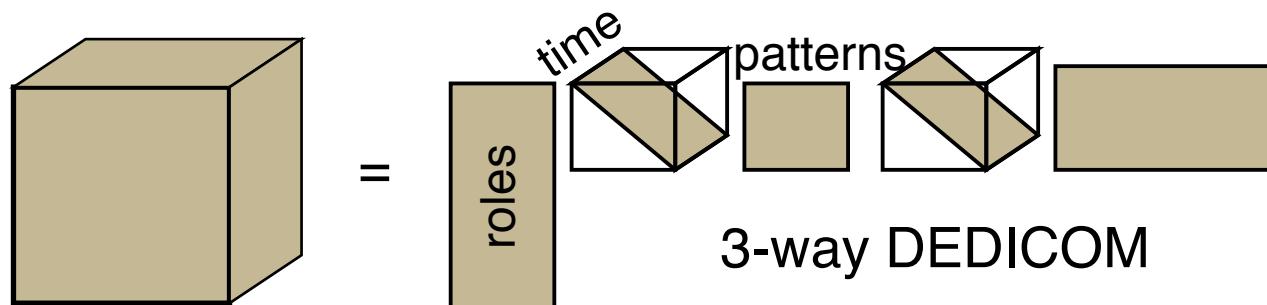


Third dimension offers more explanatory power: uncovers new latent information and reveals subtle relationships



Objective

Use DEDICOM to analyze a semantic graph of email communications changing over time



- DEcomposition into DIrectional COMponents
- Introduced in 1978 by Harshman
- Past applications
 - Study asymmetries in telephone calls among cities
 - Marketing research
 - car switching: car owners and what they buy next
 - free associations of words
 - words to describe hair in advertising shampoo:
“body” evokes “fullness” more often than “fullness”
evokes “body”
 - Asymmetric measures of world trade (import/export)
- Variations
 - Three-way DEDICOM
 - Constrained DEDICOM

DEDICOM Models & Algorithms

$$\mathbf{X} = \mathbf{A} \mathbf{R} \mathbf{A}^T$$

- Generalized Takane method (Takane, 1985; Kiers et al., 1990)
- New algorithm

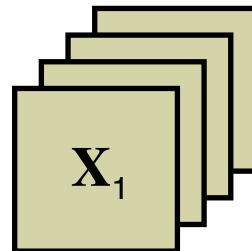
$$\mathbf{X} = \mathbf{A} \mathbf{R} \mathbf{A}^T$$

- Kiers' method (Kiers, 1993)
- New algorithm

All are “alternating” algorithms

Mathematical Notation

- Scalars a
- Vectors \mathbf{a}
- Matrices \mathbf{A}
- Tensors (3-way array) $\mathcal{D} \mathcal{X}$
 - frontal slices of \mathcal{X} : \mathbf{X}_i
- Special symbols
 - Kronecker product



$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & \dots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\mathbf{B} & \dots & a_{mn}\mathbf{B} \end{bmatrix}$$

- Hadamard product (elementwise)

$$\mathbf{A} * \mathbf{B} = \begin{bmatrix} a_{11}b_{11} & \dots & a_{1n}b_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1}b_{m1} & \dots & a_{mn}b_{mn} \end{bmatrix}$$

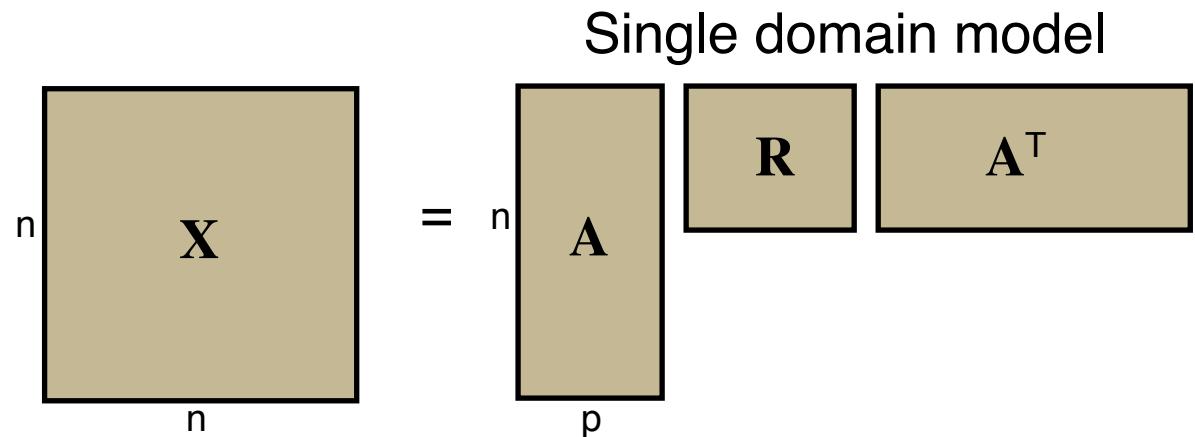
Two-way DEDICOM

$$\mathbf{X} = \mathbf{A}\mathbf{R}\mathbf{A}^T + \mathbf{E}$$

$$\mathbf{X} \approx \mathbf{A}\mathbf{R}\mathbf{A}^T$$

$$\min_{\mathbf{A}, \mathbf{R}} \left\| \mathbf{X} - \mathbf{A}\mathbf{R}\mathbf{A}^T \right\|_F^2$$

s.t. \mathbf{A} orthogonal



- \mathbf{A} ($n \times p$) is an orthogonal matrix of loadings or weights
- \mathbf{R} ($p \times p$) is a dense matrix that captures asymmetric relationships

- Decomposition is not unique
 - \mathbf{A} can be transformed with no loss of fit to the data
 - Nonsingular transformation \mathbf{Q} :

$$\mathbf{A}\mathbf{R}\mathbf{A}^T = (\mathbf{A}\mathbf{Q})(\mathbf{Q}^{-1}\mathbf{R}\mathbf{Q}^{-T})(\mathbf{A}\mathbf{Q})^T$$

- Usually “fix” \mathbf{A} with some standard rotation (e.g., VARIMAX)

New Algorithm

Solving for \mathbf{A} :

Stack data and model “side by side” in a single equation

$$\begin{aligned} (\mathbf{X} \quad \mathbf{X}^T) &= (\mathbf{A}\mathbf{R}\mathbf{A}^T \quad \mathbf{A}\mathbf{R}^T\mathbf{A}^T) \\ &= \mathbf{A} \left((\mathbf{R} \quad \mathbf{R}^T) \begin{pmatrix} \mathbf{A}^T & 0 \\ 0 & \mathbf{A}^T \end{pmatrix} \right) \\ \boxed{\mathbf{Y}} &= \boxed{\mathbf{A}} \quad \boxed{\mathbf{Z}^T} \end{aligned}$$

...and solve least-squares problem: $\min_{\mathbf{A}} \| \mathbf{Y} - \mathbf{A}\mathbf{Z}^T \|_F^2$

$$\mathbf{A}_{new} \leftarrow (\mathbf{X} \quad \mathbf{X}^T) \left((\mathbf{R} \quad \mathbf{R}^T) \begin{pmatrix} \mathbf{A}^T & 0 \\ 0 & \mathbf{A}^T \end{pmatrix} \right)^{\dagger}$$

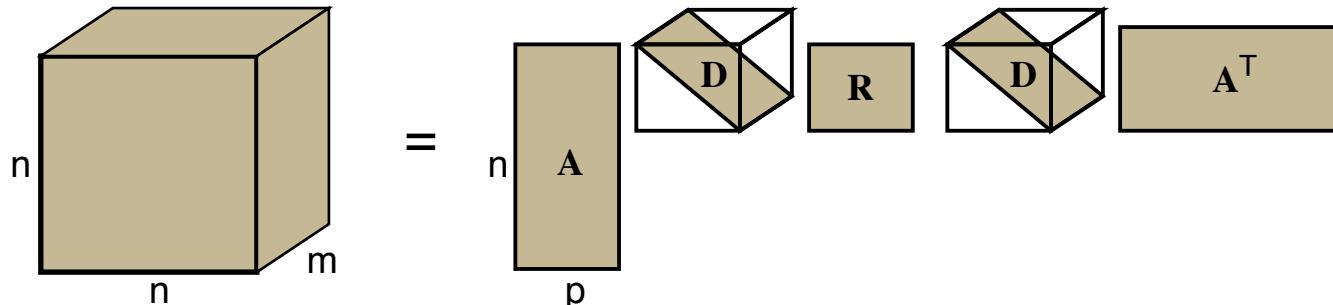
or

$$\mathbf{A}_{new} = (\mathbf{X}\mathbf{A}\mathbf{R}^T + \mathbf{X}^T\mathbf{A}\mathbf{R}) (\mathbf{R}(\mathbf{A}^T\mathbf{A})\mathbf{R}^T + \mathbf{R}^T(\mathbf{A}^T\mathbf{A})\mathbf{R})^{-1}.$$

Solving for \mathbf{R} :

$$\mathbf{R}_{new} = \mathbf{A}^{\dagger} \mathbf{X} (\mathbf{A}^T)^{\dagger}$$

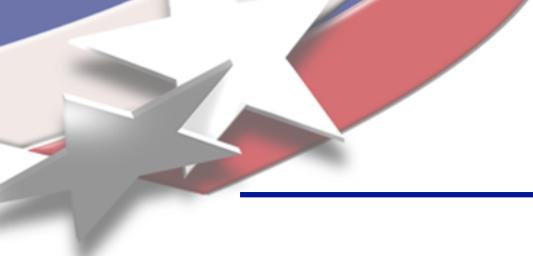
Three-way DEDICOM



$$\mathbf{X}_i = \mathbf{AD}_i\mathbf{RD}_i\mathbf{A}^T + \mathbf{E}_i \quad \text{for } i = 1, \dots, m,$$

$$\min_{\mathbf{A}, \mathbf{R}, \mathcal{D}} \sum_{i=1}^m \| \mathbf{X}_i - \mathbf{AD}_i\mathbf{RD}_i\mathbf{A}^T \|_F^2$$

- \mathbf{A} ($n \times p$) is a matrix of loadings or weights (not necessarily orthogonal)
- \mathbf{R} ($p \times p$) is a dense matrix that captures asymmetric relationships
- \mathbf{D} ($p \times p \times m$) is a tensor with diagonal frontal slices giving the weights of the columns of \mathbf{A} for each slice in third mode
- *Unique* solution with enough slices of \mathbf{X} with sufficient variation
 - i.e., no rotation of \mathbf{A} possible
 - greater confidence in interpretation of results



New Algorithm - Updating A

$$\min_{\mathbf{A}, \mathbf{R}, \mathcal{D}} \sum_{i=1}^m \| \mathbf{X}_i - \mathbf{A} \mathbf{D}_i \mathbf{R} \mathbf{D}_i \mathbf{A}^T \|_F^2$$

Solving for \mathbf{A} :

$$(\mathbf{X}_1 \quad \mathbf{X}_1^T \quad \cdots \quad \mathbf{X}_m \quad \mathbf{X}_m^T) = \mathbf{A} (\mathbf{D}_1 \mathbf{R} \mathbf{D}_1 \quad \mathbf{D}_1 \mathbf{R}^T \mathbf{D}_1 \quad \cdots \quad \mathbf{D}_m \mathbf{R} \mathbf{D}_m \quad \mathbf{D}_m \mathbf{R}^T \mathbf{D}_m) (\mathbf{I}_{2m} \otimes \mathbf{A}^T)$$

$$\boxed{\mathbf{Y}} = \boxed{\mathbf{A}} \boxed{\mathbf{Z}^T}$$

$$\mathbf{A} = \mathbf{Y} \mathbf{Z} (\mathbf{Z}^T \mathbf{Z})^{-1}$$

$$\mathbf{A} = \left[\sum_{i=1}^m (\mathbf{X}_i \mathbf{A} \mathbf{D}_i \mathbf{R}^T \mathbf{D}_i + \mathbf{X}_i^T \mathbf{A} \mathbf{D}_i \mathbf{R} \mathbf{D}_i) \right] \left[\sum_{i=1}^m (\mathbf{B}_i + \mathbf{C}_i) \right]^{-1}$$

where $\mathbf{B}_i \equiv \mathbf{D}_i \mathbf{R} \mathbf{D}_i (\mathbf{A}^T \mathbf{A}) \mathbf{D}_i \mathbf{R}^T \mathbf{D}_i$,
 $\mathbf{C}_i \equiv \mathbf{D}_i \mathbf{R}^T \mathbf{D}_i (\mathbf{A}^T \mathbf{A}) \mathbf{D}_i \mathbf{R} \mathbf{D}_i$.



New Algorithm - Updating D

$$\min_{\mathbf{D}_i} \left\| \mathbf{X}_i - \mathbf{A} \mathbf{D}_i \mathbf{R} \mathbf{D}_i \mathbf{A}^T \right\|_F^2$$

Solving for D:

Use Newton's method to solve the optimization problem for $d = \text{diag}(\mathbf{D}_i)$

$$d_{new} = d - H^{-1}g$$

Gradient: $g_k = - \sum_{i,j} \left[2(\mathbf{X} - \mathbf{A} \mathbf{D} \mathbf{R} \mathbf{D} \mathbf{A}^T) * (\mathbf{A} \mathbf{D} \mathbf{r}_k \mathbf{a}_k^T + \mathbf{a}_k \mathbf{r}_{k,:} \mathbf{D} \mathbf{A}^T) \right]_{i,j}$

Hessian: $h_{st} = -2 \sum_{i,j} \left[(\mathbf{X} - \mathbf{A} \mathbf{D} \mathbf{R} \mathbf{D} \mathbf{A}^T) * (\mathbf{a}_s r_{st} \mathbf{a}_t^T + \mathbf{a}_t r_{ts} \mathbf{a}_s^T) \right. \\ \left. - (\mathbf{A} \mathbf{D} \mathbf{r}_s \mathbf{a}_s^T + \mathbf{a}_s \mathbf{r}_{s,:} \mathbf{D} \mathbf{A}^T) * (\mathbf{A} \mathbf{D} \mathbf{r}_t \mathbf{a}_t^T + \mathbf{a}_t \mathbf{r}_{t,:} \mathbf{D} \mathbf{A}^T) \right]_{i,j}$

Use compression

QR factorization: $\mathbf{A} = \mathbf{Q} \tilde{\mathbf{A}}$,

$$\min_{\mathbf{D}_i} \left\| \mathbf{Q}^T \mathbf{X}_i \mathbf{Q} - \tilde{\mathbf{A}} \mathbf{D}_i \mathbf{R} \mathbf{D}_i \tilde{\mathbf{A}}^T \right\|_F^2$$

Smaller problem ($p \times p$)



Our Algorithm - Updating R

$$\min_{\mathbf{R}} \sum_{i=1}^m \| \mathbf{X}_i - \mathbf{A} \mathbf{D}_i \mathbf{R} \mathbf{D}_i \mathbf{A}^T \|_F^2$$

Solving for \mathbf{R} :

Use the approach in (Kiers, 1993)

$$\text{minimize: } f(\mathbf{R}) = \left\| \begin{pmatrix} \text{Vec}(\mathbf{X}_1) \\ \vdots \\ \text{Vec}(\mathbf{X}_m) \end{pmatrix} - \begin{pmatrix} \mathbf{A} \mathbf{D}_1 \otimes \mathbf{A} \mathbf{D}_1 \\ \vdots \\ \mathbf{A} \mathbf{D}_m \otimes \mathbf{A} \mathbf{D}_m \end{pmatrix} \text{Vec}(\mathbf{R}) \right\|$$

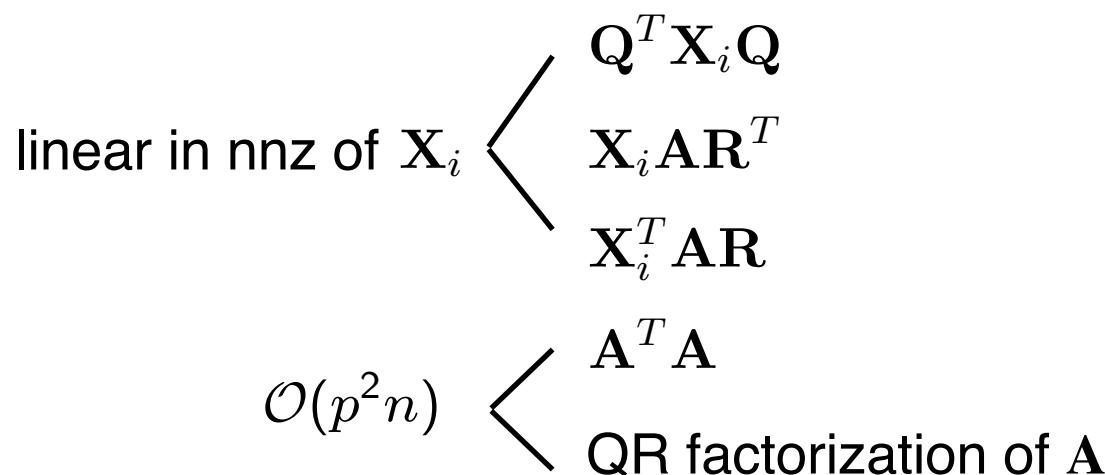
$$\text{Vec}(\mathbf{R}) = \left(\sum_{i=1}^m (\mathbf{D}_i \mathbf{A}^T \mathbf{A} \mathbf{D}_i) \otimes (\mathbf{D}_i \mathbf{A}^T \mathbf{A} \mathbf{D}_i) \right)^{-1} \sum_{i=1}^m \text{Vec}(\mathbf{D}_i \mathbf{A}^T \mathbf{X}_i \mathbf{A} \mathbf{D}_i)$$



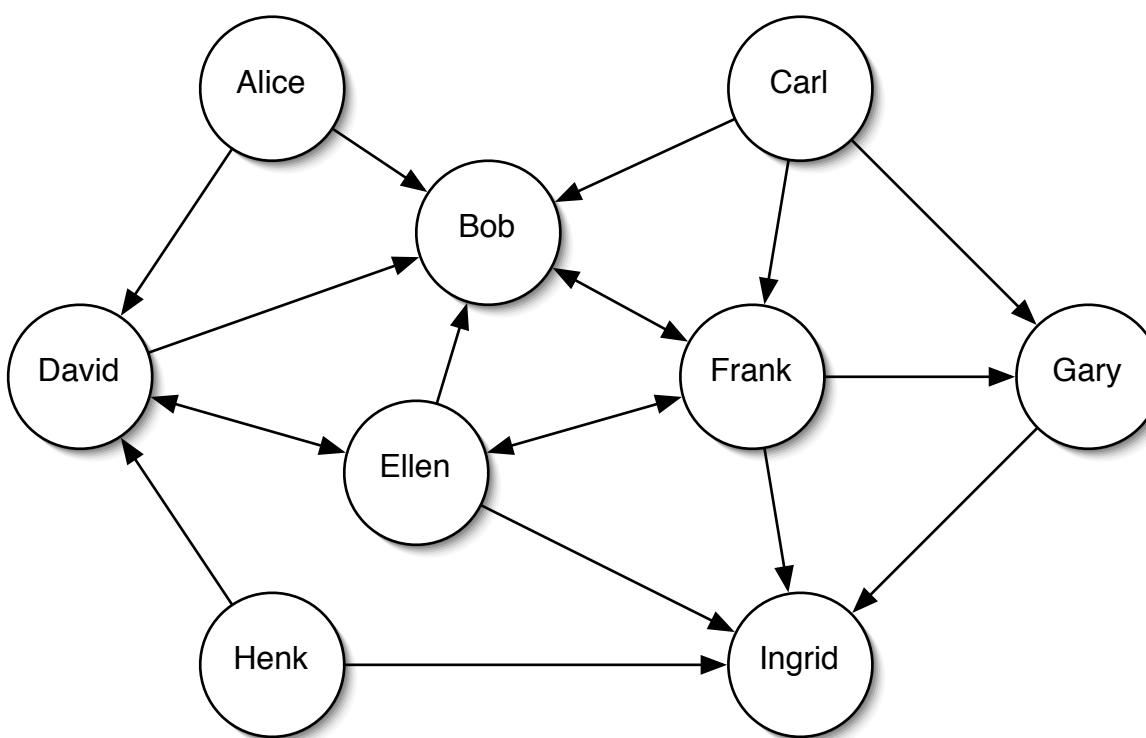
Algorithm Costs

Updating \mathbf{A} is most expensive part

Dominant costs:

$$\begin{array}{c} \mathbf{Q}^T \mathbf{X}_i \mathbf{Q} \\ \text{linear in nnz of } \mathbf{X}_i \\ \mathbf{X}_i \mathbf{A} \mathbf{R}^T \\ \mathbf{X}_i^T \mathbf{A} \mathbf{R} \\ \mathcal{O}(p^2 n) \\ \mathbf{A}^T \mathbf{A} \\ \text{QR factorization of } \mathbf{A} \end{array}$$


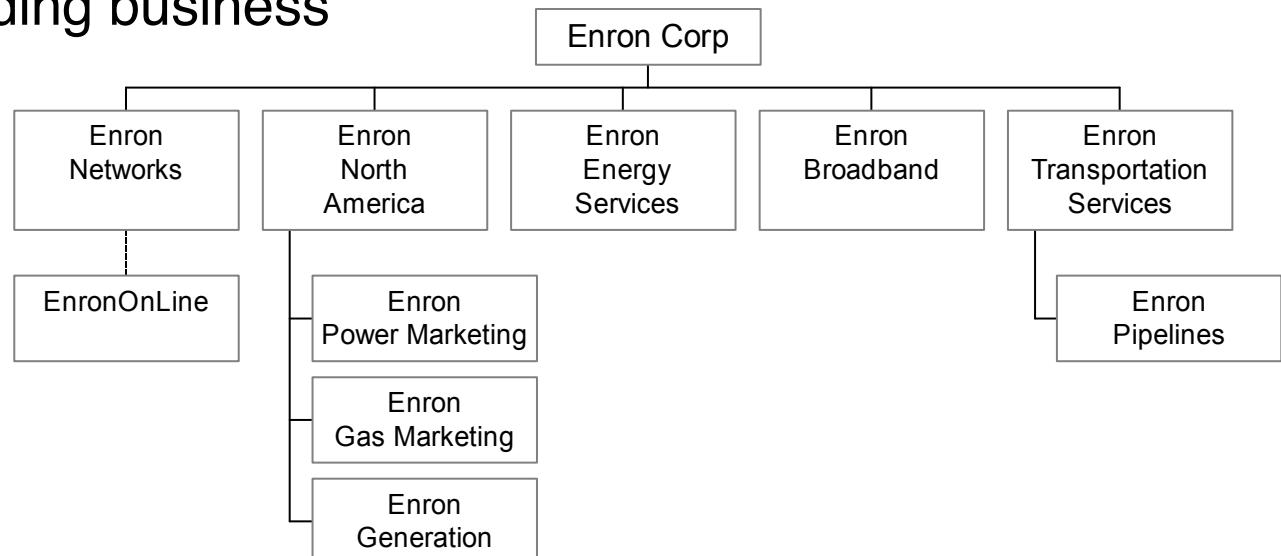
Application: Enron Email Analysis



- Links consist of email communications
- What can we learn about this network strictly from their communication patterns? (Social network analysis)

Enron Corp.

- U.S. corporation involved with creating energy markets
 - 7th largest by revenue
- EnronOnline: e-trading business
 - natural gas
 - electric power



- Investigations
 - U.S. Federal Energy Regulatory Commission (FERC)
 - energy market manipulation
 - involved energy traders
 - U.S. Securities and Exchange Commission (SEC)
 - accounting fraud
 - insider trading

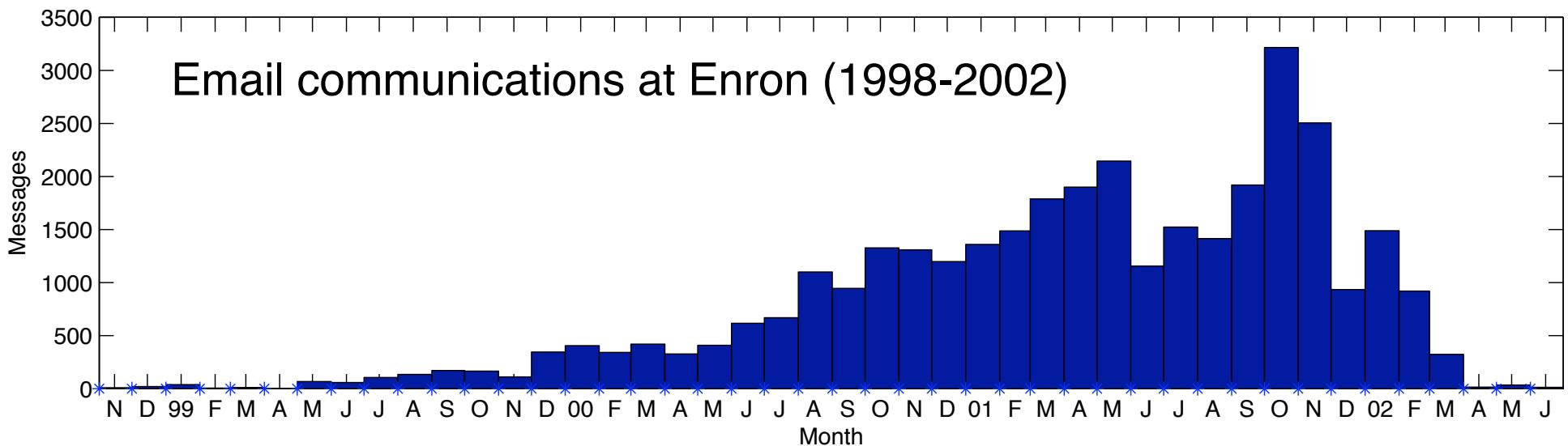


Enron Email Data

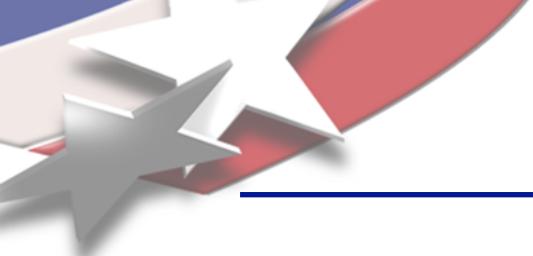
- FERC collected email of ~150 employees as evidence
 - Included emails saved in inbox, sent items, deleted items, and all other folders
- Released to the public in 2002 by FERC as part of their investigation
 - To/from, date, subject, body
 - Attachments and some names/emails removed
 - Approx. 500,000 email messages

Smaller Enron Data Set

We used a smaller data set prepared by Priebe et al.
34,427 emails among 184 employees over 44 months



- Limited information on the 184 employees
- No org chart

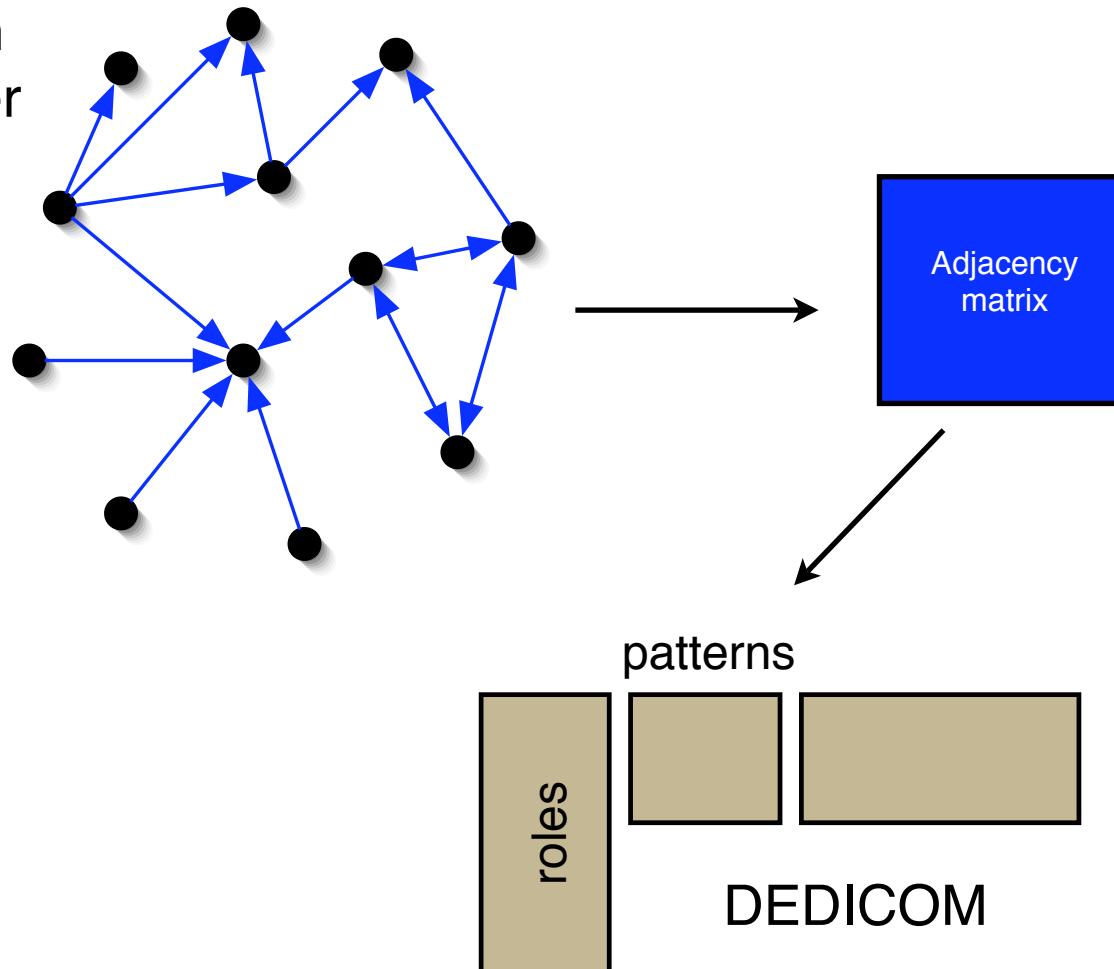


DEDICOM Experiment

- Aggregate communications
 - Sparse matrix of size 184 x 184 (3007 nonzeros)
- Time series of communication graphs
 - Sparse tensor of size 184 x 184 x 44 (9838 nonzeros)
- Weighted adjacency matrix
 - scaling: x number of messages scaled by $\log(x)+1$
 - other common choices give similar results
- Models:
 - SVD
 - 2-way DEDICOM
 - 3-way DEDICOM

Social Network Analysis

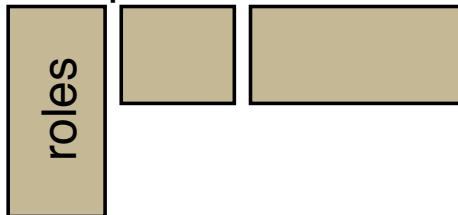
Communication graph
among employees over
all times



- Description of employees by their roles
- Aggregate communication patterns among roles

DEDICOM Results

patterns



Executives
Legal
Pipeline

Identify shared
characteristics to
label group

Execs

EMPLOYEE	DEDICOM Solution		
	1	2	3
J. Lavorato - CEO, Enron America	0.41	0.07	0.04
L. Kitchen - President, Enron Online	0.26	0.21	0.04
M. Grigsby - Director, West Desk Gas Trading	0.22	-0.01	-0.01
D. Delainey - CEO, ENA and Enron Energy Services	0.20	0.06	0.06
G. Whalley - President,	0.17	0.05	0.04
L. Taylor - Executive Assistant to Greg Whalley,	0.17	0.06	0.03
T. Jones - Employee, Financial Trading Group (ENA Legal)	-0.12	0.38	-0.02
M. Taylor - Manager, Financial Trading Group ENA Legal	-0.10	0.35	-0.01
S. Shackleton - Employee, ENA Legal	-0.13	0.31	-0.02
S. Panus - Senior Legal Specialist, ENA Legal	-0.11	0.26	-0.02
M. Heard - Senior Legal Specialist, ENA Legal	-0.10	0.24	-0.02
E. Sager - VP and Asst Legal Counsel, ENA Legal	-0.01	0.24	0.02
S. Corman - VP, Regulatory Affairs	-0.04	-0.01	0.33
K. Watson - Employee, Transwestern Pipeline Company (ETS)	-0.08	-0.03	0.32
L. Donoho - Employee, Transwestern Pipeline Company (ETS)	-0.08	-0.03	0.30
D. Fossum - VP, Transwestern Pipeline Company (ETS)?	-0.06	-0.00	0.30
M. Lokay - Admin. Asst., Transwestern Pipeline Company (ETS)	-0.07	-0.02	0.28
K. Hyatt - Director, Asset Development TW Pipeline Co. (ETS)	-0.06	-0.02	0.25
R. Hayslett - VP, Also CFO and Treasurer	-0.04	-0.01	0.23
R matrix	70.3	11.6	6.7
	15.4	68.2	5.0
	9.9	6.7	59.5

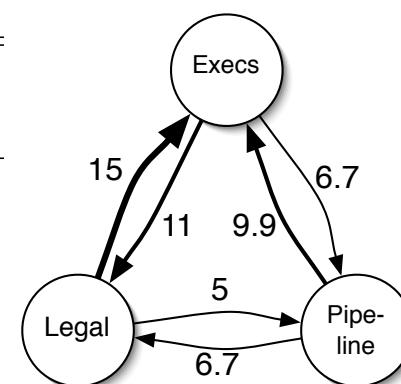
Executives

Legal

Pipeline
employees

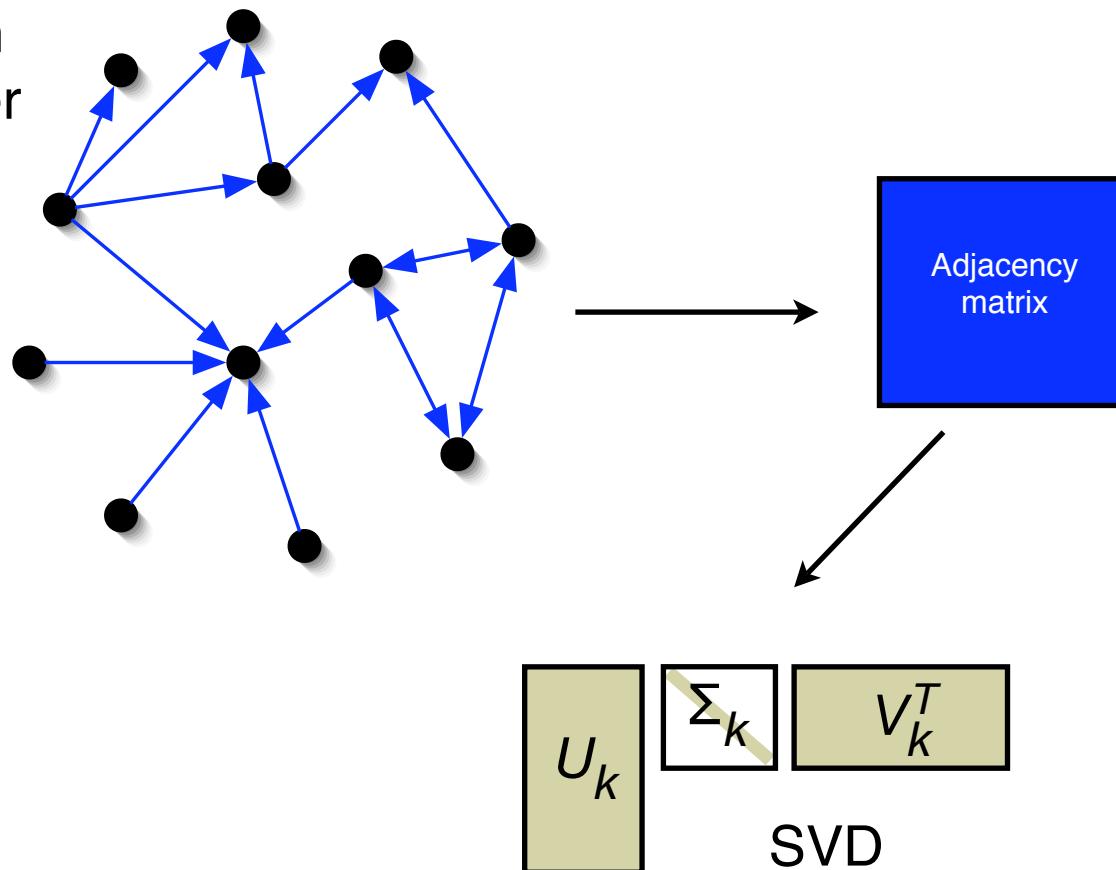
Some employees have dual roles

Pattern of communications in R matrix

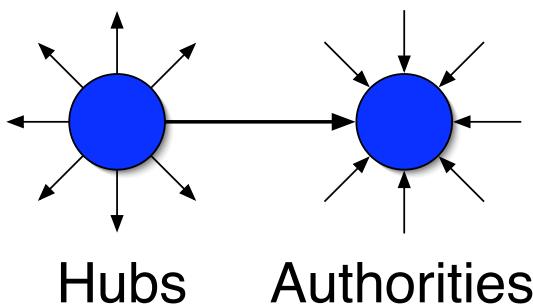


Social Network Analysis

Communication graph
among employees over
all times

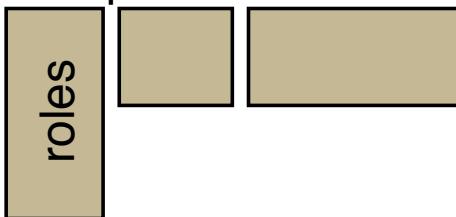


- “Hubs” and “authorities” for different roles

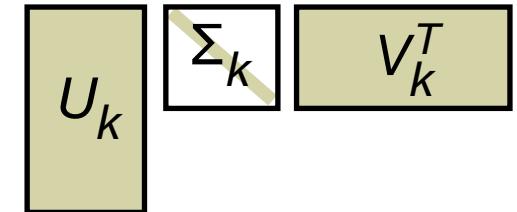


DEDICOM & SVD Results

patterns



Executives
Legal
Pipeline



		DEDICOM Solution			U (hubs)			V (authorities)		
	EMPLOYEE	1	2	3	1	2	3	1	2	3
Execs	J. Lavorato - CEO, Enron America	0.41	0.07	0.04	0.30	-0.07	-0.21	0.31	-0.09	-0.07
	L. Kitchen - President, Enron Online	0.26	0.21	0.04	0.31	0.07	-0.05	0.29	0.02	0.04
	M. Grigsby - Director, West Desk Gas Trading	0.22	-0.01	-0.01	0.16	-0.09	-0.33	0.14	-0.06	-0.20
	D. Delainey - CEO, ENA and Enron Energy Services	0.20	0.06	0.06	0.20	-0.05	-0.00	0.20	-0.05	0.03
	G. Whalley - President,	0.17	0.05	0.04	0.08	-0.02	-0.02	0.24	-0.07	0.02
	L. Taylor - Executive Assistant to Greg Whalley,	0.17	0.06	0.03	0.24	-0.05	-0.08	0.09	-0.01	-0.02
Legal	T. Jones - Employee, Financial Trading Group (ENA Legal)	-0.12	0.38	-0.02	0.17	0.36	0.13	0.10	0.24	0.10
	M. Taylor - Manager, Financial Trading Group ENA Legal	-0.10	0.35	-0.01	0.13	0.27	0.13	0.13	0.26	0.12
	S. Shackleton - Employee, ENA Legal	-0.13	0.31	-0.02	0.08	0.26	0.10	0.08	0.26	0.10
	S. Panus - Senior Legal Specialist, ENA Legal	-0.11	0.26	-0.02	0.09	0.27	0.10	0.05	0.20	0.08
	M. Heard - Senior Legal Specialist, ENA Legal	-0.10	0.24	-0.02	0.06	0.20	0.09	0.08	0.22	0.09
	E. Sager - VP and Asst Legal Counsel, ENA Legal	-0.01	0.24	0.02	0.12	0.13	0.10	0.15	0.21	0.12
Pipeline	S. Corman - VP, Regulatory Affairs	-0.04	-0.01	0.33	0.08	-0.18	0.22	0.07	-0.18	0.21
	K. Watson - Employee, Transwestern Pipeline Company (ETS)	-0.08	-0.03	0.32	0.03	-0.16	0.19	0.04	-0.18	0.22
	L. Donoho - Employee, Transwestern Pipeline Company (ETS)	-0.08	-0.03	0.30	0.03	-0.16	0.18	0.03	-0.17	0.20
	D. Fossum - VP, Transwestern Pipeline Company (ETS)?	-0.06	-0.00	0.30	0.07	-0.18	0.23	0.05	-0.13	0.16
	M. Lokay - Admin. Asst., Transwestern Pipeline Company (ETS)	-0.07	-0.02	0.28	0.03	-0.14	0.17	0.04	-0.17	0.20
	K. Hyatt - Director, Asset Development TW Pipeline Co. (ETS)	-0.06	-0.02	0.25	0.03	-0.13	0.17	0.04	-0.14	0.17
		R matrix / singular values	70.3	11.6	6.7	86.3		86.3		
			15.4	68.2	5.0		54.1		54.1	
			9.9	6.7	59.5		52.6			52.6

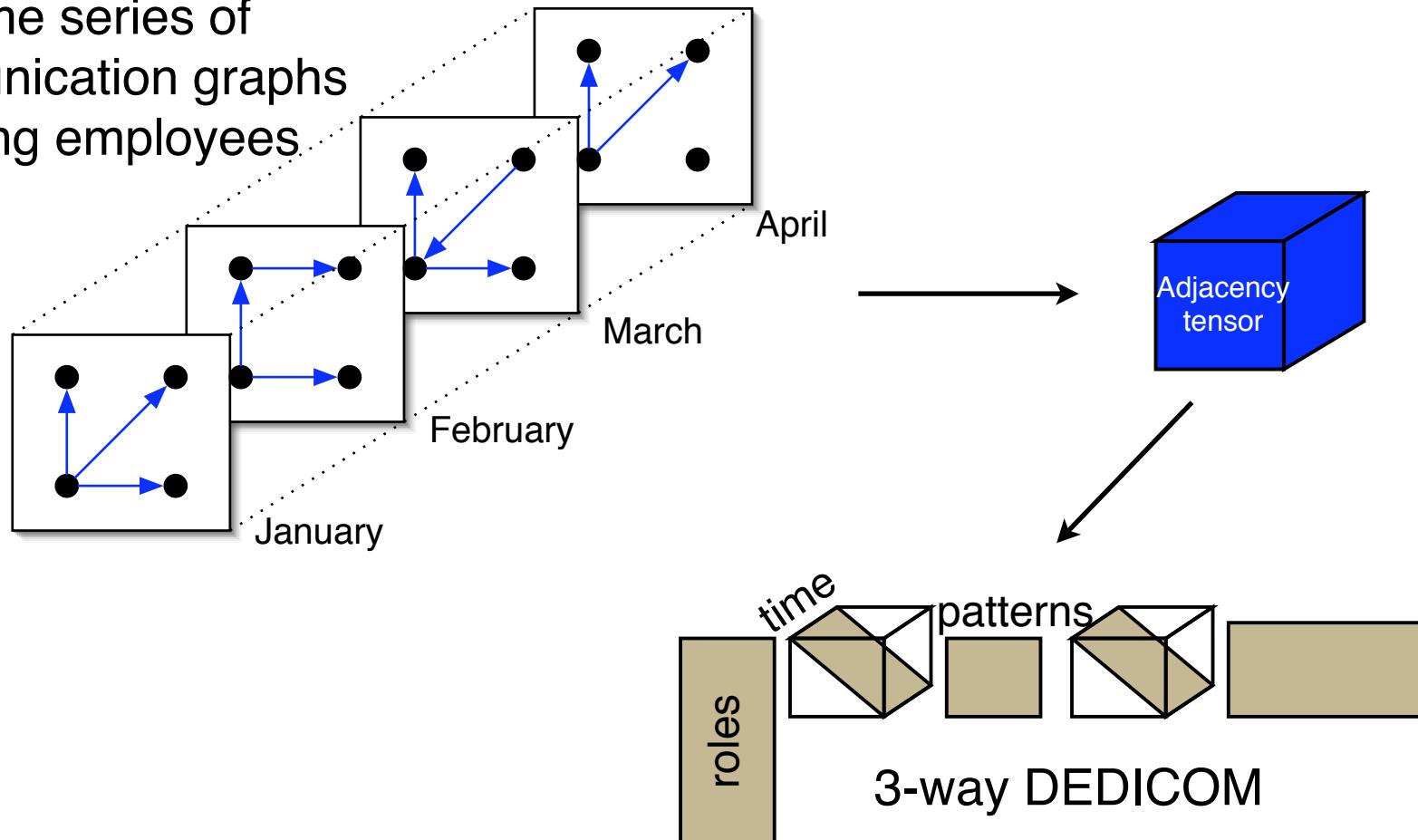
SVD: Hubs and Authorities in U and V

Roles more difficult to identify in singular vectors

No patterns of communication

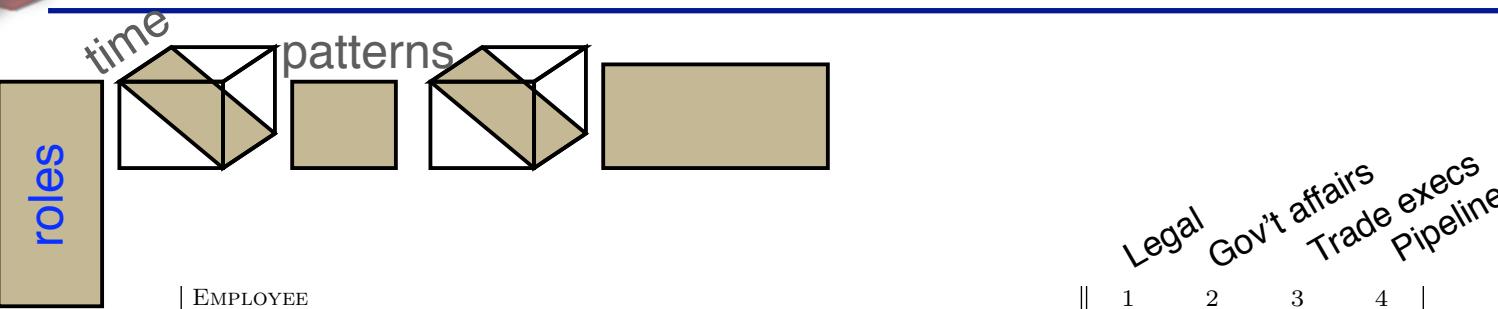
Temporal Social Network Analysis

Time series of communication graphs among employees



- Unique description of employees by their roles
- Aggregate communication patterns among roles
- Behavior over time

Roles of Employees



Legal

Gov't affairs

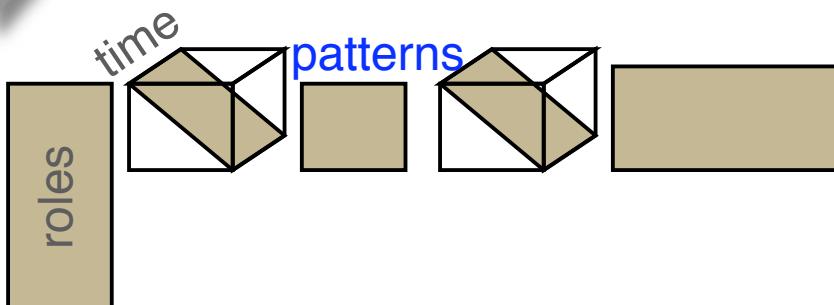
Execs - trading

Pipeline employees

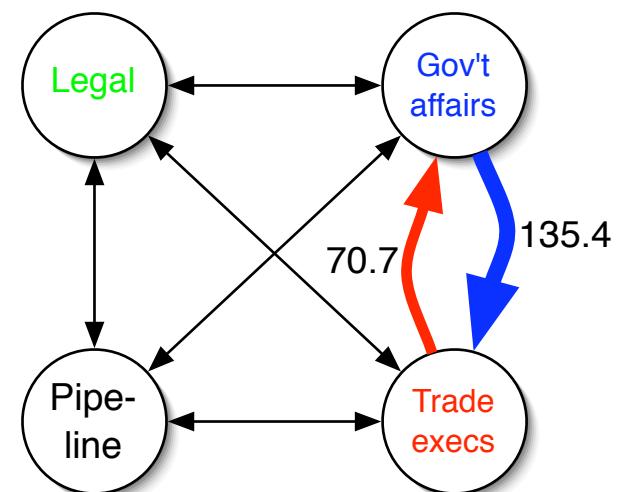
Identify shared characteristics to label group

EMPLOYEE	1	2	3	4
T. Jones - Employee, Financial Trading Group (ENA Legal)	0.64	-0.01	0.02	-0.00
S. Shackleton - Employee, ENA Legal	0.45	-0.00	-0.01	-0.00
M. Taylor - Manager, Financial Trading Group ENA Legal	0.37	0.01	0.02	-0.00
S. Bailey - Legal Assistant, ENA Legal	0.26	-0.00	-0.01	-0.00
S. Panus - Senior Legal Specialist, ENA Legal	0.26	-0.00	-0.00	-0.00
M. Heard - Senior Legal Specialist, ENA Legal	0.23	-0.00	0.00	-0.00
J. Hodge - Asst General Counsel, ENA Legal	0.13	0.03	0.01	-0.00
L. Kitchen - President, Enron Online	0.11	-0.09	0.53	0.00
S. Dickson - Employee, ENA Legal	0.09	-0.00	0.00	-0.00
E. Sager - VP and Asst Legal Counsel, ENA Legal	0.08	0.02	0.07	-0.00
J. Dasovich - Employee, Government Relationship Executive	-0.01	0.58	0.06	0.01
	0.00	0.53	-0.06	-0.01
	-0.00	0.40	0.10	-0.00
	-0.00	0.37	-0.04	-0.00
	0.03	0.16	-0.01	-0.00
	0.01	0.09	0.09	-0.00
	-0.00	0.08	-0.00	0.20
	-0.00	0.08	-0.02	-0.00
	-0.00	0.08	-0.00	0.04
	-0.00	0.08	-0.00	0.04
J. Lavorato - CEO, Enron America	0.02	-0.04	0.49	0.00
	0.00	-0.03	0.20	-0.00
	0.01	-0.01	0.19	0.00
	0.00	-0.02	0.18	0.00
	0.01	-0.05	0.18	0.00
	0.01	-0.03	0.17	0.00
	0.01	-0.02	0.16	0.00
	0.03	-0.04	0.16	-0.00
	0.00	-0.02	0.14	0.01
	-0.00	-0.00	0.01	0.59
K. Watson - Employee, Transwestern Pipeline Company (ETS)	-0.00	0.01	0.01	0.42
	-0.00	0.01	0.01	0.35
	0.00	-0.00	0.01	0.26
	-0.00	0.00	0.00	0.22
	-0.00	0.01	0.00	0.20
	-0.00	0.00	0.00	0.18
	0.00	-0.00	0.01	0.17
	0.00	-0.00	0.02	0.16
	-0.00	-0.00	0.01	0.16
	-0.00	-0.00	0.01	0.16

Communication Patterns

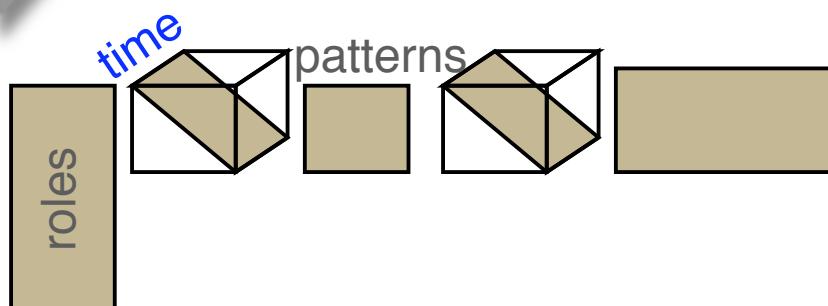


	Legal	Gov't affairs	Trade execs	Pipeline
Legal	440.2	1.6	-15.0	0.4
Government & regulatory affairs	1.6	278.3	135.4	1.6
Trade executives	-29.3	70.7	201.6	-6.2
Pipeline employees	1.4	-4.6	-7.5	172.3

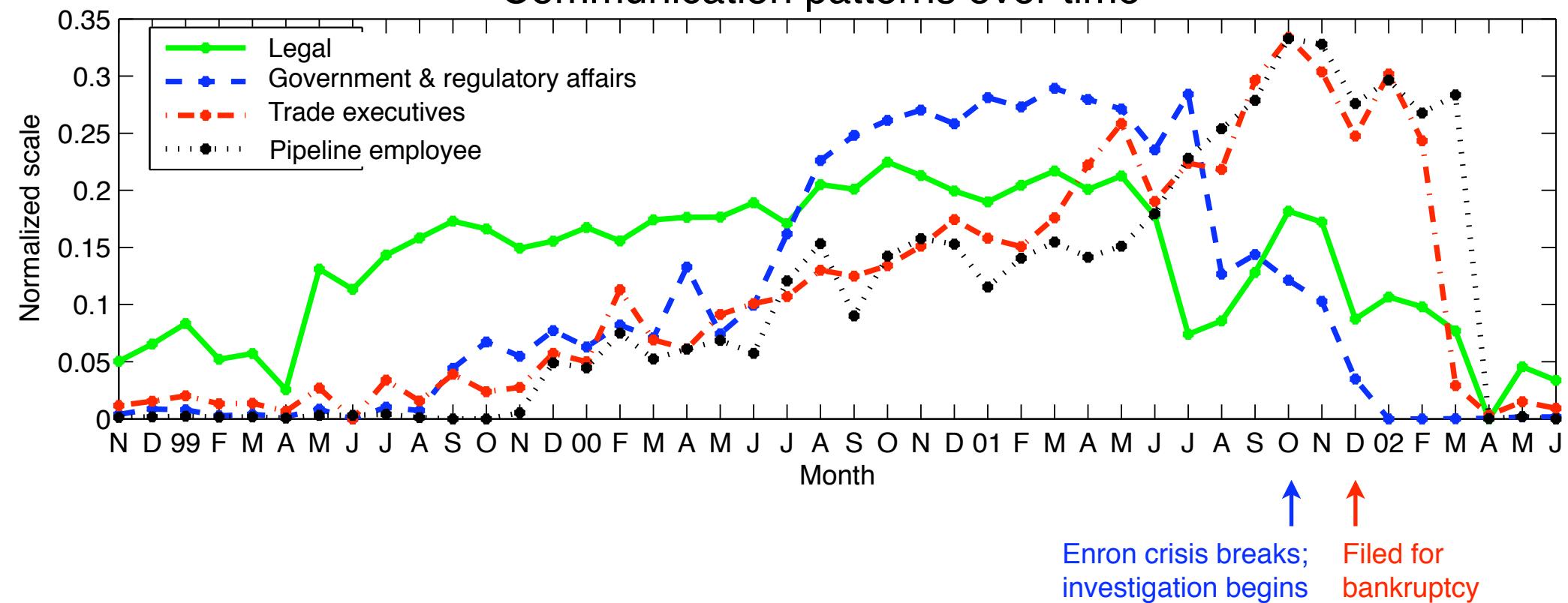


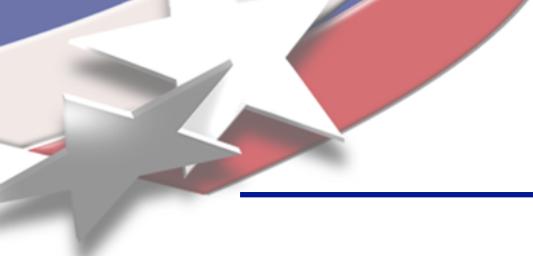
- Mostly communication within roles
- Some large exchanges
- Negative values complicates interpretation
 - Non-negative factorization being investigated

Temporal Patterns



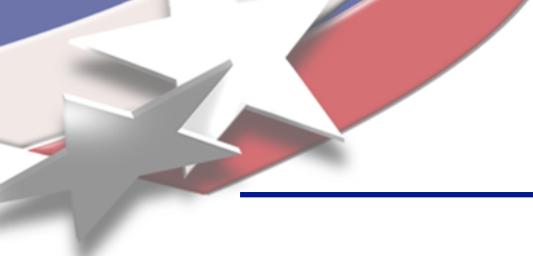
Communication patterns over time





Summary

- Improvements to DEDICOM
 - New procedure for finding **A**
 - Newton step for finding **D**
- Modifications to handle large data arrays
 - Compression
- Novel approach to social network analysis using DEDICOM
 - Roles of employees
 - Communication patterns among roles and over time
- Future research
 - Nonnegative DEDICOM
 - Constrained DEDICOM
 - PARAFAC



More Information

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- DEDICOM paper on Social Network Analysis:
 - Tech report SAND2006-2161 available
- MATLAB Tensor Toolbox:
 - <http://csmr.ca.sandia.gov/~tgkolda/TensorToolbox>
 - Tech report SAND2004-5189 available on website
 - Paper to appear in ACM Trans. Math. Softw.
 - sparse_tensor class to be released soon